VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2016**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

**STUDENT NUMBER** [ ][ ] [ ][ ][ ] [ ][ ][ ] [ ]

# SPECIALIST MATHEMATICS

## Written examination 2

**Monday 7 November 2016**

**Reading time: 11.45 am to 12.00 noon (15 minutes)**
**Writing time: 12.00 noon to 2.00 pm (2 hours)**

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| A | 20 | 20 | 20 |
| B | 6 | 6 | 60 |
| | | | Total 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

**Instructions**
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**At the end of the examination**
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## SECTION A – Multiple-choice questions

### Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1**

The cartesian equation of the relation given by $x = 3\text{cosec}^2(t)$ and $y = 4\cot(t) - 1$ is

**A.** $\dfrac{(y+1)^2}{16} - \dfrac{x^2}{9} = 1$

**B.** $(y+1)^2 = \dfrac{16(x+3)}{3}$

**C.** $\dfrac{x^2}{9} + \dfrac{(y+1)^2}{16} = 1$

**D.** $4x - 3y = 15$

**E.** $(y+1)^2 = \dfrac{16(x-3)}{3}$

**Question 2**

The implied domain of $y = \arccos\left(\dfrac{x-a}{b}\right)$, where $b > 0$ is

**A.** $[-1, 1]$

**B.** $[a - b, a + b]$

**C.** $[a - 1, a + 1]$

**D.** $[a, a + b\pi]$

**E.** $[-b, b]$

**Question 3**

The straight-line asymptote(s) of the graph of the function with rule $f(x) = \dfrac{x^3 - ax}{x^2}$, where $a$ is a non-zero real constant, is given by

**A.** $x = 0$ only.

**B.** $x = 0$ and $y = 0$ only.

**C.** $x = 0$ and $y = x$ only.

**D.** $x = 0$, $x = \sqrt{a}$ and $x = -\sqrt{a}$ only.

**E.** $x = 0$ and $y = a$ only.

**Question 4**

One of the roots of $z^3 + bz^2 + cz = 0$ is $3 - 2i$, where $b$ and $c$ are real numbers.

The values of $b$ and $c$ respectively are

**A.** 6, 13

**B.** 3, –2

**C.** –3, 2

**D.** 2, 3

**E.** –6, 13

**Question 5**

If $\text{Arg}(-1 + ai) = -\frac{2\pi}{3}$, then the real number $a$ is

**A.** $-\sqrt{3}$

**B.** $-\frac{\sqrt{3}}{2}$

**C.** $-\frac{1}{\sqrt{3}}$

**D.** $\frac{1}{\sqrt{3}}$

**E.** $\sqrt{3}$

**Question 6**

The points corresponding to the four complex numbers given by

$$z_1 = 2\text{cis}\left(\frac{\pi}{3}\right),\ z_2 = \text{cis}\left(\frac{3\pi}{4}\right),\ z_3 = 2\text{cis}\left(-\frac{2\pi}{3}\right),\ z_4 = \text{cis}\left(-\frac{\pi}{4}\right)$$

are the vertices of a parallelogram in the complex plane.

Which one of the following statements is **not** true?

**A.** The acute angle between the diagonals of the parallelogram is $\frac{5\pi}{12}$

**B.** The diagonals of the parallelogram have lengths 2 and 4

**C.** $z_1 z_2 z_3 z_4 = 0$

**D.** $z_1 + z_2 + z_3 + z_4 = 0$

**E.** $1 \le |z| \le 2$ for all four of $z_1, z_2, z_3, z_4$

**Question 7**

Given that $x = \sin(t) - \cos(t)$ and $y = \frac{1}{2}\sin(2t)$, then $\frac{dy}{dx}$ in terms of $t$ is

**A.** $\cos(t) - \sin(t)$

**B.** $\cos(t) + \sin(t)$

**C.** $\sec(t) + \operatorname{cosec}(t)$

**D.** $\sec(t) - \operatorname{cosec}(t)$

**E.** $\dfrac{\cos(2t)}{\cos(t) - \sin(t)}$

**Question 8**

Using a suitable substitution, $\int_a^b \left(x^3 e^{2x^4}\right) dx$, where $a$ and $b$ are real constants, can be written as

