VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2017**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

**STUDENT NUMBER**

# SPECIALIST MATHEMATICS

## Written examination 2

**Monday 13 November 2017**

**Reading time: 3.00 pm to 3.15 pm (15 minutes)**
**Writing time: 3.15 pm to 5.15 pm (2 hours)**

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| A | 20 | 20 | 20 |
| B | 6 | 6 | 60 |
| | | | Total 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 21 pages
- Formula sheet
- Answer sheet for multiple-choice questions

**Instructions**
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**At the end of the examination**
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## SECTION A – Multiple-choice questions

### Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1**

The implied domain of $f(x) = 2\cos^{-1}\left(\frac{1}{x}\right)$ is

**A.** $R$
**B.** $[-1, 1]$
**C.** $(-\infty, -1] \cup [1, \infty)$
**D.** $R \setminus \{0\}$
**E.** $[-1, 1] \setminus \{0\}$

**Question 2**

The solutions to $\cos(x) > \frac{1}{4}\text{cosec}(x)$ for $x \in (0, 2\pi) \setminus \{\pi\}$ are given by

**A.** $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\frac{5\pi}{12}, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

**B.** $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\frac{13\pi}{12}, \frac{17\pi}{12}\right)$

**C.** $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\pi, \frac{13\pi}{12}\right) \cup \left(\frac{13\pi}{12}, 2\pi\right)$

**D.** $x \in \left(\frac{\pi}{12}, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

**E.** $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \cup \left(\pi, \frac{13\pi}{12}\right) \cup \left(\frac{17\pi}{12}, 2\pi\right)$

**Question 3**

The number of distinct roots of the equation $(z^4 - 1)(z^2 + 3iz - 2) = 0$, where $z \in C$, is

**A.** 2
**B.** 3
**C.** 4
**D.** 5
**E.** 6

**Question 4**

The solutions to $z^n = 1 + i$, $n \in Z^+$ are given by

**A.** $2^{\frac{1}{2n}} \text{cis}\left(\frac{\pi}{4n} + \frac{2\pi k}{n}\right), \ k \in R$

**B.** $2^{\frac{1}{n}} \text{cis}\left(\frac{\pi}{4n} + 2\pi k\right), \ k \in Z$

**C.** $2^{\frac{1}{2n}} \text{cis}\left(\frac{\pi}{4} + \frac{2\pi k}{n}\right), \ k \in R$

**D.** $2^{\frac{1}{n}} \text{cis}\left(\frac{\pi}{4n} + \frac{2\pi k}{n}\right), \ k \in Z$

**E.** $2^{\frac{1}{2n}} \text{cis}\left(\frac{\pi}{4n} + \frac{2\pi k}{n}\right), \ k \in Z$

**Question 5**

On an Argand diagram, a point that lies on the path defined by $|z - 2 + i| = |z - 4|$ is

**A.** $\left(3, -\frac{1}{2}\right)$

**B.** $\left(-3, -\frac{1}{2}\right)$

**C.** $\left(-3, \frac{3}{2}\right)$

**D.** $\left(3, \frac{1}{2}\right)$

**E.** $\left(3, -\frac{3}{2}\right)$

**Question 6**

Given that $\frac{dy}{dx} = e^x \arctan(y)$, the value of $\frac{d^2y}{dx^2}$ at the point (0, 1) is

**A.** $\frac{1}{2}$

**B.** $\frac{3\pi}{8}$

**C.** $-\frac{1}{2}$

**D.** $\frac{\pi}{4}$

**E.** $-\frac{\pi}{8}$

**Question 7**

With a suitable substitution $\int_1^2 x^2\sqrt{2-x}\,dx$ can be expressed as

**A.** $-\int_1^2 \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} + u^{\frac{5}{2}}\right) du$

**B.** $\int_1^2 \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} + u^{\frac{5}{2}}\right) du$

**C.** $\int_0^1 \left(-4u^{\frac{1}{2}} + 4u^{\frac{3}{2}} - u^{\frac{5}{2}}\right) du$

**D.** $-\int_1^0 \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} + u^{\frac{5}{2}}\right) du$

**E.** $\int_1^0 \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} - u^{\frac{5}{2}}\right) du$

**Question 8**

Let $f(x) = x^3 - mx^2 + 4$, where $m, x \in R$.

