2013 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The 2013 Specialist Mathematics examination 2 comprised 22 multiple-choice questions (worth 22 marks) and five extended questions (worth 58 marks). The paper seemed accessible to students, with most making substantial attempts at all questions in Section 2.

Question 3a. was the only 'show that' question on the 2013 exam, and students were required to verify by substitution the solution to a given differential equation. Many students attempted to employ methods other than substitution, and a lack of convincing algebraic detail characterised responses.

A small number of students did not keep in mind the instruction for Section 2 that stated, 'In questions where more than one mark is available, appropriate working **must** be shown'. A number of students simply wrote answers for Questions 1a., 2b. and 3c., without giving any indication of how their answers were obtained.

A number of students did not keep in mind the other important instruction for Section 2 that stated, 'Unless otherwise specified, an **exact** answer is required to a question'. These students obtained correct exact answers, and then replaced them with decimal approximations. This happened mainly in Questions 1dii., 2f. and 3c.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included

- the use of CAS technology to perform differentiation and to evaluate definite integrals – Questions 1dii. and 3a., although the complicated form of the resultant expression for $\frac{dN}{dt}$ seemed to lead to some students not being sure of how to proceed
- the use of CAS technology to solve equations – Questions 4c., 5c. and 5e.
- an improved facility with complex number questions – Question 2
- the ability to identify and sketch relationships, and, to a lesser extent, regions in the complex plane – Question 2c.
- the ability to set up a definite integral to represent a volume of revolution – Question 1di.
- the use of pencil to sketch graphs – fewer graphs were completed in pen compared to previous years.

Areas of weakness included

- untidy working, and lack of clarity about what the student intended to be their final answer
- work being done lightly in pencil, making it difficult to read
- some poor presentation of sketch graphs: ellipses drawn with points at the x-axis intercepts (Question 1c.), circles and straight lines drawn roughly (Question 2a.) and graphs drawn inaccurately at the end points of the domain (Question 3e.)
- lack of proper vector notation, in particular the confusion of scalar 0 with null vector $\underset{\sim}{0}$ – Question 4eiii.
- uncertainty of the signs of quantities when dealing with constant acceleration formulas – Questions 5c. and 5e.

SPECIFIC INFORMATION

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	3	6	1	88	2	0	$-1 \le 3x \le 1 \Rightarrow -\frac{1}{3} \le x \le \frac{1}{3}$, so option D was correct.
2	83	9	6	1	1	0	
3	24	13	11	5	47	1	The denominator has the form $a(x+5)(x-3)$, the vertex is $(-1,-8)$, $\Rightarrow a = \frac{1}{2}$, which gave option E.
4	12	25	9	43	9	1	At the origin, the gradients of the graphs are a and b respectively, so option D was correct.
5	5	69	21	2	3	0	
6	10	23	15	36	14	1	All students were awarded mark for this question.
7	7	4	4	5	80	0	
8	16	8	61	5	10	1	$z^2 = \sqrt{2}\text{cis}\left(\frac{\pi}{4} + 2k\pi\right) \Rightarrow$ $z = 2^{1/4}\text{cis}\left(\frac{\pi}{8} + k\pi\right)$, Args are $-\frac{7\pi}{8}, \frac{\pi}{8}$, so option C was correct.
9	4	85	7	3	2	0	
10	8	9	7	72	3	0	$x = y^{3/4}$, $V = \pi\int_0^3 y^{3/2}dy$ which gives option D.
11	7	12	70	6	4	0	
12	8	6	12	6	67	1	$x = 0 \Rightarrow$ zero gradients $y = 0 \Rightarrow$ infinite gradients, and positive gradients in first quadrant, so option E was correct.
13	57	10	21	7	5	1	
14	3	89	3	2	3	0	
15	5	4	7	22	62	0	Option E as $(\underset{\sim}{u} + \underset{\sim}{w}).\underset{\sim}{v} = 6$
16	63	4	24	4	5	1	
17	5	57	20	11	5	1	$\underset{\sim}{b}$ was the only vector with an $\underset{\sim}{i}$ component, so option B was correct.
18	4	5	10	65	15	1	
19	15	16	57	7	4	1	$100 = -2t + 4.9t^2$, which solves to give option C.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
20	66	17	6	7	3	1	
21	6	6	77	7	2	1	
22	11	18	19	13	38	1	$m\ddot{x} = mg - kv^2 \Rightarrow mv\frac{dv}{dx} = mg - kv^2$, which leads to option E.

Section 2

Question 1a.

Marks	0	1	2	Average
%	7	4	89	**1.8**

$\frac{x-1}{3} = \cos(t)$, $\frac{y+2}{2} = \sin(t)$, gives $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$

This question was well done, with nearly all students realising that t had to be eliminated. Of the few students who gave a result with y as the subject, most omitted the $\pm$ from their answer.

