GENERAL COMMENTS

In the 2014 Specialist Mathematics examination 1, students were required to respond to eight short-answer questions worth a total of 40 marks.

In the comments on specific questions in the next section, common errors are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. A particular concern is the need for students to read the questions carefully as responses to several questions indicated that students had not done this.

Another concern is the clarity of answers and the manner in which students set out their mathematics. Students should be reminded that if an assessor is not certain as to what a student's answer is conveying, that assessor cannot award marks. Furthermore, if an assessor is unable to follow a student's working (or reasoning), full marks will not be awarded. Equals signs should be placed between quantities that are equal – the working should not appear to be a collection of disjointed statements. If there are inconsistencies in the student's working, full marks will not be awarded. For example, if an equals sign is placed between quantities that are not equal, full marks will not be awarded.

Students are reminded that they should sketch graphs with care and include details such as a reasonable scale, correct domain, asymptotes and asymptotic behaviour. Smoothly drawn curves are expected.

Areas of strength included:

- finding the magnitude of a vector
- use of the dot product
- conversion from parametric equations to a Cartesian equation
- finding the speed, given the displacement vector
- use of the conjugate root theorem
- use of a double-angle formula
- use of the product rule.

Areas of weakness included:

- failing to read the question carefully enough – this included not answering the question, proceeding further than required or not giving the answer in the specified form. The latter was common and particularly evident in Questions 3a., 3b., 4, 8ai. and 8c. Students should be reminded that good examination technique includes rereading the question after it has been answered to ensure that they have answered what was required and that they have given their answer in the correct form
- algebraic skills. Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common. Poor use of brackets was also common
- arithmetic skills. Difficulty with arithmetic was evident in several questions. The inability to evaluate expressions, especially those involving fractions or surds, was common
- notation, especially the omission of the *dx* or equivalent in integration
- showing a given result. This was required in Questions 2a., 6a., 7b. and 8b. In such questions, the onus is on students to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether or not they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 4)
- recognising the need to use the product rule when differentiating implicitly (Question 4)
- recognising the need to use the product rule when differentiating (Question 7b.)
- recognising the method of integration required (Questions 5b., 6 and 7c.)
- knowing the exact values for circular functions (Questions 1b., 5c., 7c., and 8)
- giving answers in the required form (Question 5c.).

In this examination, students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology. Students are expected to be able to simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence. Many students made algebraic or numerical slips at the end of an answer, which meant the final mark could not be awarded. This was especially unfortunate when they had a correct answer and there was no need for further simplification.

There were several cases where incorrect working fortuitously led to a correct answer. Students should be reminded that in questions where working is required, the final answer mark may not be awarded if it is not supported by relevant and correct working.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	21	79	**0.8**

$\hat{\underset{\sim}{a}} = \frac{1}{\sqrt{6}}\left(\sqrt{3}\,\underset{\sim}{i} - \underset{\sim}{j} - \sqrt{2}\,\underset{\sim}{k}\right)$

Students answered this question well. The most common errors involved finding the magnitude to be 6 or $\sqrt{6}$ but not finding the unit vector. Some students made arithmetic errors and, finding the magnitude to be $\sqrt{5}$ or $\sqrt{7}$, obtained the correct unit vector but then incorrectly rationalised the denominators.

Question 1b.

Marks	0	1	2	Average
%	49	9	42	**1**

$\theta = \frac{\pi}{4} = 45°$

Common errors included not using the dot product or direction cosines, but instead using a 'tan' argument from a right-angled triangle to get $\pm\frac{\pi}{6}$ or $\pm\frac{\pi}{3}$. Many students simplified surds incorrectly, with the most common error being $\frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{2}$. Several errors were made with exact trigonometric values.

Question 1c.

Marks	0	1	2	Average
%	10	12	78	**1.7**

$m = 6 + 5\sqrt{2}$

Most students answered this question well, although many equated the dot product to 1 or -1. Sign errors were quite common when rearranging the equation to find *m*.

Question 2a.

