VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2016 VCE Specialist Mathematics 2 examination report

General comments

The 2016 Specialist Mathematics examination 2 comprised 20 multiple-choice questions (worth a total of 20 marks) and six extended-answer questions (worth a total of 60 marks).

There were four questions (Questions 3c., 3d., 4a. and 5a.) where students needed to establish a given result with instructions to 'show that' or 'verify'. In these cases all steps that led to the given result or that verified the given statement needed to be clearly and logically set out. Students needed to provide a convincing and clear sequence of steps to obtain full marks.

Answers were generally given in required forms, although a few students moved from a correct exact answer to an incorrect decimal approximation in Question 1e. or gave an exact answer where a decimal approximation was required in Question 5c.

Students should note the instruction at the beginning of Section B that they must show appropriate working in questions where more than one mark is available.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- the use of technology to solve equations that give both real and complex solutions, and evaluate definite integrals
- facility with kinematics questions – most of Question 5 was attempted well
- facility with statistics questions – most of Question 6 was attempted well.

Areas of weakness included:

- poor presentation of graphs and graphs drawn roughly or over incorrect domains (Questions 1c., 2e. and 4b.) – use of a ruler would be beneficial for sketching lines
- inaccurate transfer of graphs from technology to a scaled set of axes, particularly failing to preserve shape and endpoints (Question 1c.)
- use of standard approaches or formulas to find areas within geometric figures (Question 2d.)
- familiarity with complex number concepts, particularly expressions involving arguments and sketching a ray on an Argand diagram (Questions 2e. and 2f.)
- finding and working with the concentration in a mixing problem (Questions 3b. and 3e.).

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Multiple-choice questions

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	11	12	11	4	61	1	$\frac{x}{3}=\text{cosec}^2(t), \frac{y+1}{4}=\cot(t)$ then eliminate *t*
2	3	84	8	4	1	0	$-1\le\frac{x-a}{b}\le 1$
3	12	6	71	7	4	0	$f(x)=x-\frac{a}{x}$
4	8	7	10	6	68	1	$z=3+2i$ is also a solution.
5	72	6	7	4	10	0	
6	7	13	57	15	8	1	$\lvert z_1z_2z_3z_4\rvert=2\times1\times2\times1\neq0$
7	37	8	17	16	21	1	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{\cos(2t)}{\cos(t)+\sin(t)}$ $=\frac{\cos^2(t)-\sin^2(t)}{\cos(t)+\sin(t)}=\cos(t)-\sin(t)$
8	2	5	9	73	12	0	$u=x^4$ leads to option D.
9	5	9	55	15	16	0	$y_0=0,\ y_1=0.6,\ y_2=1.272$
10	6	65	14	11	4	0	
11	3	10	10	75	2	1	$\underset{\sim}{a}.\hat{\underset{\sim}{b}}=\frac{10+\alpha^3}{\sqrt{17+\alpha^4}}=\frac{74}{\sqrt{273}}$
12	4	8	72	4	11	0	$\underset{\sim}{a}-\underset{\sim}{b}=(m-2)\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k},\quad (\underset{\sim}{a}-\underset{\sim}{b}).\underset{\sim}{b}=0$
13	63	23	7	4	3	0	$a=\frac{F}{m}=\frac{\sqrt{12^2+9^2}}{5}=3$
14	6	41	10	7	36	1	Draw a diagram showing forces. The ratio of force magnitudes is not the ratio of string lengths.
15	3	23	11	58	5	0	$a=\frac{d}{dx}\left(\frac{1}{2}v^2\right)=2x^3-6x,\quad F=3a$
16	14	7	28	46	3	1	It is simplest to consider vertical and horizontal components of velocity separately.
17	5	12	10	9	63	1	$\Delta p=mv_2-mv_1$ for straight line motion.
18	61	5	22	8	3	1	Note that $\text{Var}(X_1+X_2+X_3)\neq\text{Var}(3X)$
19	7	78	9	4	2	1	$\left(210-1.96\times\frac{16}{\sqrt{100}},210+1.96\times\frac{16}{\sqrt{100}}\right)$
20	5	4	14	7	68	1	$\text{E}(\bar{X})=20, \text{sd}(\bar{X})=\frac{2}{\sqrt{25}}=\frac{2}{5}$

Section B

Question 1a.