Question 1b.

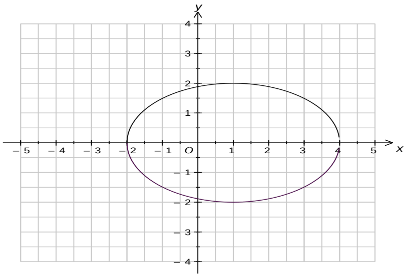
Marks	0	1	2	3	Average
%	25	15	19	41	**1.8**

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2\cos(t) \times \frac{1}{-3\sin(t)}$, $-\frac{2\sqrt{3}}{3} = -\frac{2}{3}\cot(t)$, $t = \frac{\pi}{6}, \frac{7\pi}{6}$

This question was fairly well done; however, a number of students gave only one answer for t, and others gave extra solutions outside the specified domain. Some students used implicit differentiation and then attempted to solve one equation for both x and y. The general solution for t appeared occasionally.

Question 1c.

Marks	0	1	2	3	Average
%	2	3	12	82	**2.8**



The ellipse is shown above. The y-axis intercepts are $\pm\frac{4\sqrt{2}}{3}$.

This question was well done. Many ellipses were drawn roughly, with 'points' at the x-intercepts and imprecise semi-axis lengths. Some students gave decimal approximations for the y-intercepts instead of exact values.

Question 1di.

Marks	0	1	2	Average
%	7	14	79	**1.7**

$$\int_1^3 \pi\left(4-\frac{4}{9}(x-1)^2\right)dx$$

This question was well done. The main errors were the omission of π, incorrect rearrangement of the equation of the ellipse for the integrand and some integrals missing the dx.

Question 1dii.

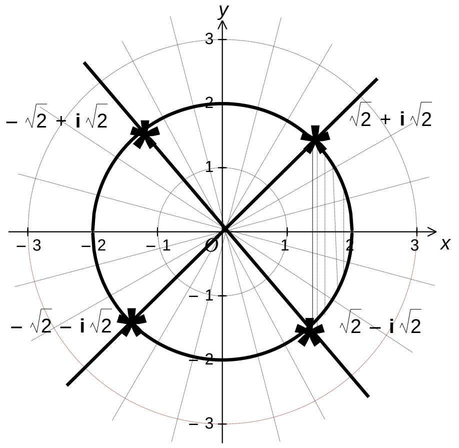
Marks	0	1	Average
%	29	71	0.7

$$\frac{184\pi}{27}$$

This question was fairly well done, with the most common error being the decimal approximation instead of the exact value.

Question 2a.

Marks	0	1	2	3	Average
%	17	18	46	20	1.7



The majority of students did not draw the line $y=-x$. Most got the circle and the line $y=x$. Some students drew more than one circle, and straight lines were sometimes drawn roughly.

Question 2b.

Marks	0	1	2	3	Average
%	44	6	35	15	1.2

Solving $y=\pm x$ and $x^2+y^2=4$ gives $x=\pm\sqrt{2}$, and so $z=\pm\sqrt{2}\pm i\sqrt{2}$.

This question was only moderately well done. Most students found only two solutions for z, $z=\sqrt{2}+i\sqrt{2}$ and $z=-\sqrt{2}-i\sqrt{2}$.

Question 2c.

Marks	0	1	2	Average
%	32	17	50	1.2

Other roots are $\sqrt{2}-i\sqrt{2}$, $-\sqrt{2}+i\sqrt{2}$, $-\sqrt{2}-i\sqrt{2}$.
All four roots are plotted and labelled on the diagram in Question 2a.

Major errors involved not plotting the roots, not labelling the plotted roots, and plotting the roots on the incorrect circle and in other locations.

Question 2d.

Marks	0	1	Average
%	40	60	0.6

$\left(z-\sqrt{2}-i\sqrt{2}\right)\left(z+\sqrt{2}-i\sqrt{2}\right)\left(z+\sqrt{2}+i\sqrt{2}\right)\left(z-\sqrt{2}+i\sqrt{2}\right)$

A number of students confused factors and roots. Some wrote down the correct factors but not as a product as required.

Question 2e.

Marks	0	1	Average
%	42	58	0.6

Segment as shaded on the diagram in Question 2a.

Frequent errors included segments shaded in other quadrants, the major segment shaded, shading of an annulus, and inaccurate borders and shading of the defined region.

Question 2f.

Marks	0	1	2	Average
%	44	15	41	1

$A=\frac{1}{4}\times \pi\times 2^2-\frac{1}{2}\times 2\times 2,\ A=\pi-2$

This question was done reasonably well by students who shaded the correct region in Question 2e. Some students gave a decimal approximation. A frequent error was $A=\frac{1}{2}(\pi-2)$, where only half the segment was sketched, or where integration was used without doubling the integral.