The **gradient** of $f$ will always be strictly increasing for

**A.** $x \geq 0$

**B.** $x \geq \frac{m}{3}$

**C.** $x \leq \frac{m}{3}$

**D.** $x \geq \frac{2m}{3}$

**E.** $x \leq \frac{2m}{3}$

**Question 9**

Consider $\frac{dy}{dx} = 2x^2 + x + 1$, where $y(1) = y_0 = 2$.

Using Euler's method with a step size of 0.1, an approximation to $y(0.8) = y_2$ is given by

**A.** 0.94

**B.** 1.248

**C.** 1.6

**D.** 2.4

**E.** 2.852

**Question 10**

A function $f$, its derivative $f'$ and its second derivative $f''$ are defined for $x \in R$ with the following properties.

$$f(a) = 1, f(-a) = -1$$

$$f(b) = -1, f(-b) = 1$$

$$\text{and } f''(x) = \frac{(x+a)^2(x-b)}{g(x)}, \text{ where } g(x) < 0$$

The coordinates of any points of inflection of $|f(x)|$ are

**A.** $(-a, 1)$ and $(b, 1)$

**B.** $(b, -1)$

**C.** $(-a, -1)$ and $(b, -1)$

**D.** $(-a, 1)$

**E.** $(b, 1)$

**Question 11**

The vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + d\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$, where $d$ is a real constant, are linearly dependent if

**A.** $d = -10$

**B.** $d \in R \setminus \{-14\}$

**C.** $d = -14$

**D.** $d \in R \setminus \{-10\}$

**E.** $d \in R$

**Question 12**

Let $\underset{\sim}{r}(t) = \left(1 - \sqrt{a}\sin(t)\right)\underset{\sim}{i} + \left(1 - \frac{1}{b}\cos(t)\right)\underset{\sim}{j}$ for $t \geq 0$ and $a, b \in R^+$ be the path of a particle moving in the cartesian plane.

The path of the particle will always be a circle if

**A.** $ab^2 = 1$

**B.** $a^2b = 1$

**C.** $ab^2 \neq 1$

**D.** $ab = 1$

**E.** $a^2b \neq 1$

**Question 13**

Given the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + 12\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, the vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is

**A.** $-\frac{14}{3}$

**B.** $-\frac{14}{3}(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k})$

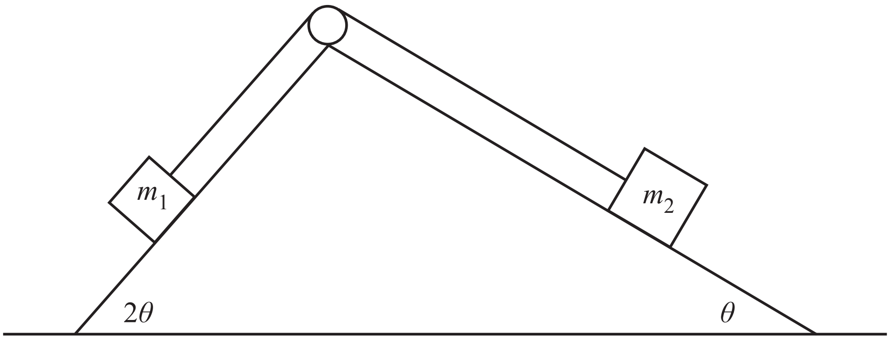
**C.** $-\frac{14}{9}(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k})$

**D.** $-\frac{14}{13}$

**E.** $-\frac{14}{169}(3\underset{\sim}{i} - 4\underset{\sim}{j} + 12\underset{\sim}{k})$

**Question 14**

Two particles with mass $m_1$ kilograms and $m_2$ kilograms are connected by a taut light string that passes over a smooth pulley. The particles sit on smooth inclined planes, as shown in the diagram below.




If the system is in equilibrium, then $\frac{m_1}{m_2}$ is equal to

**A.** $\frac{\sec(\theta)}{2}$

**B.** $2\sec(\theta)$

**C.** $2\cos(\theta)$

**D.** $\frac{1}{2}$

**E.** 1

**Question 15**

A body has displacement of $3\underset{\sim}{i} + \underset{\sim}{j}$ metres at a particular time. The body moves with constant velocity and two seconds later its displacement is $-\underset{\sim}{i} + 5\underset{\sim}{j}$ metres.

