2017 VCE Further Mathematics 1 examination report

General comments

Overall, students seemed well prepared for Further Mathematics examination 1 in 2017. However, students seemed to struggle with questions involving elementary application of the key skills and key knowledge from the study design. Routine practice of these skills is important.

Specific information

The tables below indicate the percentage of students who chose each option. The correct answers are indicated by shading.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Core

In 2017, the Core section comprised two components: Data analysis (Questions 1–16) and Recursion and financial modelling (Questions 17–24).

Question	% A	% B	% C	% D	% E	% No answer
1	1	2	4	93	0	0
2	2	58	38	2	0	0
3	1	3	7	88	2	0
4	7	22	4	62	5	0
5	21	76	2	1	1	0
6	6	3	12	8	71	0
7	4	8	36	6	46	0
8	3	43	6	39	9	0
9	52	13	11	10	13	0
10	58	14	13	9	6	1
11	5	10	21	59	4	0
12	2	6	10	81	2	1
13	4	51	32	2	10	0
14	3	12	6	53	26	0
15	10	5	10	64	10	0
16	32	11	22	27	6	1
17	82	9	3	2	4	0
18	2	3	2	92	1	0
19	5	15	61	8	9	1
20	15	14	11	46	13	1
21	22	10	9	5	53	1
22	15	65	7	4	8	0
23	3	35	15	38	7	1
24	4	58	16	15	6	1

Data analysis

Students generally answered questions in this section very well, particularly questions that required standard, routine calculations (Questions 1, 3, 5 and 15). Questions that required the use or analysis of graphical information were less well answered (Questions 2, 8, 13 and 14). Questions requiring analysis and interpretation of statistical information were answered particularly well (Questions 6 and 12).

Question 2

Students were required to interpret the provided boxplot and choose the correct five-number summary for the data. While many students did this correctly, a large number of students ignored the outlier points when determining the maximum and minimum values for the five-number summary, leading to the choice of option C, which was incorrect. Even though the points are identified as outliers, they are still valid data points within the data set and must be used as maximum and minimum values if appropriate.

Question 4

The use of a $\log_{10}$ scale on the horizontal axis for the histogram caused difficulty for some students. Many students identified the location of the median correctly (between 3 and 4 on the horizontal axis) but some did not convert these $\log_{10}$ values to actual area values (1000 and 10 000), leading to the choice of option B, which was incorrect.

Question 7

This question required students to identify the type of variable for each of those given in the question. While almost half of the students did so correctly, many incorrectly chose option C, evidently because they incorrectly assumed that the variable number of moths was numerical because it involved numbers. The use of numbers in a variable definition does not automatically make the variable numerical, and students should be careful to analyse whether the numbers are referring to categories for that variable, as was the case in this question.

Question 8

Students were asked to identify the equation of the least squares line drawn on a graph that contained 13 points. While it was not possible to determine exact slope and intercept values from the graph, students should have been able to approximate these values. Many students incorrectly assumed that the intercept value of the line was 17.4, read directly from the graph; however, this is only possible if the horizontal axis begins at value zero. Students are encouraged to look very carefully at graphs before choosing what might seem to be the obvious answer.

Question 13

This question presented students with a time series graph and required them to identify the correct description. Around half of the students answered this question correctly; however, many others incorrectly identified the time series graph as showing seasonality. The existence of ‘peaks’ and ‘troughs’ is not enough information to determine seasonality; they must exist with regular intervals of time between them. Students needed to observe that the peaks did not occur with this regularity and thus reject the existence of seasonality.

Question 16

This question required students to analyse the percentage change when adjusting a value for seasonality. While some students answered this question correctly, many had difficulty completing this analysis. A sales figure would be divided by the seasonal index of 1.6, which is the equivalent of multiplying by 0.625, since $\frac{1}{1.6} = 0.625$. The deseasonalised value will be 62.5% of the original value, which means the original value has been reduced by 100% – 62.5% = 37.5%

Recursion and financial modelling

Overall, students completed the questions in this section very well. The questions students found most challenging involved more complex calculations (Questions 21, 23 and 24) or writing a recurrence relation (Question 20).

Question 20

This question required students to determine the multiplying factor in the recurrence relation, as well as determine the value of interest-only repayment.

The multiplying factor is $1+\left(\frac{\frac{3.6}{12}}{100}\right)=1.003$

The repayment is $\left(\frac{\frac{3.6}{12}}{100}\right)\times 225\ 000=675$

These calculations led to the correct answer of option D.