Question 3a.

Marks	0	1	2	Average
%	46	24	30	0.9

Differentiation of the solution with respect to t gives $\frac{1}{N}\frac{dN}{dt}=1.2e^{-0.4t}$, and substituting into the left side of the differential equation gives

left side $=1.2e^{-0.4t}+0.4\times\left(6-3e^{-0.4t}\right)-2.4\Rightarrow$ left side = $1.2e^{-0.4t}+2.4-1.2e^{-0.4t}-2.4$, $\Rightarrow$ left side = 0 = right side.

The question was not done well. Many students employed a variety of approaches different to the required method of substitution.

Question 3b.

Marks	0	1	Average
%	18	82	0.8

$t=0\Rightarrow\log_e(N)=3,\ N=e^3,\ N\simeq 20$

This question was quite well done; however, a number of students left their answer as e^3, instead of giving an answer to the nearest integer.

Question 3c.

Marks	0	1	2	Average
%	33	12	55	1.2

As $t\to\infty$, $\log_e(N)\to 6$, limiting number is 403

This question was fairly well done, with still some students giving the answer as e^6**,** rather than the nearest integer, 403. A number of students did not show their working.

Question 3di.

Marks	0	1	2	Average
%	82	9	9	**0.3**

$$\frac{d^2N}{dt^2} = \frac{d}{dN}\left(\frac{dN}{dt}\right)\times\frac{dN}{dt} = \left(0.4\times\left(6-\log_e(N)\right)+0.4N\times-\frac{1}{N}\right)\times\frac{dN}{dt},\ \frac{d^2N}{dt^2} = \left(2-0.4\log_e(N)\right)0.4N\left(6-\log_e(N)\right)$$

This question required the use of both the chain and product rules. Very few students answered this question correctly. A number of students found $\frac{d^2N}{dt^2}$ in terms of t, while others found $\frac{d^2t}{dN^2}$ and attempted to invert the result, which showed little understanding of the properties of second derivatives. A common error was to find $\frac{d}{dN}\left(\frac{dN}{dt}\right)$.

Question 3dii.

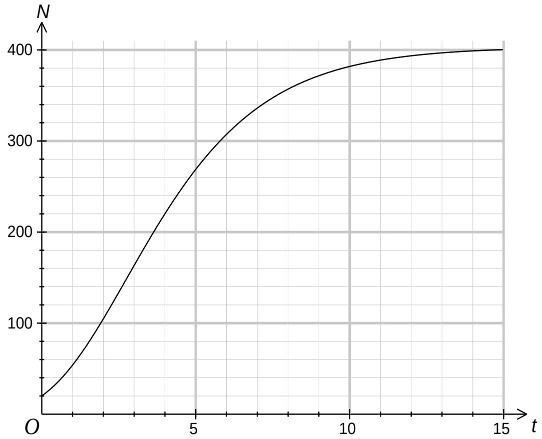
Marks	0	1	2	Average
%	43	19	38	**1**

$\frac{d^2N}{dt^2} = 0 \Rightarrow 2-0.4\log_e(N) = 0$, solving gives $N = 148$, so the point of inflection is at $t = 2.7,\ N = 148$

The majority of students realised that the second derivative needed to be equated to zero. However, there was a small group who equated the first derivative to zero. Some students obtained the correct result independently of their efforts in part i.

Question 3e.

Marks	0	1	2	Average
%	37	25	39	**1**



This question was moderately well done. Major errors included inaccurate position of endpoints, inaccurate placement of the point of inflection and lack of change in concavity.

Question 4a.

Marks	0	1	Average
%	20	80	0.8

$\hat{\underset{\sim}{b}} = \frac{1}{4}\left(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j} + 2\sqrt{3}\underset{\sim}{k}\right)$

This question was well done, with most students knowing how to find the unit vector. Most errors related to finding the magnitude of $\underset{\sim}{b}$.

Question 4b.

Marks	0	1	2	3	Average
%	25	22	13	40	1.7

The parallel result $\underset{\sim}{a}.\hat{\underset{\sim}{b}}\,\hat{\underset{\sim}{b}} = \frac{1}{4}\left(\frac{-7\sqrt{3}}{3} + \sqrt{3} - 4\sqrt{3}\right)\frac{1}{4}\left(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j} + 2\sqrt{3}\underset{\sim}{k}\right) = -\frac{\sqrt{3}}{3}\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$

The perpendicular result $\underset{\sim}{a} - \underset{\sim}{a}.\hat{\underset{\sim}{b}}\,\hat{\underset{\sim}{b}} = -2\sqrt{3}\underset{\sim}{i} + 2\underset{\sim}{j}$

Most students knew the formulas to apply; however, the surd calculations proved to be a problem for many. A small number attempted to find the resolutes from first principles. A few students had the resolutes the wrong way around, finding the resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$, etc.