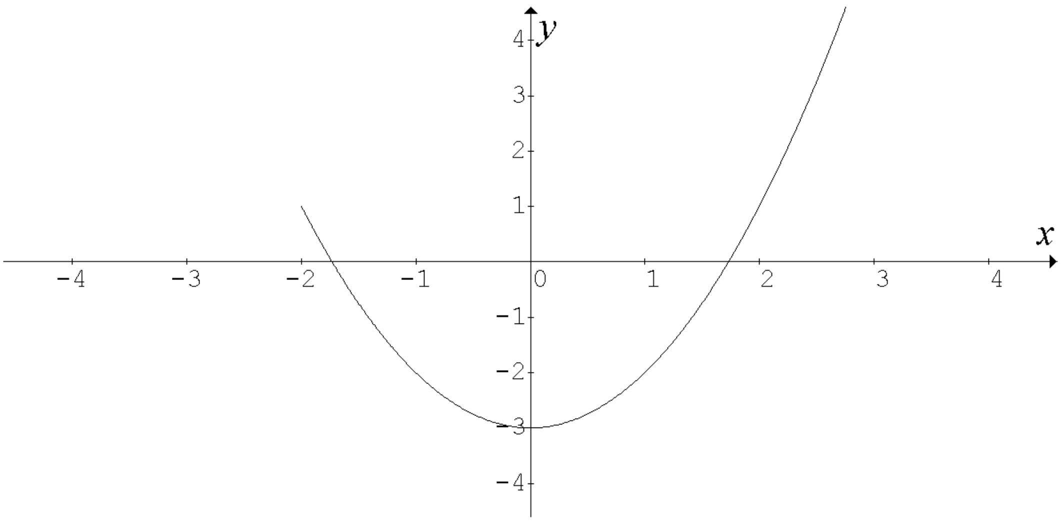
Marks	0	1	Average
%	11	89	**0.9**

Students had to show the given result $y = x^2 - 3$.

This question was well answered. Most students were able to obtain the given result. There were, however, some unconvincing arguments, often due to insufficient steps being shown. Some students made algebraic errors.

Question 2b.

Marks	0	1	2	Average
%	22	36	42	1.2



with $(-2,\ 1)$, $(-\sqrt{3},\ 0)$, $(\sqrt{3},\ 0)$, $(0,\ -3)$ labelled.

Most students knew that a parabola was required. Some graphs were poorly drawn and did not give a smoothly drawn curve that had symmetry properties. Many graphs were drawn with a wrong domain, the most common being the whole parabola or $x \geq 0$, which may have been due to a confusion with $t \geq 0$. Many students did not choose a number scale, primarily on the vertical axis, which caused the graph to be distorted and therefore led to an incorrect answer.

Question 2c.

Marks	0	1	2	Average
%	20	7	73	1.5

$\left|\underset{\sim}{\dot{r}}(1)\right| = \sqrt{5}$

This question was generally answered well but many students substituted $t = 1$ into the displacement vector or differentiated incorrectly. Common errors for the velocity were $t\,\underset{\sim}{i} + (2t-4)\,\underset{\sim}{j}$ and $-2\,\underset{\sim}{i} + (2t-4)\,\underset{\sim}{j}$. Many students gave the term $2t - 4\,\underset{\sim}{j}$ without any brackets. Some derivatives did not include $\underset{\sim}{i}$ or $\underset{\sim}{j}$. There was also an arithmetic issue involving the square of a negative, so the answer $\sqrt{3}$ was common. Some students gave the answer as the velocity vector rather than finding its magnitude to give the speed. Others made the question more difficult by finding the magnitude of the velocity vector prior to substituting in $t = 1$ rather than the other way around. Several made errors when expanding inside the square root.

Question 3a.

Marks	0	1	2	Average
%	24	9	67	1.5

$z^2 + 1$

Most students identified the need to use the conjugate root theorem but some then gave $z^2 - 1$ as their answer. Confusion between solutions and factors was often evident. Several students gave the conjugate root or factor and then did no further work. Some students seemed not to realise that they had completed what was required and found the second quadratic factor. Some quoted $z = \pm 1$ as solutions rather than $z = \pm i$.