Alternatively, students may have realised that the terms in the sequence of balance values generated by the recurrence relation would be constant at $225 000 because each repayment only pays the interest. The balance of the loan after each repayment will be $225 000. Students could have calculated a few terms of each recurrence relation using recursion with technology. Option D was the only recurrence relation that resulted in this constant sequence.

Question 21

Although this was a standard application of unit-cost depreciation, almost half of the students were unable to determine the depreciation per page printed as required. The correct answer could be determined by calculating the overall decrease in value of the printer ($680 − $125 = $555) and then dividing by the number of pages (4 × 1920). Students who divided by 1920 only would get the result 0.289, which may have been incorrectly interpreted as 3 cents, leading many students to choose option A.

Question 23

This question required the interpretation of an amortisation table and the calculation of a missing payment value. Many students seemed to assume that the payment value would be constant at $100 (option B), despite the question stating that the payment may vary. The solution required four steps:

1. Calculation of the missing principal addition by subtracting the balance after payment 19 from the balance after payment 20:

$7500.00 – $7233.83 = $266.17

2. Calculation of the interest rate. This could have been achieved by finding any of the interest payments from the table as a percentage of the previous balance:

$\frac{27.91}{6977.50}\times 100\% = 0.4$ or $\frac{28.42}{7105.41}\times 100\% = 0.3999\ ...$

The very small difference in these two values is due to the rounding of all values in the amortisation table. Either could be used to determine the interest at payment number 20.

3. Calculation of the interest at payment number 20. This could have been achieved by finding the percentage from step 2 of the balance after payment number 19:

$$\frac{0.4}{100} \times \$7233.83 = \$28.94 \quad \text{or} \quad \frac{0.3999}{100} \times \$7233.83 = \$28.92$$

Any level of rounding would result in an interest value of approximately $28 or $29.

4. Calculation of the payment. This could have been achieved by subtracting the interest from the principal addition, both calculated above. If students used more decimal places in the interest rate, their answer would still be closest to the correct answer of $237.

Students must be familiar with the calculations that are used to create an amortisation table and be able to use these to complete questions such as this.

Question 24

This question required the use of a TVM solver application. Many students were able to do so successfully, although it seemed that problems involving changing conditions were particularly challenging for some students.

This question could have been solved by determining the future value of the loan after the first 10 years, using the following TVM entries:

N = 120 (10 years)

I% = 4.35

PV = 245 000

PMT = –1800

FV = ?

P/Y = 12

C/Y = 12

to result in a future value of –$108 219.16

The negative indicates that this amount must still be paid back to the bank.

The second stage of the solution required the determination of the new interest rate using the following TVM entries:

N = 60 (5 years)

I% = ?

PV = 108 219.16 (This value is entered as positive here. It represents the new principal value.)

PMT = –2000

FV = 0 (for fully repaid loan)

P/Y = 12

P/Y = 12

to result in an interest rate of 4.14275… and the correct answer of option B.

Understanding of the sign convention for TVM use is very important, as is the careful tracking of values used in subsequent calculations.

Section B – Modules

Students were required to complete questions from two of the four modules.

The selection of modules by students in 2017 is shown is shown in the table below.

Module	% 2017
Matrices	89
Networks and decision mathematics	51
Geometry and measurement	29
Graphs and relations	31

Module 1 – Matrices

Students completed most of the questions in this module very well. The questions that students found most challenging involved some symbolic analysis (Question 6) and the use of transition matrices or recurrence relations (Questions 7 and 8).

Question	% A	% B	% C	% D	% E	% No Answer
1	1	98	0	0	1	0
2	1	2	3	4	90	0
3	14	8	63	10	4	1
4	5	9	74	7	5	1
5	74	4	10	10	2	0
6	6	13	12	58	10	1
7	36	17	24	12	10	1
8	30	13	20	19	17	1

Question 6

This question defined the elements of two matrices in terms of a rule based on the row and column values. It required students to identify the sum of these two matrices.

Many students were able to answer this question correctly.

The question was most easily answered by adding the two rules together to identify the rule for the sum of the two matrices:

$$a_{ij} + b_{ij} = 2i + j + i - j = 3i$$

The elements in the sum depend only on the row reference, *i*.
The first row elements ($i = 1$) will be $3 \times 1 = 3$.
The second row elements ($i = 2$) will be $3 \times 2 = 6$.
The third row elements ($i = 3$) will be $3 \times 3 = 9$.

leading to the correct answer, option D.