Marks	0	1	Average
%	7	93	1

$(1.11, 5.95)$

This question was answered very well. A small number of students gave the coordinates for the point of inflection rather than the stationary point.

Question 1b.

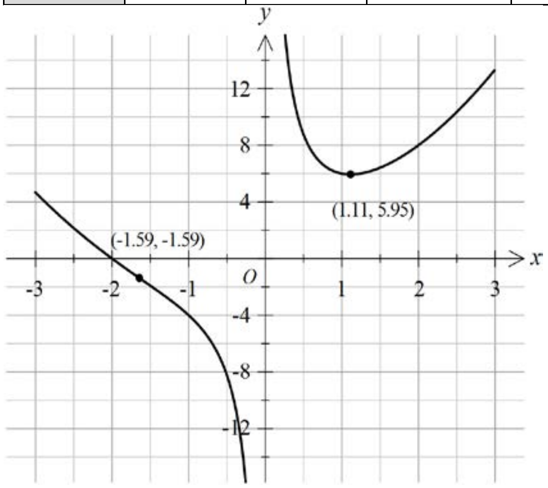
Marks	0	1	2	Average
%	7	15	78	1.7

$(-1.59, -1.59)$

This question was generally answered well. The most common error resulted from substituting a rounded *x* value, yielding an incorrect *y* value of –1.58. Some students left off the negative sign of the *y* value.

Question 1c.

Marks	0	1	2	3	Average
%	6	19	29	46	2.2



Students missed out on marks for ignoring the domain of the function or a lack of accuracy in the placement of the endpoints. Students are advised to use their technology as a tool to support the sketching of an accurate graph rather than simply copying a roughly correct shape from a screen. Students generally followed the instruction to label particular points but these points were not always plotted with appropriate accuracy. Careful attention to the axes scale is required.

Question 1di.

Marks	0	1	Average
%	27	73	0.8

$$\int_{-3}^{-0.5} \sqrt{1+\left(\frac{2x^3+x^2-4}{x^2}\right)^2}\,dx$$

This question was answered fairly well. A variety of equivalent correct forms were presented. A common error was an integrand containing the square of $f(x)$ rather than the square of $f'(x)$. Other errors included incorrect terminals, sign errors within the integrand and expressions that appeared to represent the volume of a solid of revolution.

Question 1dii.

Marks	0	1	Average
%	26	74	0.8

13.18

The majority of students who answered Question 1di. correctly were able to answer Question 1dii. correctly. A number of students who wrote an incorrect expression for Question 1di. went on to evaluate a correct expression and gained the mark for Question 1dii.

Question 1e.

Marks	0	1	Average
%	66	34	0.4

$a=\pi,\quad b=\frac{14}{3},\quad c=-\frac{33}{4}=-8.25$

Question 1e. was not answered well. A number of students incorrectly gave decimal approximations for the value of *b*. Students must note and follow the general instructions given at the start of Section B. Some students interchanged the values of *b* and *c*, but this would only be correct if they wrote $a=-\pi$.

Question 2a.

Marks	0	1	2	Average
%	14	14	72	1.6

$y=x+2$

The majority of correct answers resulted from substituting $z=x+yi$ into the expression provided. Very few students used a perpendicular bisector approach at this stage. The most common error was a negative gradient.

Question 2b.