**A.** $\int_a^b \left(e^{2u}\right) du$

**B.** $\int_{a^4}^{b^4} \left(e^{2u}\right) du$

**C.** $\frac{1}{8}\int_a^b \left(e^{u}\right) du$

**D.** $\frac{1}{4}\int_{a^4}^{b^4} \left(e^{2u}\right) du$

**E.** $\frac{1}{8}\int_{8a^3}^{8b^3} \left(e^{u}\right) du$

**Question 9**

If $f(x) = \frac{dy}{dx} = 2x^2 - x$, where $y_0 = 0 = y(2)$, then $y_3$ using Euler's formula with step size 0.1 is

**A.** $0.1\,f(2)$

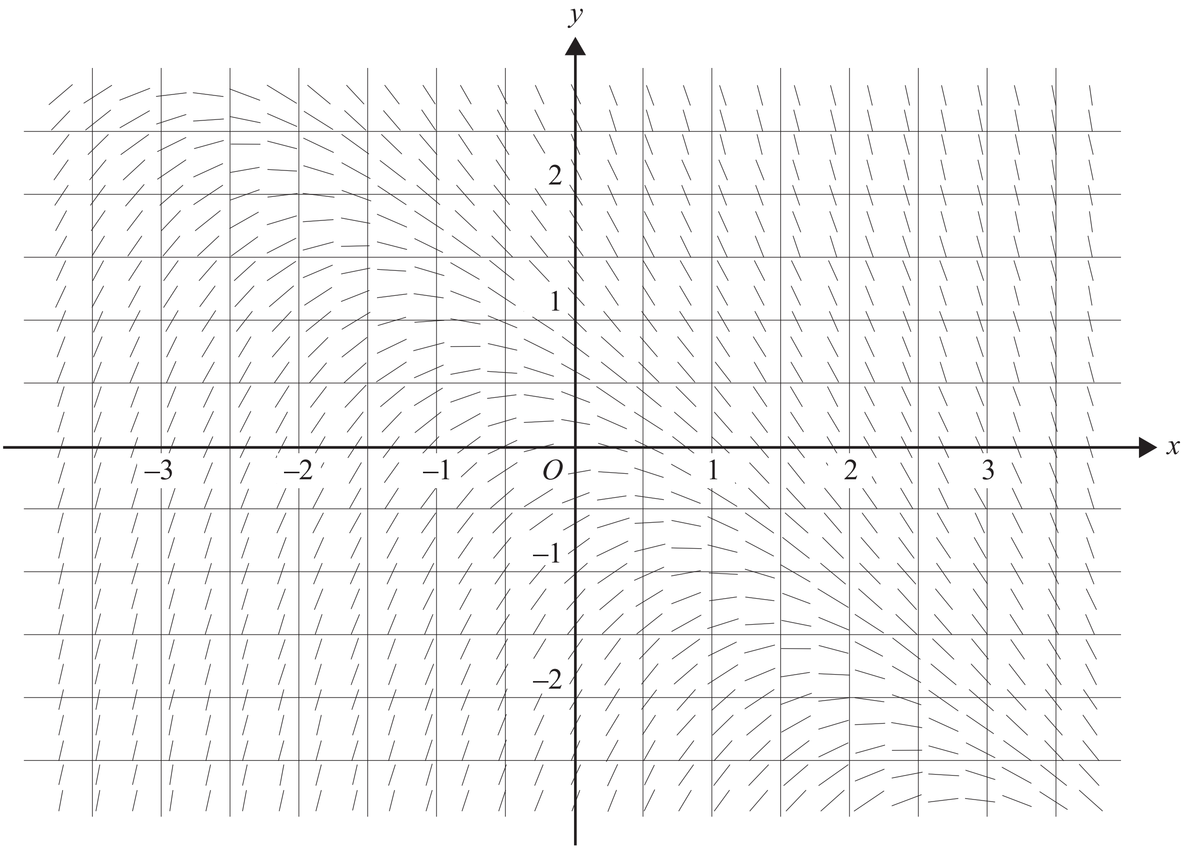
**B.** $0.6 + 0.1\,f(2.1)$

**C.** $1.272 + 0.1\,f(2.2)$

**D.** $2.02 + 0.1\,f(2.3)$

**E.** $2.02 + 0.1\,f(2.2)$

**Question 10**




The direction field for the differential equation $\frac{dy}{dx} + x + y = 0$ is shown above.

A solution to this differential equation that includes (0, –1) could also include

**A.** (3, –1)

**B.** (3.5, –2.5)

**C.** (–1.5, –2)

**D.** (2.5, –1)

**E.** (2.5, 1)

**Question 11**

Let $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \alpha\underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} - \underset{\sim}{j} + \alpha^2\underset{\sim}{k}$, where $\alpha$ is a real constant.

If the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $\frac{74}{\sqrt{273}}$, then $\alpha$ equals