Question 7

Many students struggled to answer this question correctly. The critical information in the question was the fact that the number of each type of fish in the farm remained constant, so that the matrices F_0 and F_1 were the same.

To solve the question, an equation of the given format needed to be constructed and solved to find matrix B.

$$F_{n+1} = \begin{bmatrix} 0.65 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0.20 & 0.95 & 0 \\ 0.10 & 0.05 & 0.05 & 1 \end{bmatrix} \times F_n + B$$

Given that the number of each type of fish is constant, F_{n+1} and F_n will be equal to $F_0 = \begin{bmatrix} 50000 \\ 10000 \\ 7000 \\ 0 \end{bmatrix}$.

$$\begin{bmatrix} 50000 \\ 10000 \\ 7000 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.65 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0.20 & 0.95 & 0 \\ 0.10 & 0.05 & 0.05 & 1 \end{bmatrix} \times \begin{bmatrix} 50000 \\ 10000 \\ 7000 \\ 0 \end{bmatrix} + B$$

after which the equation is solved to find $B = \begin{bmatrix} 17500 \\ -10000 \\ -1650 \\ -5850 \end{bmatrix}$

At this stage of solution, the question required students to correctly interpret the signs of the elements in matrix *B*. A positive value means fish are added; a negative value means fish are sold or removed. The element in the third row of this matrix shows that 1650 adult fish must be sold if the number of each fish remains constant, leading to the correct answer, option A.

Question 8

Many students seemed to struggle with the complexity of this question. Two of the unknown values in the matrix *T* could be found from the elementary concept of transition matrices – that is, the column values must add to one.

$X = 1 – 0.3 – 0.2 = 0.5$

$Y = 1 – 0.2 – 0.2 = 0.6$

The solution then required students to set up a matrix equation:

$$\begin{bmatrix} 0.3 & 0.2 & V \\ 0.2 & 0.2 & W \\ X & Y & Z \end{bmatrix} \begin{bmatrix} 40 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} 29 \\ 13 \\ 33 \end{bmatrix}$$

and then multiply to generate the following equations:

$0.3 \times 40 + 0.2 \times 15 + V \times 20 = 29$

$0.2 \times 40 + 0.2 \times 15 + W \times 20 = 13$

$0.5 \times 40 + 0.6 \times 15 + Z \times 20 = 33$

The solutions of these equations led to the remaining unknown values:

$V = 0.7$, $W = 0.1$ and $Z = 0.2$.

An alternative calculation for the value of *Z* could have been $Z = 1 – 0.7 – 0.1 = 0.2$

Once the values of *X*, *Y*, *V*, *W* and *Z* were found, the five options needed to be checked. The only option that was not true was option A.

Module 2 – Networks and decision mathematics

Student understanding of elementary graph terminology was clearly demonstrated in Questions 1, 2 and 3, with most students answering these questions correctly. In other questions, many students were challenged by the routine application of standard calculation techniques and careful observation of graph elements.

Question	% A	% B	% C	% D	% E	% No answer
1	4	92	0	2	2	0
2	9	1	83	4	3	0
3	92	2	1	4	1	0
4	4	14	10	65	7	0
5	8	20	51	13	7	1
6	8	16	26	20	29	1
7	9	27	18	26	19	1
8	13	44	17	19	7	1

Question 4

The critical path analysis in Question 4 involved standard forward-scanning calculations. While there was some complexity of the activity network, students should be able to apply standard routine calculations to graphs such as this with care.

Question 6

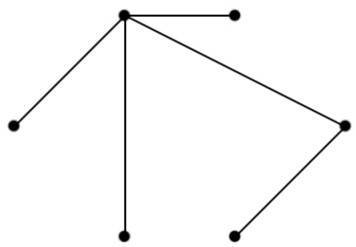
Students should be familiar with scenarios involving adding or removing an edge from a graph to enable an Eulerian trail. This question relied on the knowledge that a graph will have an Eulerian trail if exactly two of the vertices of that graph have an odd degree. The graph presented to the students had four odd-degree vertices and so an edge between any two of these odd-degree vertices could be removed. In this graph, there were five such edges.

Most students answered option B, C or D, suggesting that simple counting errors contributed to the low success of this question.