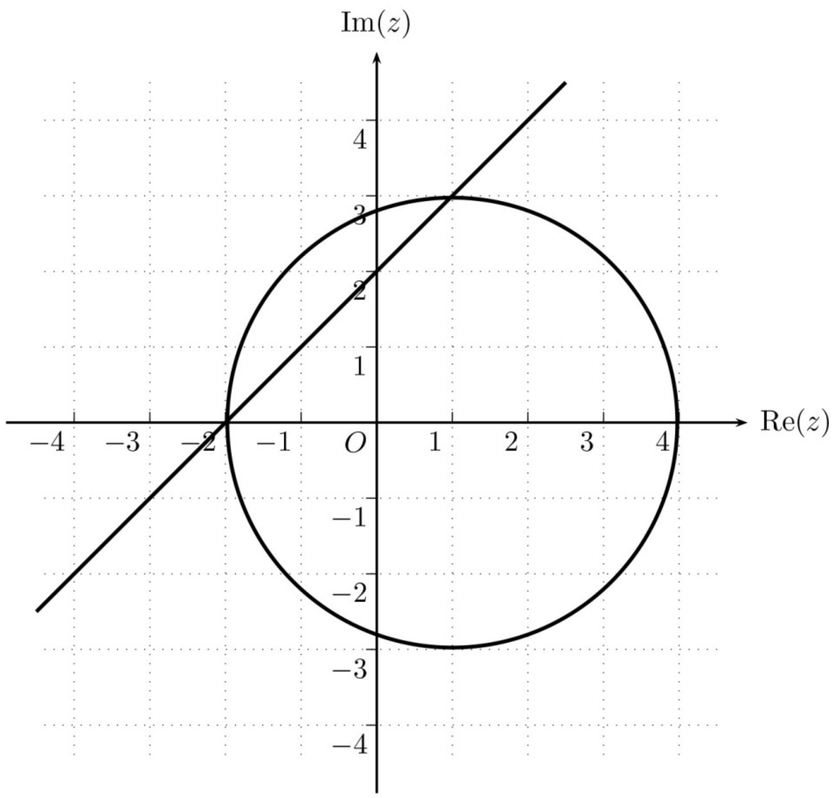
Marks	0	1	2	Average
%	17	18	66	1.5

$|x + yi - 1| = |x + yi + 2 - 3i|$, $(x-1)^2 + (x+2)^2 = 9$, $(-2, 0)$, $(1, 3)$

Most students were able to find cartesian expressions for the circle and the line and then apply substitution or use technology to find the required points.

Question 2c.

Marks	0	1	2	Average
%	11	12	77	1.7



This question was generally well answered. The circle was sketched correctly in almost all cases. However, the line was not always placed with sufficient accuracy. Some students who were unable to find the correct equation in Question 2a. were able to use a perpendicular bisector approach to draw a correct line.

Question 2d.