The velocity, in ms$^{-1}$, of the body is

**A.** $2\underset{\sim}{i} + 6\underset{\sim}{j}$

**B.** $-2\underset{\sim}{i} + 2\underset{\sim}{j}$

**C.** $-4\underset{\sim}{i} + 4\underset{\sim}{j}$

**D.** $4\underset{\sim}{i} - 4\underset{\sim}{j}$

**E.** $\underset{\sim}{i} + 3\underset{\sim}{j}$

**Question 16**

An object of mass 20 kg, initially at rest, is pulled along a rough horizontal surface by a force of 80 N acting at an angle of 40° upwards from the horizontal. A friction force of 20 N opposes the motion.

After the pulling force has acted for 5 s, the magnitude of the momentum, in kg ms$^{-1}$, of the object is closest to

**A.** 10

**B.** 40

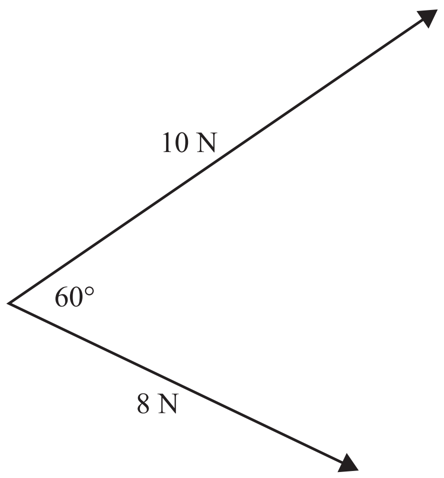
**C.** 160

**D.** 210

**E.** 4100

**Question 17**

Forces of 10 N and 8 N act on a body as shown below.




The resultant force acting on the body will, correct to one decimal place, have

**A.** magnitude 15.6 N and act at 26.3° to the 10 N force.

**B.** magnitude 9.2 N and act at 49.1° to the 10 N force.

**C.** magnitude 15.6 N and act at 33.7° to the 10 N force.

**D.** magnitude 9.2 N and act at 70.9° to the 10 N force.

**E.** magnitude 15.6 N and act at 49.1° to the 10 N force.

**Question 18**

$U$ and $V$ are independent normally distributed random variables, where $U$ has a mean of 5 and a variance of 1, and $V$ has a mean of 8 and a variance of 1. The random variable $W$ is defined by $W = 4U - 3V$.

In terms of the standard normal variable $Z$, $\Pr(W > 5)$ is equivalent to

**A.** $\Pr\left(Z > \frac{9\sqrt{7}}{7}\right)$

**B.** $\Pr(Z < 1.8)$

**C.** $\Pr\left(Z < \frac{9\sqrt{7}}{7}\right)$

**D.** $\Pr(Z > 0.2)$

**E.** $\Pr(Z > 1.8)$

**Question 19**

A confidence interval is to be used to estimate the population mean $\mu$ based on a sample mean $\bar{x}$.

To decrease the width of a confidence interval by 75%, the sample size must be multiplied by a factor of

**A.** 2

**B.** 4

**C.** 9

**D.** 16

**E.** 25

**Question 20**

In a one-sided statistical test at the 5% level of significance, it would be concluded that

**A.** $H_0$ should not be rejected if $p = 0.04$

**B.** $H_0$ should be rejected if $p = 0.06$

**C.** $H_0$ should be rejected if $p = 0.03$

**D.** $H_0$ should not be rejected if $p \neq 0.05$

**E.** $H_0$ should not be rejected if $p = 0.01$

**END OF SECTION A**

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## SECTION B

### Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1** (11 marks)