Question 3b.

Marks	0	1	2	3	Average
%	30	7	9	54	**1.9**

$z = \pm i,\ 2 \pm \sqrt{2}\, i$

Students made many sign errors and other algebraic errors in finding the second quadratic factor. Some students, having found this factor, gave $-2 \pm \sqrt{2}\, i$, $2 \pm 2i$ or $2 \pm \sqrt{10}\, i$ as the solution. Some confusion between solutions and factors was evident.

Question 4

Marks	0	1	2	3	Average
%	14	9	29	48	**2.1**

$\frac{4}{9}$

Many students answered this question well. Most recognised the need for implicit differentiation and attempted to use the product rule. The most common differentiation errors were $\frac{d}{dy}(e^y) = ye^y$ and $\frac{d}{dy}(y) = 0$, or occasionally $= 1$. Some students rearranged the equation prior to attempting to find the derivative. On most occasions this either led to complications or was an incomplete attempt. A number of students were unable to take the $\frac{dy}{dx}$ terms to one side of the equation or made algebraic errors in doing so. Many students did not substitute in the given values. Some who did substitute in the given values made numerical errors or were unable to simplify $e^{-3}e^3$. Several students correctly found the gradient of the tangent and then did no further work. Some students found the equation of the normal, which was not required.

Question 5a.

Marks	0	1	Average
%	19	81	**0.8**

$a = 48$

This question was answered well, with most students understanding that a double-angle formula was required. Some applied the double-angle formula incorrectly. Common incorrect answers were 16, 96 and 192.

Question 5b.

Marks	0	1	2	3	Average
%	20	11	22	48	**2.0**

$I = -8\int_{\frac{\sqrt{3}}{2}}^{0} u^2\ du = 8\int_{0}^{\frac{\sqrt{3}}{2}} u^2\ du$

This question was answered reasonably well, but the result from Question 5a. was often not used. There were many errors in notation, with dx and du often missing. When performing the required substitution, several students used $u = \sin(6x)$, $u = \sin(3x)$, $u = \cos(3x)$ or $u = \cos^2(6x)$ rather than $u = \cos(6x)$. These attempts led to a more complicated solution and were rarely successful. Many who used $u = \cos(6x)$ then stated $\frac{du}{dx} = -\sin(6x)$, $6\sin(6x)$ and sometimes $-6\sin(x)$ or hybrids of these. Others failed to change the terminals. Many students made simplification errors and errors in exact values for circular functions.

Question 5c.

Marks	0	1	Average
%	55	45	**0.5**

$\sqrt{3}$

This question was answered well by students who completed Question 5b. successfully. There were some instances of poor arithmetic. Some students correctly obtained $\sqrt{3}$ but then stated $k = \sqrt{3}$. Typical errors included substituting the terminals in the incorrect order or being unable to correctly evaluate $\left(\frac{\sqrt{3}}{2}\right)^3$.

Question 6a.

Marks	0	1	Average
%	14	86	**0.9**

Students were required to show the given result $\frac{a}{a-4} = 1 + \frac{4}{a-4}$.

This question was answered well, but many students did not know how a verification or proof should be set out. Some arguments were not convincing, and some eventually showed that $a = a$ or similar.

Question 6b.

Marks	0	1	2	3	4	Average
%	15	35	2	11	37	**2.2**

$V = \pi\left(1 + \log_e\left(\frac{5}{3}\right)\right)$

Many students did not use the result from Question 6a. Those who used the result from Question 6a. generally answered this question well. Those who did not answer this question well commonly used partial fractions, incorrectly attempting $\frac{x^2}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$. Students are reminded that it is often necessary or beneficial to use the results from earlier parts of a question in the latter parts. Many students performed the division in this part, missing the prompt given. Several did not use partial fractions at all, giving the log of the denominator as their answer. A number of arithmetic and simplification errors were seen. Most students remembered to include π in their integral, but some did not square the expression for *y*.