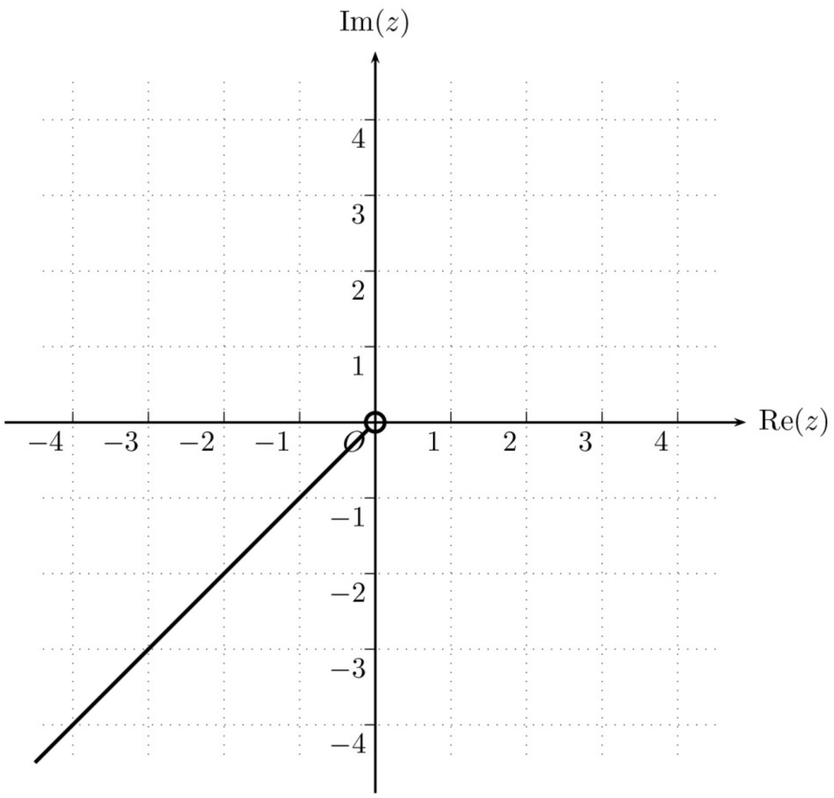
Marks	0	1	2	Average
%	48	13	39	0.9

$A=\frac{3}{4}\times\pi\times3^2+\frac{1}{2}\times3\times3,\ A=\frac{27\pi}{4}+\frac{9}{2}$

Students found this question more difficult than previous parts of Question 2. A correct answer was most easily found by adding a right-angled triangle to three-quarters of a circle. Subtracting the minor segment area from the circle area was a common approach. Some students correctly used a segment area formula and a larger number set up elaborate definite integrals to find the area, occasionally successfully, but this was not an efficient approach. Sign and factorisation errors meant that some students moved from a correct approach and evaluation to an incorrect final answer.

Question 2e.

Marks	0	1	Average
%	66	34	0.4



Many students were not able to sketch the required ray. Some students sketched a line but did not restrict their ray appropriately, either including or extending past the origin.

Question 2f.

Marks	0	1	2	Average
%	85	9	7	0.3

$\left(-1,-\frac{3}{4}\right)\cup\left(\frac{1}{4},1\right]$

Students found this question demanding, with few students giving a fully correct answer. Many students did not respond to this question. Common incorrect answers contained multiples of π or included the endpoint $\alpha=-1$. Some students did not note that the principal value of the argument was used in the question. Of the correct answers, a variety of correct notations were presented.

Question 3a.

Marks	0	1	2	3	Average
%	29	16	13	42	1.7

$\int\frac{1}{x}dx=-\int\frac{1}{20+t}dt$, $\log_e(x)=-\log_e(20+t)+c$, $c=\log_e(400)$, $x=\frac{400}{20+t}$

A range of errors prevented students from achieving full marks for this question. There were many instances where students did not separate variables correctly, offering attempts such as

$\int(20+t)\,dt=\int -x\,dx\Rightarrow 20t+\frac{t^2}{2}=-\frac{x^2}{2}+c$. Some students did not write down or evaluate the constant. Errors with constants were common among students who added a constant to both sides of the expression before attempting to find its value.

A small number of students used definite integrals from 0 to *t* and 20 to *x* on the sides.

Question 3b.

Marks	0	1	Average
%	66	34	0.4

$\frac{y}{100+10t}$

Many students did not demonstrate an understanding of what was required by this question. Students frequently found an expression for $\frac{dy}{dt}$ rather than the concentration at time *t*.

Question 3c.

Marks	0	1	2	Average
%	51	9	40	0.9

$\frac{dy}{dt}=20\times\frac{1}{60}-\frac{y}{100+10t}\times 10$, $\frac{dy}{dt}+\frac{y}{10+t}=\frac{1}{3}$

This 'show that' question required students to obtain the expression $\frac{dy}{dt}=\frac{1}{3}-\frac{y}{10+t}$ by logical steps. Some students incorrectly started with the given expression with no explanation of its origin. Students frequently did not seem to realise that work done for Question 3b. was useful here.

Question 3d.