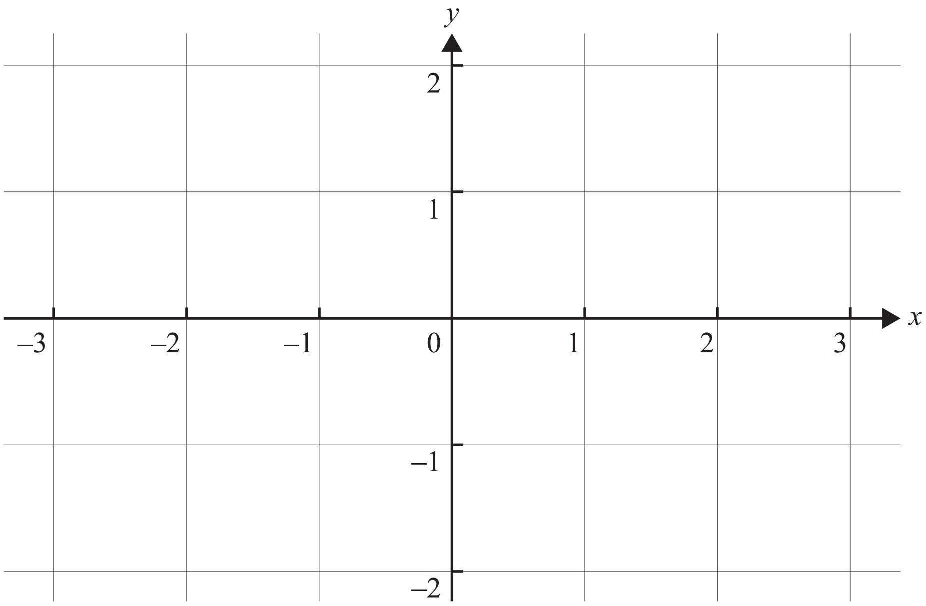
Let $f: D \to R,\ f(x) = \dfrac{x}{1+x^3}$, where $D$ is the maximal domain of $f$.

**a.** **i.** Find the equations of any asymptotes of the graph of $f$. 1 mark

**ii.** Find $f'(x)$ and state the coordinates of any stationary points of the graph of $f$, correct to two decimal places. 2 marks

**iii.** Find the coordinates of any points of inflection of the graph of $f$, correct to two decimal places. 2 marks

**b.** Sketch the graph of $f(x) = \dfrac{x}{1+x^3}$ from $x = -3$ to $x = 3$ on the axes provided below, marking all stationary points, points of inflection and intercepts with axes, labelling them with their coordinates. Show any asymptotes and label them with their equations. 3 marks




**c.** The region $S$, bounded by the graph of $f$, the $x$-axis and the line $x = 3$, is rotated about the $x$-axis to form a solid of revolution. The line $x = a$, where $0 < a < 3$, divides the region $S$ into two regions such that, when the two regions are rotated about the $x$-axis, they generate solids of equal volume.

**i.** Write down an equation involving definite integrals that can be used to determine $a$. 2 marks

**ii.** Hence, find the value of $a$, correct to two decimal places. 1 mark

**Question 2** (10 marks)

A helicopter is hovering at a constant height above a fixed location. A skydiver falls from rest for two seconds from the helicopter. The skydiver is subject only to gravitational acceleration and air resistance is negligible for the first two seconds. Let downward displacement be positive.

**a.** Find the distance, in metres, fallen in the first two seconds. 2 marks

**b.** Show that the speed of the skydiver after two seconds is 19.6 $ms^{-1}$. 1 mark

After two seconds, air resistance is significant and the acceleration of the skydiver is given by $a = g - 0.01v^2$.

**c.** Find the limiting (terminal) velocity, in $ms^{-1}$, that the skydiver would reach. 1 mark

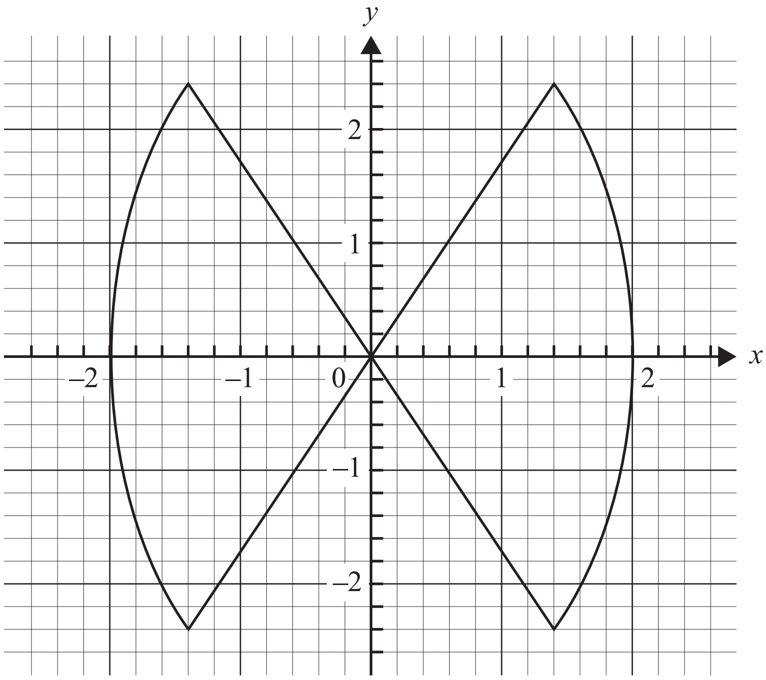
**d.** **i.** Write down an expression involving a definite integral that gives the time taken for the skydiver to reach a speed of 30 $ms^{-1}$. 2 marks