**A.** 1

**B.** 2

**C.** 3

**D.** 4

**E.** 5

**Question 12**

If $\underset{\sim}{a} = -2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = -m\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$, where $m$ is a real constant, the vector $\underset{\sim}{a} - \underset{\sim}{b}$ will be perpendicular to vector $\underset{\sim}{b}$ where $m$ equals

**A.** 0 only

**B.** 2 only

**C.** 0 or 2

**D.** 4.5

**E.** 0 or –2

**Question 13**

A particle of mass 5 kg is subject to forces $12\underset{\sim}{i}$ newtons and $9\underset{\sim}{j}$ newtons.
If no other forces act on the particle, the magnitude of the particle's acceleration, in $ms^{-2}$, is

**A.** 3

**B.** $2.4\underset{\sim}{i} + 1.8\underset{\sim}{j}$

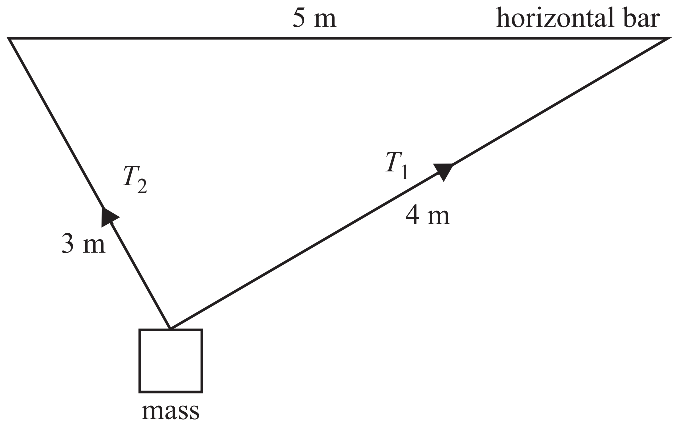
**C.** 4.2

**D.** 9

**E.** $60\underset{\sim}{i} + 45\underset{\sim}{j}$

**Question 14**

Two light strings of length 4 m and 3 m connect a mass to a horizontal bar, as shown below.
The strings are attached to the horizontal bar 5 m apart.




Given the tension in the longer string is $T_1$ and the tension in the shorter string is $T_2$, the ratio of the tensions $\frac{T_1}{T_2}$ is

**A.** $\frac{3}{5}$

**B.** $\frac{3}{4}$

**C.** $\frac{4}{5}$

**D.** $\frac{5}{4}$

**E.** $\frac{4}{3}$

**Question 15**

A variable force of $F$ newtons acts on a 3 kg mass so that it moves in a straight line. At time $t$ seconds, $t \geq 0$, its velocity $v$ metres per second and position $x$ metres from the origin are given by $v = 3 - x^2$.