Question 7a.

Marks	0	1	Average
%	96	4	**0.1**

$[0, \infty)$

This question was answered poorly by most students. Few realised that *x* and the arctan function are both positive for the same values, negative for the same values and zero for the same values. Incorrect responses included $R, \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$ and $\left(-\frac{3\pi x}{2}, \frac{3\pi x}{2}\right)$. Many students seemed to use the product of the range of each of the 'parts', some ignored one part and others found the product of the range of one part and the variable *x*. Some ignored the presence of one of the two functions involved.

Question 7b.

Marks	0	1	Average
%	9	91	**0.9**

Students had to show the given result $f'(x) = 3\arctan(2x) + \frac{6x}{1+4x^2}$.

This question was well answered. Most students were able to obtain the given result. There were, however, some unconvincing arguments, often because insufficient steps were shown. A few students used $\tan^{-1}(2x)$ and then confused inverses with reciprocals.

Question 7c.

Marks	0	1	2	3	Average
%	27	19	28	26	**1.6**

$\frac{\sqrt{3}\pi}{6} - \frac{\pi}{8} - \frac{1}{4}\log_e(2)$

Most students who used the result from Question 7b. made good attempts at this question. Some ignored the word 'hence'. Most attempted to apply this method but many made algebraic errors. When attempting to integrate $\frac{6x}{1+4x^2}$, some gave $3\arctan(2x)$ or similar, others made the correct substitution but made errors in either changing the terminals or with the arithmetic.

Question 8ai.

Marks	0	1	2	Average
%	21	35	44	**1.3**

$T_1\cos(\theta) = 5g = 49$, $T_1\sin(\theta) = T_2$, $T_1 = 49\sec(\theta)$

This question was not answered well. Some students labelled their diagram incorrectly, swapping T_1 and T_2, while some included a normal force. There was inconsistency in the use of positive direction in resolving forces. Some students did not resolve forces as required but used another method. Several students used 5 for the weight force rather than $5g$ and it was common for sin and cos to be confused. It was also common to see only the horizontal resolution attempted. Most went on to write down T_1 and T_2 in terms of θ.

Question 8aii.

Marks	0	1	Average
%	32	68	**0.7**

$T_2 = 49\tan(\theta)$

This question was reasonably well answered by students who gave correct answers to Question 8ai. Most errors were due to previous mistakes and $5g = 48$ was a common error.

Question 8b.

Marks	0	1	Average
%	60	40	**0.4**

Students had to show the given result $\tan(\theta) < \sec(\theta)$.

Responses to this question were mixed, with many good solutions and a large number of unconvincing arguments often because insufficient steps were shown. Several students simply substituted a few values in for θ and then asserted that the result was therefore true for all values. Others attempted to demonstrate the result with a graph. Neither approach was sufficient. Several students argued from $\tan(\theta) < \sec(\theta)$ and eventually showed that $\sin(\theta) < 1$.

Question 8c.

Marks	0	1	2	3	Average
%	37	14	25	25	**1.4**

$T_2 < T_1$ since $\tan(\theta) < \sec(\theta)$ so $\max(\theta) = \frac{\pi}{3}$

Many students realised that if the string attached to the ceiling with tension T_1 broke, both strings would break. Many students correctly found that $\max(\theta) = \frac{\pi}{3}$ but failed to justify that this would cause both strings to break. A number of students gave the answer $\cos^{-1}\left(\frac{5g}{98}\right)$ or $\cos^{-1}\left(\frac{1}{2}\right)$ but were unable to simplify. Some only considered the horizontal string and gave $\tan^{-1}(2)$ as the answer. Others correctly asserted that $\tan^{-1}(2) > \frac{\pi}{3}$ and completed the question successfully. Some found that $\frac{\pi}{3} \le \theta \le \frac{\pi}{2}$ and concluded that the answer was $\frac{\pi}{2}$. A number of students assumed that $T_2 + T_1 = 192$. Some students made errors with exact trigonometric values, some students gave an inequality as the answer and some used $g = 10$.