**ii.** Hence, find the time, in seconds, taken to reach a speed of 30 $ms^{-1}$, correct to the nearest tenth of a second. 1 mark

**e.** Write down an expression involving a definite integral that gives the distance through which the skydiver falls to reach a speed of 30 $ms^{-1}$. Find this distance, giving your answer in metres, correct to the nearest metre. 3 marks

**Question 3** (10 marks)

A brooch is designed using inverse circular functions to make the shape shown in the diagram below.




The edges of the brooch in the first quadrant are described by the piecewise function

$$f(x)=\begin{cases}3\arcsin\left(\dfrac{x}{2}\right), & 0\le x\le\sqrt{2}\\[2ex] 3\arccos\left(\dfrac{x}{2}\right), & \sqrt{2}<x\le 2\end{cases}$$

**a.** Write down the coordinates of the corner point of the brooch in the first quadrant. 1 mark

**b.** Specify the piecewise function that describes the edges in the third quadrant. 1 mark

**c.** Given that each unit in the diagram represents one centimetre, find the area of the brooch. Give your answer in square centimetres, correct to one decimal place. 3 marks

**d.** Find the acute angle between the edges of the brooch at the origin. Give your answer in degrees, correct to one decimal place. 3 marks

**e.** The perimeter of the brooch has a border of gold.

Show that the length of the gold border needed is given by a definite integral of the form $\int_0^2 \left(\sqrt{a + \frac{b}{4 - x^2}}\right) dx$, where $a, b \in R$. Find the values of $a$ and $b$. 2 marks

**Question 4** (10 marks)

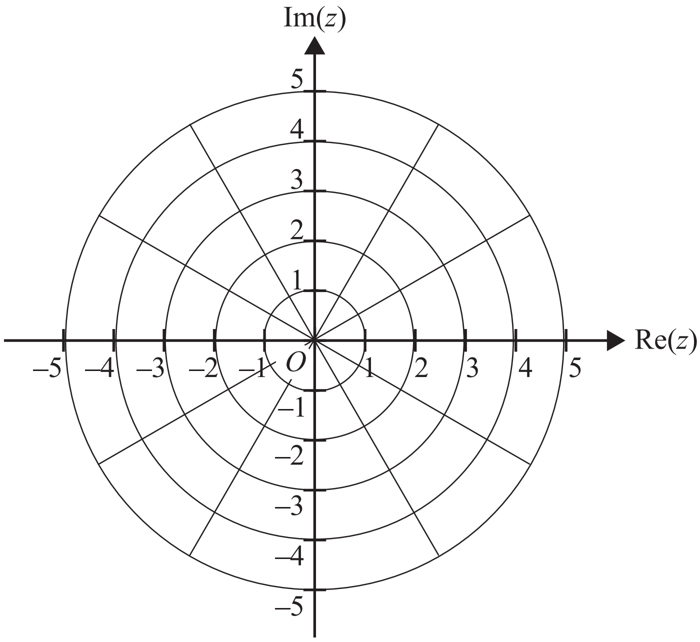
**a.** Express $-2-2\sqrt{3}i$ in polar form. 1 mark

**b.** Show that the roots of $z^2 + 4z + 16 = 0$ are $z=-2-2\sqrt{3}i$ and $z=-2+2\sqrt{3}i$. 1 mark

**c.** Express the roots of $z^2 + 4z + 16 = 0$ in terms of $2-2\sqrt{3}i$. 1 mark

**d.** Show that the cartesian form of the relation $|z|=|z-(2-2\sqrt{3}i)|$ is $x-\sqrt{3}y-4=0$. 2 marks

**e.** Sketch the line represented by $x-\sqrt{3}y-4=0$ and plot the roots of $z^2+4z+16=0$ on the Argand diagram below. 2 marks