It follows that

**A.** $F = -2x$

**B.** $F = -6x$

**C.** $F = 2x^3 - 6x$

**D.** $F = 6x^3 - 18x$

**E.** $F = 9x - 3x^3$

**Question 16**

A cricket ball is hit from the ground at an angle of 30° to the horizontal with a velocity of 20 ms$^{-1}$. The ball is subject only to gravity and air resistance is negligible.

Given that the field is level, the horizontal distance travelled by the ball, in metres, to the point of impact is

**A.** $\frac{10\sqrt{3}}{g}$

**B.** $\frac{20}{g}$

**C.** $\frac{100\sqrt{3}}{g}$

**D.** $\frac{200\sqrt{3}}{g}$

**E.** $\frac{400}{g}$

**Question 17**

A body of mass 3 kg is moving to the left in a straight line at 2 ms$^{-1}$. It experiences a force for a period of time, after which it is then moving to the right at 2 ms$^{-1}$.

The change in momentum of the particle, in kg ms$^{-1}$, in the direction of the final motion is

**A.** $-6$

**B.** 0

**C.** 4

**D.** 6

**E.** 12

**Question 18**

Oranges grown on a citrus farm have a mean mass of 204 grams with a standard deviation of 9 grams. Lemons grown on the same farm have a mean mass of 76 grams with a standard deviation of 3 grams.

The masses of the lemons are independent of the masses of the oranges.

The mean mass and standard deviation, in grams, respectively of a set of three of these oranges and two of these lemons are

**A.** 764, $3\sqrt{29}$

**B.** 636, 12

**C.** 764, $\sqrt{33}$

**D.** 636, $3\sqrt{10}$

**E.** 636, 33

**Question 19**

A random sample of 100 bananas from a given area has a mean mass of 210 grams and a standard deviation of 16 grams.

Assuming the standard deviation obtained from the sample is a sufficiently accurate estimate of the population standard deviation, an approximate 95% confidence interval for the mean mass of bananas produced in this locality is given by

**A.** (178.7, 241.3)

**B.** (206.9, 213.1)

**C.** (209.2, 210.8)

**D.** (205.2, 214.8)

**E.** (194, 226)

**Question 20**

The lifetime of a certain brand of batteries is normally distributed with a mean lifetime of 20 hours and a standard deviation of two hours. A random sample of 25 batteries is selected.

The probability that the mean lifetime of this sample of 25 batteries exceeds 19.3 hours is

**A.** 0.0401

**B.** 0.1368

**C.** 0.6103

**D.** 0.8632

**E.** 0.9599

**END OF SECTION A**

**TURN OVER**

## SECTION B

### Instructions for Section B

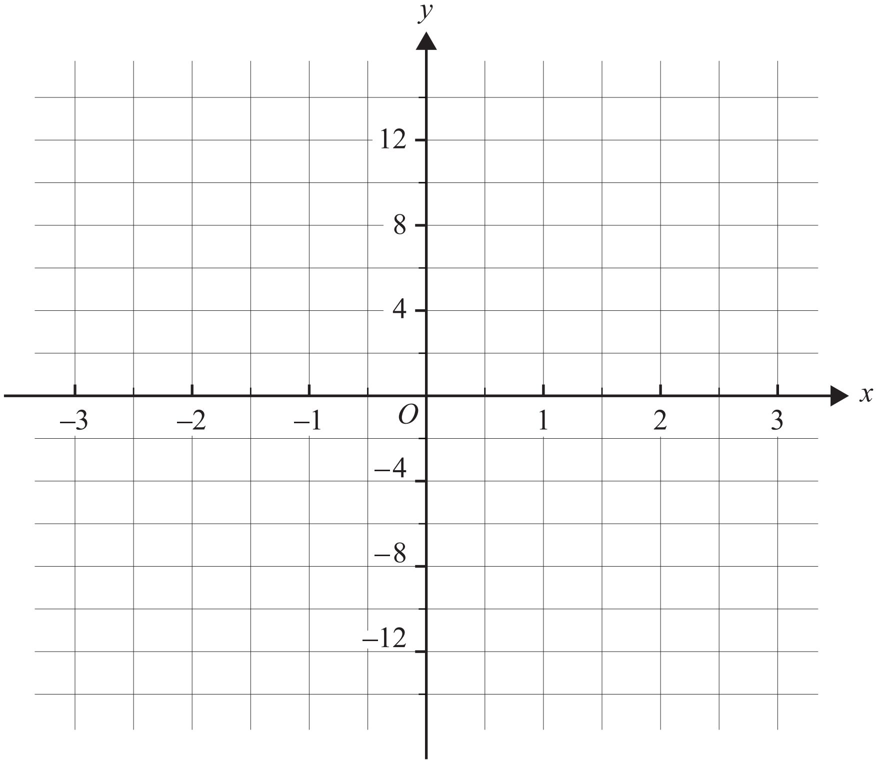
Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1** (9 marks)