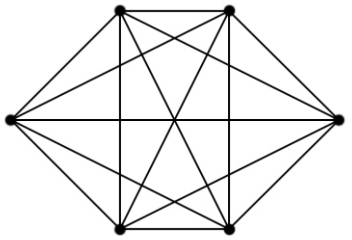
Question 7

This question was not completed well. An effective way to solve this question was to check each of the options separately, with the aid of a diagram.

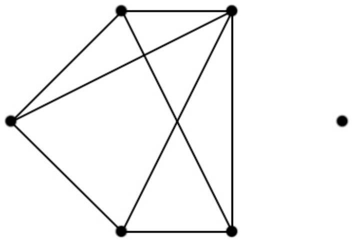
Option A was true because the number of edges in a tree is one less than the number of vertices. A tree with six vertices will have five edges, as shown below. Many different trees are possible but they will all have five edges.



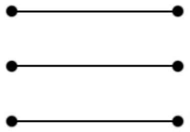
Option B was true because a complete graph of six vertices will have 15 edges, as shown below.



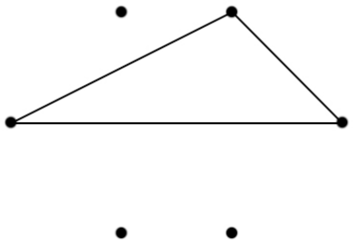
Option C was true because a graph can be drawn that satisfies the given conditions. There are many alternatives, but one possible graph is shown below.



Option D was not true and was therefore the correct answer to the question. A bipartite graph can be drawn that has six vertices and less than nine edges as shown below.



Option E was true. A cycle has no repeated edges and so, in order to begin and end at the same vertex, it must have at least three vertices and three edges, as shown below.



Question 8

The solution can be obtained by substitution of the three different values of x from the available options, that is $x = 1$, $x = 2$ and $x = 3$.

Cut E will always have capacity 27.
Cut D will always have capacity 32 (since the edge with weight x is not included in the calculation)

If $x = 1$: Cut A will have capacity 26
Cut B will have capacity 24
Cut C will have capacity 25

If $x = 2$: Cut A will have capacity 27
Cut B will have capacity 25
Cut C will have capacity 26

If $x = 3$: Cut A will have capacity 28
Cut B will have capacity 26
Cut C will have capacity 27

Each of the options could then be checked.

- Option A: maximum flow will be Cut B, not Cut A.
- Option B: maximum flow will be Cut B as stated (correct answer).
- Option C: maximum flow will be Cut B, not Cut C.
- Option D: maximum flow will be Cut B, not Cut D.
- Option E: maximum flow will be Cut B, not Cut E.

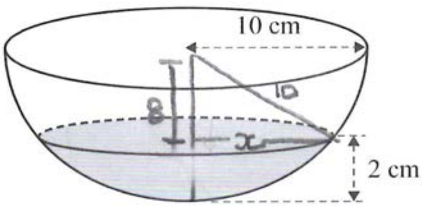
Module 3 – Geometry and measurement

Questions involving elementary application of the key skills from this module were answered correctly by the majority of students. The spherical geometry question (Question 6) and the application of the sine rule (Question 7) caused difficulty for many students. Question 8 was also challenging for many students.

Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	4	92	1	0
2	1	3	2	92	2	0
3	13	63	16	3	4	2
4	4	80	6	5	3	1
5	6	2	81	6	5	0
6	9	54	23	6	7	1
7	17	7	8	10	57	1
8	35	24	25	11	5	1

Question 6

This question provided a diagram of a hemispherical bowl containing water. In order to calculate the required radius of the surface of that water, students needed to apply Pythagoras's theorem to the triangle, as shown in the diagram below.



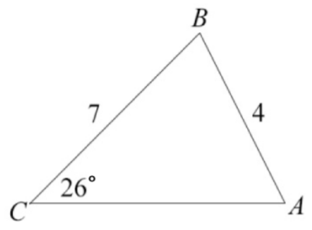
$10^2 = 8^2 + x^2$
$x^2 = 100 - 64$
$x^2 = 36$
$x = 6$
leading to the correct answer, option B.

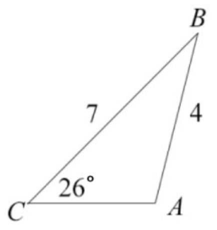
Students should have carefully noted the existence of multiple radius measurements within the hemisphere (10 cm). These lengths are often the key to solving spherical geometry questions such as this.