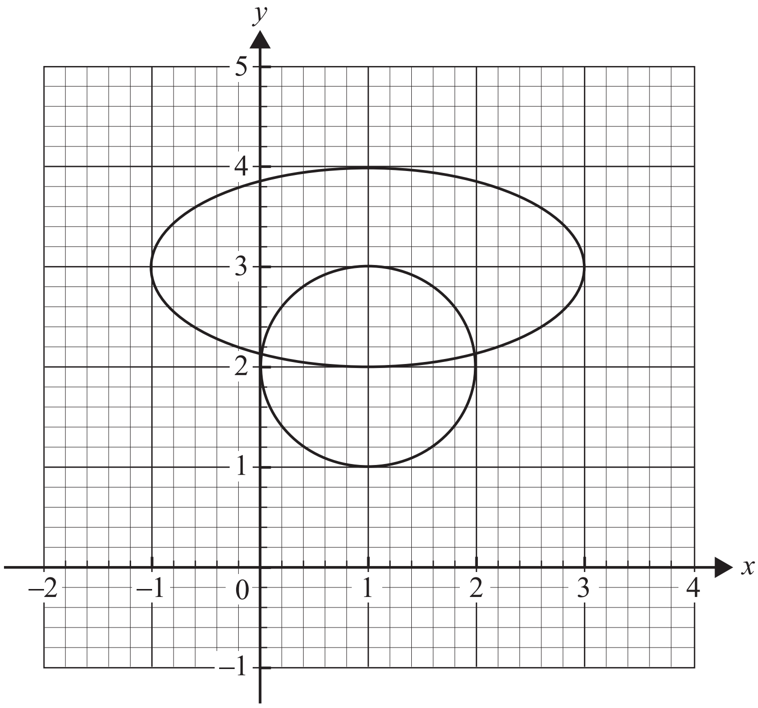

**f.** The equation of the line passing through the two roots of $z^2+4z+16=0$ can be expressed as $|z-a|=|z-b|$, where $a, b \in C$.

Find $b$ in terms of $a$. 1 mark

**g.** Find the area of the major segment bounded by the line passing through the roots of $z^2+4z+16=0$ and the major arc of the circle given by $|z|=4$. 2 marks

**Question 5** (10 marks)

On a particular morning, the position vectors of a boat and a jet ski on a lake $t$ minutes after they have started moving are given by $\underset{\sim}{r}_B(t) = (1 - 2\cos(t))\underset{\sim}{i} + (3 + \sin(t))\underset{\sim}{j}$ and $\underset{\sim}{r}_J(t) = (1 - \sin(t))\underset{\sim}{i} + (2 - \cos(t))\underset{\sim}{j}$ respectively for $t \geq 0$, where distances are measured in kilometres. The boat and the jet ski start moving at the same time. The graphs of their paths are shown below.




**a.** On the diagram above, mark the initial positions of the boat and the jet ski, clearly identifying each of them. Use arrows to show the directions in which they move. 2 marks

**b.** **i.** Find the first time for $t > 0$ when the speeds of the boat and the jet ski are the same. 2 marks

**ii.** State the coordinates of the boat at this time. 1 mark

**c.** **i.** Write down an expression for the distance between the jet ski and the boat at any time *t*. 1 mark

**ii.** Find the minimum distance separating the boat and the jet ski. Give your answer in kilometres, correct to two decimal places. 1 mark

**d.** On another morning, the boat's position vector remained the same but the jet skier considered starting from a different location with a new position vector given by $\underset{\sim}{r}(t) = (1 - \sin(t))\underset{\sim}{i} + (a - \cos(t))\underset{\sim}{j}$, $t \geq 0$, where $a$ is a real constant. Both vessels are to start at the same time.

Assuming the vessels would collide shortly after starting, find the time of the collision and the value of *a*. 3 marks

**Question 6** (9 marks)

A dairy factory produces milk in bottles with a nominal volume of 2 L per bottle. To ensure most bottles contain at least the nominal volume, the machine that fills the bottles dispenses volumes that are normally distributed with a mean of 2005 mL and a standard deviation of 6 mL.

**a.** Find the percentage of bottles that contain at least the nominal volume of milk, correct to one decimal place. 1 mark

Bottles of milk are packed in crates of 10 bottles, where the nominal total volume per crate is 20 L.