**a.** Find the stationary point of the graph of $f(x) = \frac{4 + x^2 + x^3}{x}$, $x \in R \setminus \{0\}$. Express your answer in coordinate form, giving values correct to two decimal places. 1 mark

**b.** Find the point of inflection of the graph given in **part a.** Express your answer in coordinate form, giving values correct to two decimal places. 2 marks

**c.** Sketch the graph of $f(x)=\dfrac{4+x^2+x^3}{x}$ for $x\in[-3,\ 3]$ on the axes below, labelling the turning point and the point of inflection with their coordinates, correct to two decimal places. 3 marks




A glass is to be modelled by rotating the curve that is the part of the graph where $x\in[-3,\ -0.5]$ about the $y$-axis, to form a solid of revolution.

**d.** **i.** Write down a definite integral, in terms of $x$, which gives the length of the curve to be rotated. 1 mark

**ii.** Find the length of this curve, correct to two decimal places. 1 mark

**e.** The volume of the solid formed is given by $V=a\int_c^b x^2\,dy$.

Find the values of $a$, $b$ and $c$. Do **not** attempt to evaluate this integral. 1 mark

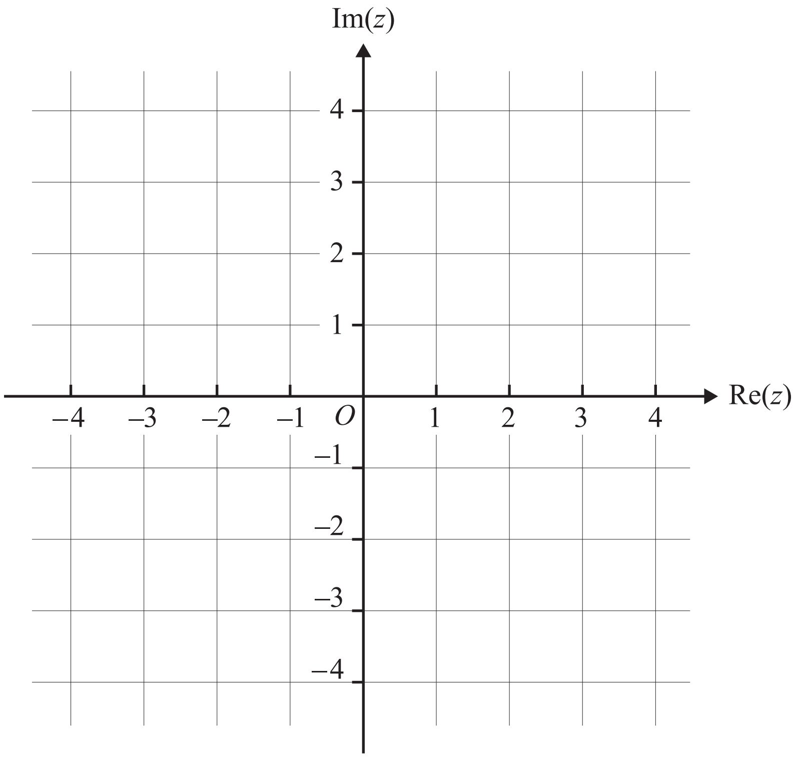
**Question 2** (11 marks)

A line in the complex plane is given by $|z-1|=|z+2-3i|$, $z \in C$.