Question 4c.

Marks	0	1	2	Average
%	17	19	64	1.5

$\left(m\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right).\left(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j} + 2\sqrt{3}\underset{\sim}{k}\right) = \sqrt{m^2+5} \times 4 \times \cos\left(\frac{2\pi}{3}\right)$ solves to give $m = \frac{\sqrt{3}}{3}$

Most students were able to set up the formula, and then solve for *m* using their calculator. A small number of students did not eliminate the solution which gave vector $\underset{\sim}{a}$.

Question 4d.

Marks	0	1	2	Average
%	29	27	44	1.2

$\left(\frac{\sqrt{3}}{3}\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right).\left(-\frac{7\sqrt{3}}{3}\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right) = \sqrt{\frac{64}{3}} \times \sqrt{\frac{16}{3}} \times \cos(\theta)$, $\theta = 75.5^\circ$

Most students applied the correct method, but a large number made errors in computation along the way. Some students gave the answer in radians, and others did not round correctly to one decimal place.

Question 4ei.

Marks	0	1	Average
%	8	92	0.9

$\overrightarrow{AN} = \underset{\sim}{u} + \frac{1}{2}\underset{\sim}{v}$

This question was very well done. The main error was to give the negative of the correct result.

Question 4eii.

Marks	0	1	2	Average
%	15	23	63	**1.5**

$\overrightarrow{CM} = -\underset{\sim}{u} + \frac{1}{2}\left(\underset{\sim}{u} + \underset{\sim}{v}\right) = \frac{1}{2}\underset{\sim}{v} - \frac{1}{2}\underset{\sim}{u}$, $\overrightarrow{BP} = -\frac{1}{2}\underset{\sim}{u} - \underset{\sim}{v}$

This question was reasonably well done, with the main errors relating to signs.

Question 4eiii.

Marks	0	1	Average
%	90	10	**0.1**

$\underset{\sim}{0}$

This question was not well done. The most common error was the omission of the tilde for this null vector. The majority of students had scalar 0 as the result of the vector calculation, rather than the vector $\underset{\sim}{0}$. This is a conceptual error – a sum of vectors is a vector.

Question 5a.

Marks	0	1	2	3	Average
%	14	33	15	38	**1.8**

$\dot{\underset{\sim}{r}}(t) = 7.5\underset{\sim}{i} - \frac{5\pi}{3}\cos\left(\frac{\pi t}{6}\right)\underset{\sim}{j}$, Min speed = 7.5 m/s, Max speed = 9.1 m/s

Common errors included the omission of $\underset{\sim}{j}$, and not identifying which were the minimum and maximum speeds. Some students gave only one speed and neglected to say whether it was the minimum or the maximum.

Question 5b.

Marks	0	1	2	Average
%	26	59	15	**0.9**

$\ddot{\underset{\sim}{r}}(t) = \frac{5\pi^2}{18}\sin\left(\frac{\pi t}{6}\right)\underset{\sim}{j}$, acceleration is zero for $t = 6n$, where $n \in Z^+$ ($n \in Z^+ \cup \{0\}$ was also accepted)

Many students had $n \in Z$. Correct chain rule differentiation was also a problem for some students.

Question 5c.

Marks	0	1	2	Average
%	59	12	29	**0.7**

$-5 = 15\sin(30^\circ)t - 4.9t^2$, $t = 2.03$ (s)

The major error with this question related to consistency of signs. A number of students broke the motion into two stages, causing more work for themselves and making the question more complex. Some students confused the 10 m length of the ramp with the speed of 15 m/s in their calculations.

Question 5d.

Marks	0	1	Average
%	61	39	**0.4**

$15\cos(30^\circ) \times 2.03 = 26$ m

The main error with this question was the incorrect horizontal component of velocity.

Question 5e.

Marks	0	1	2	3	Average
%	59	11	7	23	**1**

$$-\frac{1}{8\sqrt{3}} \times mg\cos(30^{\circ}) - mg\sin(30^{\circ}) = ma\,,\ \ a = -\frac{9g}{16}\,,\ \ -10^{2} = 2\times -\frac{9g}{16}\times s\,,\ \ s = 9.1\,\text{m}$$

This question was reasonably well done by students who attempted it. A number had the weight force down the plane and friction acting in opposite directions. Others took the direction down the plane as positive, and had difficulty dealing with the negative value of *s* obtained.

Question 5f.

Marks	0	1	Average
%	66	34	**0.4**

$$mg\sin(30^{\circ}) - \mu mg\cos(30^{\circ}) = 0\,,\ \ \mu = \frac{1}{\sqrt{3}}\ \text{ or }\ \mu = \frac{\sqrt{3}}{3}$$

Negative values for μ were sometimes seen, and occasionally unsimplified expressions for μ were given.