**b.** Show that the total volume of milk contained in each crate varies with a mean of 20 050 mL and a standard deviation of $6\sqrt{10}$ mL. 2 marks

**c.** Find the percentage, correct to one decimal place, of crates that contain at least the nominal volume of 20 L. 1 mark

**d.** Regulations require at least 99.9% of crates to contain at least the nominal volume of 20 L.

Assuming the mean volume dispensed by the machine remains 2005 mL, find the maximum allowable standard deviation of the bottle-filling machine needed to achieve this outcome. Give your answer in millilitres, correct to one decimal place. 3 marks

**e.** A nearby dairy factory claims the milk dispensed into its 2 L bottles varies normally with a mean of 2005 mL and a standard deviation of 2 mL.

When authorities visit the nearby dairy factory and check a random sample of 10 bottles of milk, they find the mean volume to be 2004 mL.

Assuming that the standard deviation of 2 mL is correct, carry out a one-sided statistical test and determine, **stating a reason**, whether the nearby dairy's claim should be accepted at the 5% level of significance. 2 marks

**END OF QUESTION AND ANSWER BOOK**

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2017**

# SPECIALIST MATHEMATICS

## Written examination 2

### FORMULA SHEET

**Instructions**

| |
|---|
| This formula sheet is provided for your reference.<br>A question and answer book is provided with this formula sheet. |

| |
|---|
| **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.** |

# Specialist Mathematics formulas

## Mensuration

| | |
|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder | $2\pi rh$ |
| volume of a cylinder | $\pi r^2 h$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid | $\frac{1}{3}Ah$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| area of a triangle | $\frac{1}{2}bc\sin(A)$ |
| sine rule | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule | $c^2 = a^2 + b^2 - 2ab\cos(C)$ |

## Circular functions

| | |
|---|---|
| $\cos^2(x) + \sin^2(x) = 1$ | |
| $1 + \tan^2(x) = \sec^2(x)$ | $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$ |
| $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ | $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$ |
| $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ | $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ |
| $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ | $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$ |
| $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$ | |
| $\sin(2x) = 2\sin(x)\cos(x)$ | $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$ |

## Circular functions – continued

| Function | $\sin^{-1}$ or arcsin | $\cos^{-1}$ or arccos | $\tan^{-1}$ or arctan |
|---|---|---|---|
| **Domain** | $[-1, 1]$ | $[-1, 1]$ | $R$ |
| **Range** | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$ | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

## Algebra (complex numbers)

| | |
|---|---|
| $z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$ | |
| $\lvert z \rvert = \sqrt{x^2 + y^2} = r$ | $-\pi < \text{Arg}(z) \le \pi$ |
| $z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ | $\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$ |
| $z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem) | |

## Probability and statistics

| | |
|---|---|
| for random variables $X$ and $Y$ | $\text{E}(aX + b) = a\text{E}(X) + b$<br>$\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$<br>$\text{var}(aX + b) = a^2\text{var}(X)$ |
| for independent random variables $X$ and $Y$ | $\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$ |
| approximate confidence interval for $\mu$ | $\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$ |
| distribution of sample mean $\bar{X}$ | mean $\text{E}(\bar{X}) = \mu$<br>variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$ |

## Calculus

| | |
|---|---|
| $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ | $\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$ |
| $\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$ | $\int e^{ax}dx=\frac{1}{a}e^{ax}+c$ |
| $\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$ | $\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$ |
| $\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$ | $\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$ |
| $\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$ | $\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$ |
| $\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$ | $\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$ |
| $\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$ | $\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$ |
| $\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$ |
| $\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$ |
| | $\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$ |
| | $\int (ax+b)^{-1}dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$ |
| product rule | $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ |
| quotient rule | $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ |
| chain rule | $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ |
| Euler's method | If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$ |
| acceleration | $a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ |
| arc length | $\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$ |

## Vectors in two and three dimensions

| |
|---|
| $\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$ |
| $\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$ |
| $\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$ |
| $\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$ |

## Mechanics

| | |
|---|---|
| momentum | $\underset{\sim}{p}=m\underset{\sim}{v}$ |
| equation of motion | $\underset{\sim}{R}=m\underset{\sim}{a}$ |

**END OF FORMULA SHEET**