**a.** Find the equation of this line in the form $y = mx + c$. 2 marks

**b.** Find the points of intersection of the line $|z-1|=|z+2-3i|$ with the circle $|z-1|=3$. 2 marks

**c.** Sketch both the line $|z-1|=|z+2-3i|$ and the circle $|z-1|=3$ on the Argand diagram below. 2 marks




**d.** The line $|z-1|=|z+2-3i|$ cuts the circle $|z-1|=3$ into two segments.

Find the area of the major segment. 2 marks

**e.** Sketch the ray given by $\text{Arg}(z)=-\frac{3\pi}{4}$ on the Argand diagram in **part c.** 1 mark

**f.** Write down the range of values of $\alpha$, $\alpha \in R$, for which a ray with equation $\text{Arg}(z) = \alpha\pi$ intersects the line $|z-1|=|z+2-3i|$. 2 marks

**Question 3** (11 marks)

A tank initially has 20 kg of salt dissolved in 100 L of water. Pure water flows into the tank at a rate of 10 L/min. The solution of salt and water, which is kept uniform by stirring, flows out of the tank at a rate of 5 L/min.

If $x$ kilograms is the amount of salt in the tank after $t$ minutes, it can be shown that the differential equation relating $x$ and $t$ is $\frac{dx}{dt} + \frac{x}{20+t} = 0$.

**a.** Solve this differential equation to find $x$ in terms of $t$. 3 marks

A second tank initially has 15 kg of salt dissolved in 100 L of water. A solution of $\frac{1}{60}$ kg of salt per litre flows into the tank at a rate of 20 L/min. The solution of salt and water, which is kept uniform by stirring, flows out of the tank at a rate of 10 L/min.

**b.** If $y$ kilograms is the amount of salt in the tank after $t$ minutes, write down an expression for the **concentration**, in kg/L, of salt in the second tank at time $t$. 1 mark

**c.** Show that the differential equation relating $y$ and $t$ is $\frac{dy}{dt}+\frac{y}{10+t}=\frac{1}{3}$. 2 marks

**d.** Verify by differentiation and substitution into the left side that $y=\frac{t^2+20t+900}{6(10+t)}$ satisfies the **differential equation** in **part c.** Verify that the given solution for $y$ also satisfies the **initial condition**. 3 marks

**e.** Find when the **concentration** of salt in the second tank reaches 0.095 kg/L. Give your answer in minutes, correct to two decimal places. 2 marks

**CONTINUES OVER PAGE**

**Question 4** (10 marks)

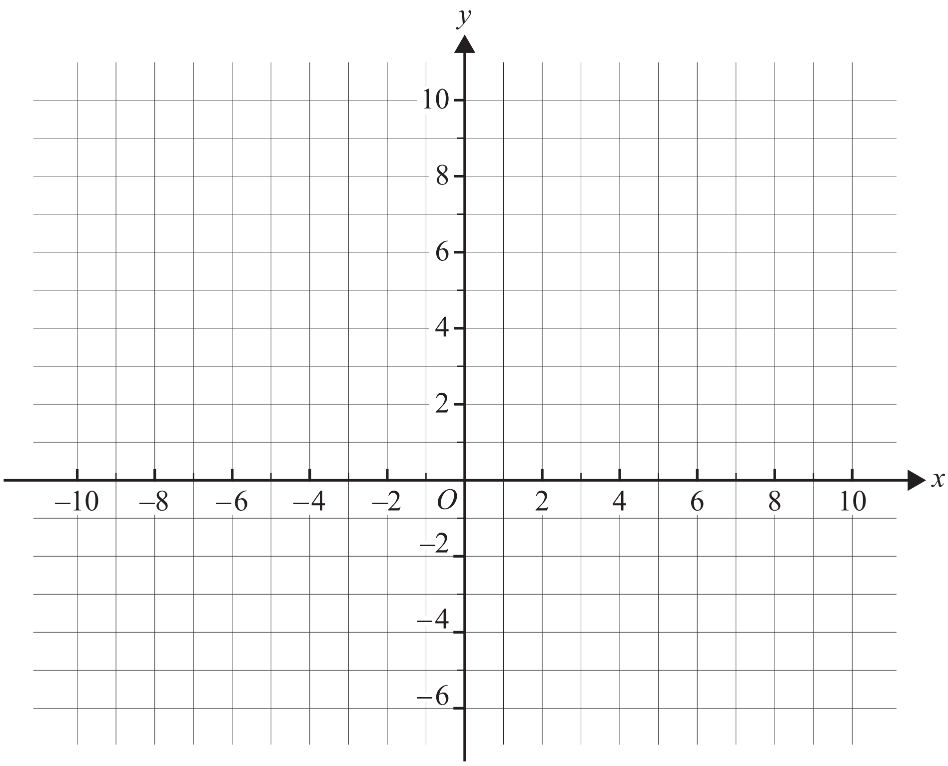
Two ships, *A* and *B*, are observed from a lighthouse at origin *O*. Relative to *O*, their position vectors at time *t* hours after midday are given by