Marks	0	1	2	3	Average
%	41	20	22	17	1.2

$\frac{dy}{dt}=\frac{t^2+20t-700}{6(10+t)^2}$, $\text{LHS}=\frac{t^2+20t-700}{6(10+t)^2}+\frac{t^2+20t+900}{6(10+t)^2}=\frac{2t^2+40t+200}{6(10+t)^2}=\frac{2(10+t)^2}{6(10+t)^2}=\frac{1}{3}=\text{RHS}$

Initial condition: $t=0,\ y=\frac{900}{6\times 10}=15$

Most students were able to find a correct expression for the derivative. It was not always clear how expressions for the left side simplified to the right side. Verification that the given solution satisfied the initial conditions was often absent.

Question 3e.

Marks	0	1	2	Average
%	69	8	23	0.6

$\frac{t^2+20t+900}{6(10+t)(100+10t)}=0.095$, $t=3.05$

Many students did not attempt this question. It was common to see $\frac{dy}{dt}=0.095$ rather than using the concentration in the equation.

Question 4a.

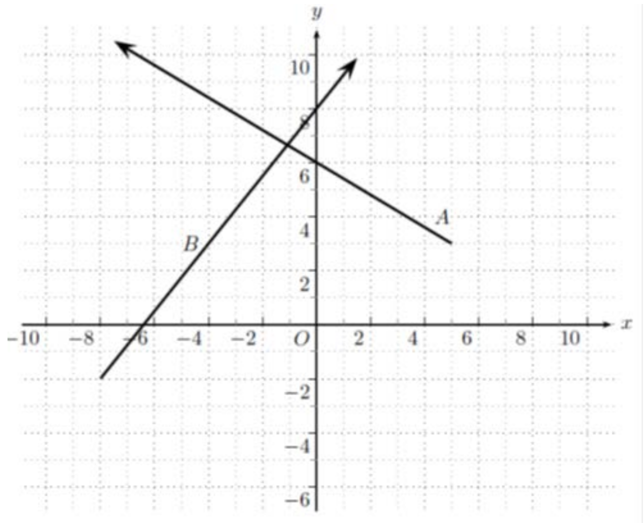
Marks	0	1	2	Average
%	23	18	58	1.4

$5(1-t)=4(t-2),\ t=\frac{13}{9},\ 3(1+t)=5t-2,\ t=\frac{5}{2}$, different times so particles do not collide.

A variety of correct approaches were used by students but the approach above led to success most frequently. Many students did not find both times correctly due to simple algebraic errors.

Question 4b.

Marks	0	1	2	3	Average
%	25	19	19	36	1.7



The majority of students found correct cartesian expressions for the paths. Many students did not take note of when the vector functions applied and consequently plotted the paths over incorrect domains. The instruction to show the direction of motion was usually followed.

Question 4c.

Marks	0	1	2	Average
%	69	7	24	0.6

$\theta = 97.7^{\circ}$

Students who found velocity vectors before finding the angle between them using a scalar product were more successful than those who used the gradients from the cartesian expressions. Some students found the angle between position vectors at chosen times, which indicated that a greater appreciation of the meaning of position and velocity vectors is required. Occasionally students gave the acute angle between the paths.

Question 4di.

Marks	0	1	2	Average
%	64	9	27	0.7

$\underset{\sim}{d} = (13-9t)\underset{\sim}{i} + (5-2t)\underset{\sim}{j}$, $|\underset{\sim}{d}|^2 = (13-9t)^2 + (5-2t)^2$, $t = 1.494$

Students who found a displacement vector frequently went on to find the correct time. Some students used an expression for the difference between position vector magnitudes. Students are reminded that the instruction to give the answer correct to three decimal places must be followed to gain full marks. A rational answer did not suffice here.

Question 4dii.

Marks	0	1	Average
%	71	29	0.3

2.06

Many students who did not attempt Question 4di. did not attempt this question.

Question 5a.