Question 7

Given that this question required students to draw their own diagrams from given information, it was very well completed, with over half of the students answering it correctly. The question required an understanding of the ambiguous nature of the sine rule and subsequent application of trigonometric rules to determine valid angles in order to determine the angle that was not valid from the five options.

From the given information, two different triangles could have been drawn.





The sine rule could have been applied to determine ∠*CAB*:

$$\frac{\sin(A)}{7} = \frac{\sin(26)}{4}$$

$$\sin(A) = \frac{7 \times \sin(26)}{4}$$

$$A = 50° \text{ or } 130°$$

leading to the third angle in the triangles as

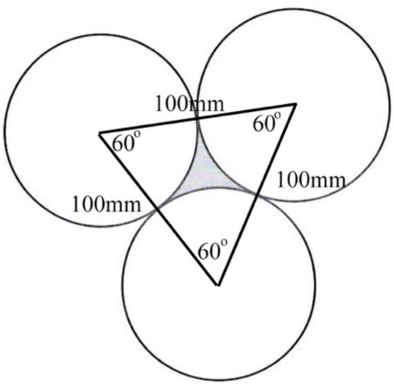
$B = 180° - 26° - 50° = 104°$ or $B = 180° - 26° - 130° = 24°$

The only angle that was not possible in these triangles was therefore 144° (option E).

Question 8

This question appeared to be challenging for a number of students, possibly because the solutions required multiple steps of calculation.

When the centres of the circles are joined together, an equilateral triangle is formed.



This triangle has side lengths of 100 mm (twice the radius of the circle) and internal angles of 60°.

The area of the triangle can be calculated:

Area triangle = $\frac{1}{2} \times 100 \times 100 \times sin(60°)$

The area of one sector of a circle inside that triangle can be calculated:

Area sector = $\frac{60}{360} \times \pi \times 50^2$

The shaded area is the space left behind when three sectors are removed from the triangle:

Shaded area = area triangle – 3 $\times$ area sector

Shaded area = $\frac{1}{2} \times 100 \times 100 \times sin(60°) - 3 \times \frac{60}{360} \times \pi \times 50^2$ = 403.136 …

The correct answer was option A as the calculated answer was closest to 403.

Module 4 – Graphs and relations

Overall, students completed this module very well. Many students were challenged by the question involving inequalities (Question 5) and the application of the sliding line technique for linear programming (Question 8).

Question	% A	% B	% C	% D	% E	% No answer
1	6	69	6	6	14	0
2	1	7	89	2	1	0
3	6	77	5	10	1	1
4	3	1	4	19	72	1
5	19	18	10	31	22	1
6	3	7	79	7	3	1
7	9	14	50	19	7	1
8	16	14	14	44	12	1

Question 1

The easiest solution method was to plot the points on a graph to recognise the line as horizontal through $y = 4$. Students who relied on an algebraic method should have determined the same result. Students must ensure that elementary skills are thoroughly practised and understood.

Question 5

Some students likely had difficulty interpreting the inequality information from this question. One way to approach the solution of this question was to consider the number of children enrolled (y), broken into groups of 15 $\left(\frac{y}{15}\right)$. There must be one teacher ($1x$) for each of these groups, but there could be more, resulting in the inequality $x \geq \frac{y}{15}$. This was not one of the five options, but could have been rearranged as $y \leq 15x$, which was the correct answer, option D.

Question 8

Many students seemed to have understood the application of the sliding-line technique to this question. Students should practise this technique well.

The gradient of the objective function can be determined as $\frac{m}{n}$. Each answer option needed to be checked individually by substituting the given values of m and n to find each gradient. Drawing or imagining a line with this gradient on the graph allows the given condition to be checked. The question required the identification of the option that was **not** true.

For example, the incorrect option B could have been checked with a line of gradient $\frac{1}{6}$. A ruler placed on the graph and passing through (0,1) and (6,0) could have been slid along the graph to show that the minimum value of *Z* would occur at point *B* as suggested. The statement in option B was true and therefore was not the required answer to the question.

The correct option was option D. This option could have been checked with a line of gradient $\frac{2}{6} = \frac{1}{3}$, passing through (0,1) and (3,0). The maximum value should be at point B, but the statement said it was at point D. This was the option statement that was not true and thus was the correct answer to the question.