$$\underset{\sim}{r}_A = 5(1-t)\underset{\sim}{i} + 3(1+t)\underset{\sim}{j}$$
$$\underset{\sim}{r}_B = 4(t-2)\underset{\sim}{i} + (5t-2)\underset{\sim}{j}$$

where displacements are measured in kilometres.

**a.** Show that the two ships will not collide, clearly stating your reason. 2 marks

**b.** Sketch and label the path of each ship on the axes below. Show the direction of motion of each ship with an arrow. 3 marks



**c.** Find the obtuse angle between the paths of the two ships. Give your answer in degrees, correct to one decimal place. 2 marks

**d.** **i.** Find the value of $t$, correct to three decimal places, when the ships are closest. 2 marks

**ii.** Find the minimum distance between the two ships, in kilometres, correct to two decimal places. 1 mark

**Question 5** (10 marks)

A model rocket of mass 2 kg is launched from rest and travels vertically up, with a vertical propulsion force of $(50 - 10t)$ newtons after $t$ seconds of flight, where $t \in [0, 5]$. Assume that the rocket is subject only to the vertical propulsion force and gravity, and that air resistance is negligible.

**a.** Let $v$ ms$^{-1}$ be the velocity of the rocket $t$ seconds after it is launched.

Write down an equation of motion for the rocket and show that $\frac{dv}{dt} = \frac{76}{5} - 5t$. 1 mark

**b.** Find the velocity, in ms$^{-1}$, of the rocket after five seconds. 2 marks

**c.** Find the height of the rocket after five seconds. Give your answer in metres, correct to two decimal places. 2 marks

**d.** After five seconds, when the vertical propulsion force has stopped, the rocket is subject only to gravity.

Find the maximum height reached by the rocket. Give your answer in metres, correct to two decimal places. 2 marks

**e.** Having reached its maximum height, the rocket falls directly to the ground.

Assuming negligible air resistance during this final stage of motion, find the time for which the rocket was in flight. Give your answer in seconds, correct to one decimal place. 3 marks

**Question 6** (9 marks)

The mean level of pollutant in a river is known to be 1.1 mg/L with a standard deviation of 0.16 mg/L.

**a.** Let the random variable $\bar{X}$ represent the mean level of pollutant in the measurements from a random sample of 25 sites along the river.

Write down the mean and standard deviation of $\bar{X}$. 2 marks

After a chemical spill, the mean level of pollutant from a random sample of 25 sites is found to be 1.2 mg/L.

To determine whether this sample provides evidence that the mean level of pollutant has increased, a statistical test is carried out.