Marks	0	1	Average
%	61	39	0.4

$50-10t-2g=2a,\ 25-5t-9.8=\frac{dv}{dt},\ \left(\frac{dv}{dt}=15.2-5t\right),\ \frac{dv}{dt}=\frac{76}{5}-5t$

As this was a 'show that' question, students needed to show clear progress from an equation of motion to the required differential equation. Some students did not include the weight force in the initial equation; drawing a diagram showing forces could have benefited these students.

Question 5b.

Marks	0	1	2	Average
%	13	7	81	1.7

$v=\frac{76}{5}t-\frac{5}{2}t^2,\ v=13.5$

Most students found the velocity by integration. A small number incorrectly used a constant acceleration formula.

Question 5c.

Marks	0	1	2	Average
%	31	9	61	1.3

$h=\frac{76}{10}t^2-\frac{5}{6}t^3,\ h=85.83$

Success in Question 5b. generally translated to success in Question 5c., although a greater number of students used a constant acceleration in Question 5c. An answer correct to two decimal places was required, but this instruction was ignored by some students.

Question 5d.

Marks	0	1	2	Average
%	50	10	39	0.9

$x=9.298$, Max. height $=85.83+9.30=95.13$

Students used a variety of correct approaches. Some students used calculus, while others used constant acceleration formulas. The most common misconception arising in this question was not realising that acceleration was now constant, and some students proceeded to use their equation for the displacement that was obtained in Question 5c. Students should note the instructions at the beginning of Section B where they are told to use $g=9.8$, not 10.

Question 5e.

Marks	0	1	2	3	Average
%	44	27	7	21	1.1

$t_1 = 5, t_2 = \frac{13.5}{9.8} = 1.378, t_3 = 4.41, \text{Total time} = 10.8$

Of the students who made progress in this question, the majority used equations of constant acceleration. A few realised that it was possible to describe the whole of this section of the motion using the equation $-\frac{515}{6} = \frac{27}{2}t - 4.9t^2$. A number who treated the motion in separate sections added only some parts of the motion to get the total time. Inconsistent signs caused some difficulty.

Question 6a.

Marks	0	1	2	Average
%	8	12	80	1.7

$\text{E}\left(\bar{X}\right) = 1.1, \text{sd}\left(\bar{X}\right) = 0.032\left(= \frac{4}{125}\right)$

Students handled this very well. Almost all students stated the correct mean.

Question 6b.

Marks	0	1	2	Average
%	19	5	75	1.6

$\text{H}_0: \ \mu = 1.1, \quad \text{H}_1: \ \mu > 1.1$

This question was answered very well by the majority of students. Some students failed to use appropriate notation or state the hypotheses clearly. The alternate hypothesis was occasionally written for a two-tail test, that is $\text{H}_1: \ \mu \neq 1.1$.

Question 6ci.

Marks	0	1	2	Average
%	23	15	62	1.4

$p = \Pr\left(\bar{X} > 1.2 \mid \mu = 1.1\right), \ p = 0.0009$

Transcription errors caused some students to miss out on marks, with answers such as 0.009 occurring. High-scoring answers using the *z*-distribution were prevalent.

Question 6cii.

Marks	0	1	Average
%	35	65	0.7

$p < 0.05$, reject the null hypothesis at the 5% level of significance, supports the contention.

A correct response to Question 6ci. was generally followed by a correct answer to this question. Some students did not explicitly test at the 5% level of significance. Two-tail approaches appeared occasionally.

Question 6d.

Marks	0	1	Average
%	57	43	0.5

$\bar{x}_c = 1.153$

Correct answers were obtained by the majority of students who attempted this question. Rounding errors caused some students to miss out on the mark.

Question 6e.

Marks	0	1	Average
%	54	46	0.5

$\Pr\left(\bar{X} < 1.163 \mid \mu = 1.2\right) = 0.124$

Correct answers were obtained by the majority of students who attempted this question. Rounding errors caused some students to miss out on the mark.