**b.** Write down suitable hypotheses $H_0$ and $H_1$ to test whether the mean level of pollutant has increased. 2 marks

**c.** **i.** Find the *p* value for this test, correct to four decimal places. 2 marks

**ii.** State with a reason whether the sample supports the contention that there has been an increase in the mean level of pollutant after the spill. Test at the 5% level of significance. 1 mark

**d.** For this test, what is the smallest value of the sample mean that would provide evidence that the mean level of pollutant has increased? That is, find $\bar{x}_c$ such that $\Pr\left(\bar{X} > \bar{x}_c \mid \mu = 1.1\right) = 0.05$. Give your answer correct to three decimal places. 1 mark

**e.** Suppose that for a level of significance of 2.5%, we find that $\bar{x}_c = 1.163$. That is, $\Pr\left(\bar{X} > 1.163 \mid \mu = 1.1\right) = 0.025$

If the mean level of pollutant in the river, $\mu$, is in fact 1.2 mg/L after the spill, find $\Pr\left(\bar{X} < 1.163 \mid \mu = 1.2\right)$. Give your answer correct to three decimal places. 1 mark

**END OF QUESTION AND ANSWER BOOK**

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2016**

# SPECIALIST MATHEMATICS

## Written examination 2

### FORMULA SHEET

**Instructions**

| |
|---|
| This formula sheet is provided for your reference.<br>A question and answer book is provided with this formula sheet. |

| |
|---|
| **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.** |

# Specialist Mathematics formulas

## Mensuration

| | |
|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder | $2\pi rh$ |
| volume of a cylinder | $\pi r^2 h$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid | $\frac{1}{3}Ah$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| area of a triangle | $\frac{1}{2}bc\sin(A)$ |
| sine rule | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule | $c^2 = a^2 + b^2 - 2ab\cos(C)$ |

## Circular functions

| | |
|---|---|
| $\cos^2(x) + \sin^2(x) = 1$ | |
| $1 + \tan^2(x) = \sec^2(x)$ | $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$ |
| $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ | $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$ |
| $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ | $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ |
| $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ | $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$ |
| $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$ | |
| $\sin(2x) = 2\sin(x)\cos(x)$ | $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$ |

## Circular functions – continued

| **Function** | $\sin^{-1}$ or arcsin | $\cos^{-1}$ or arccos | $\tan^{-1}$ or arctan |
|---|---|---|---|
| **Domain** | $[-1, 1]$ | $[-1, 1]$ | $R$ |
| **Range** | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$ | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

## Algebra (complex numbers)

| | |
|---|---|
| $z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$ | |
| $\lvert z\rvert = \sqrt{x^2 + y^2} = r$ | $-\pi < \text{Arg}(z) \leq \pi$ |
| $z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ | $\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$ |
| $z^n = r^n \,\text{cis}(n\theta)$ (de Moivre's theorem) | |

## Probability and statistics

| | |
|---|---|
| for random variables $X$ and $Y$ | $\text{E}(aX + b) = a\text{E}(X) + b$<br>$\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$<br>$\text{var}(aX + b) = a^2\text{var}(X)$ |
| for independent random variables $X$ and $Y$ | $\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$ |
| approximate confidence interval for $\mu$ | $\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$ |
| distribution of sample mean $\overline{X}$ | mean $\text{E}\left(\overline{X}\right) = \mu$<br>variance $\text{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$ |

## Calculus

| | |
|---|---|
| $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$ |
| $\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ |
| $\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$ |
| $\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ | $\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$ |
| $\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ | $\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$ |
| $\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ | $\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$ |
| $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$ |
| $\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$ |
| $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$ |
| | $\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \ n \neq -1$ |
| | $\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$ |
| product rule | $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ |
| quotient rule | $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ |
| chain rule | $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ |
| Euler's method | If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$ |
| acceleration | $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ |
| arc length | $\int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$ |

## Vectors in two and three dimensions

| |
|---|
| $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ |
| $\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$ |
| $\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$ |
| $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$ |

## Mechanics

| | |
|---|---|
| momentum | $\underset{\sim}{p} = m\underset{\sim}{v}$ |
| equation of motion | $\underset{\sim}{R} = m\underset{\sim}{a}$ |

**END OF FORMULA SHEET**