

# 2018 VCE Further Mathematics 2 examination report

## General comments

Students were required to complete:

- a compulsory Core section of Data Analysis (worth 24 marks)
- a compulsory Core section of Recursion and financial modelling (worth 12 marks)
- two selected modules (worth 12 marks each).

The selection of modules by students in 2018 is shown in the table below.

Module	% 2018
Matrices	88
Networks and decision mathematics	50
Geometry and measurement	29
Graphs and relations	33

Students are expected to be familiar with the formula sheet provided with the examination.

Many students gave answers that did not relate to the question asked. Students are strongly encouraged to read the question again after they have completed their answer to ensure that they are answering appropriately. For example, in Question 1f. of the Core section when asked to describe the shape of the distribution, some students gave additional information concerning the centre and spread. Attention needed to be given to the type of statistical description required.

Many questions on the examination were worth one mark, and with these questions the mark was awarded for a correct answer. Generally there was no need to put the answer into a sentence. For example, in Question 2e. of the Core the required answer was 85%, but students were not required to describe the meaning of this percentage.

Some questions required the further application of a previous answer. If the previous answer was incorrect, the student may have been eligible for consequential error consideration. For this to apply, working out needed to show a correct substitution of the previous reasonable, but incorrect, answer into a relevant calculation. The resulting answer then needed to match that substitution and also be a reasonable answer. For example, in Question 3b. of the Geometry and Measurement module, a student who used Pythagoras's theorem appropriately using their incorrect answer from Question 3a. was able to be awarded the consequential mark.

For questions worth more than one mark, students are strongly advised to show working. An incorrect answer on its own will not be awarded any marks in a two-mark question; however, often

a method mark can be awarded for an intelligible attempt. For example, in Question 2d. of the Core section, an answer of +1.8 without any working was not awarded any marks.

When a descriptive answer is required, brevity is encouraged in the response.

Some questions on the examination asked students to ‘show that’ a particular answer could be obtained. Students must work towards the given result with all relevant steps shown in their response. The answer the student is asked to show may be relevant in a subsequent part of the question.

Rounding of answers to a specified level of accuracy is an important skill that students must be able to demonstrate. When completing the equation of a least squares line, some students did not understand the difference between decimal places rounding and significant figures rounding.

In the Recursion and financial modelling module students sometimes continued working from a correct answer to an incorrect answer because they rounded to the nearest 5 cents or 10 cents. For example, in Question 5b. of the Core section, the required answer was \$5484.23 and some students incorrectly rounded further to \$5484.20 or \$5484.25.

In questions where no instruction to round was given, an exact answer was required as rounding did not apply. For example, in Question 2c. of the Core section the required answer was 63.8% and since rounding did not apply an answer of 64% could not be accepted.

## Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

## Core

### Data analysis

#### Question 1a.

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	28	72	<b>0.7</b>

3 (city, congestion level, size)

The definition of a categorical variable was not always understood.

**Question 1b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	29	71	

2 (congestion level, size)

Most students were able to answer this question correctly, but 1 and 3 were also common answers.

**Question 1c.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	20	80	

Newcastle-Sunderland and Liverpool

It was evident that some students did not read the question carefully as they gave the full list of all cities with a medium level of congestion instead of giving only the large cities with a medium level of congestion as required by the question.

**Question 1d.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>Average</b>
<b>%</b>	1	5	94	

<b>Congestion level</b>	<b>City size</b>	
	<b>Small</b>	<b>Large</b>
high	4	2
medium	4	2
low	8	3
<b>Total</b>	16	7

**Question 1e.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	19	81	

25%

This question was answered well, although some students gave 17% by taking the percentage of all 23 cities.

**Question 1f.**

Marks	0	1	Average
%	13	87	0.9

Positively skewed

This question was answered well; however, some students went on to comment on the centre and spread, which was not required and made it difficult to determine if the student understood which had been asked for. The mark could not be awarded in this case.

**Question 1g.**

Marks	0	1	Average
%	19	81	0.8

52.5

This question was answered well; however, some students added the relationship between the data value and the upper fence, which was not part of this question.

**Question 2a.**

Marks	0	1	2	Average
%	30	29	41	1.1

Median percentage congestion level for morning peak period

52%

Median percentage congestion level for evening peak period

56%

This question was not answered well. Some students gave the correct median percentage for the morning peak period but not for the evening peak period as they found the median 'x' value and then wrote the 'y' value of that data point as their median 'y' value.

**Question 2b.**

Marks	0	1	Average
%	10	90	0.9

Evening congestion level

**Question 2c.**

Marks	0	1	Average
%	22	78	0.8

63.8%

Rounding did not apply, so an answer of 64% was not accepted.

**Question 2d.**

Marks	0	1	2	Average
%	36	21	42	1.1

-1.8

Some students did not answer this question well. Many students did not find the actual value of 50 from the graph and used 47 instead. Some found 51.814 but did not know how to proceed further.

**Question 2e.**

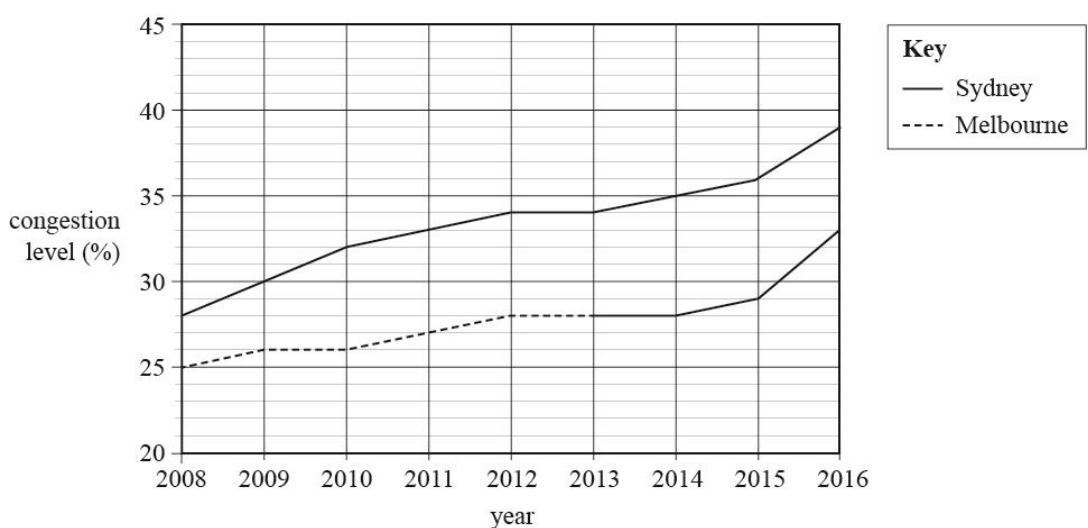
Marks	0	1	Average
%	41	59	0.6

85%

92% was a very common incorrect answer.

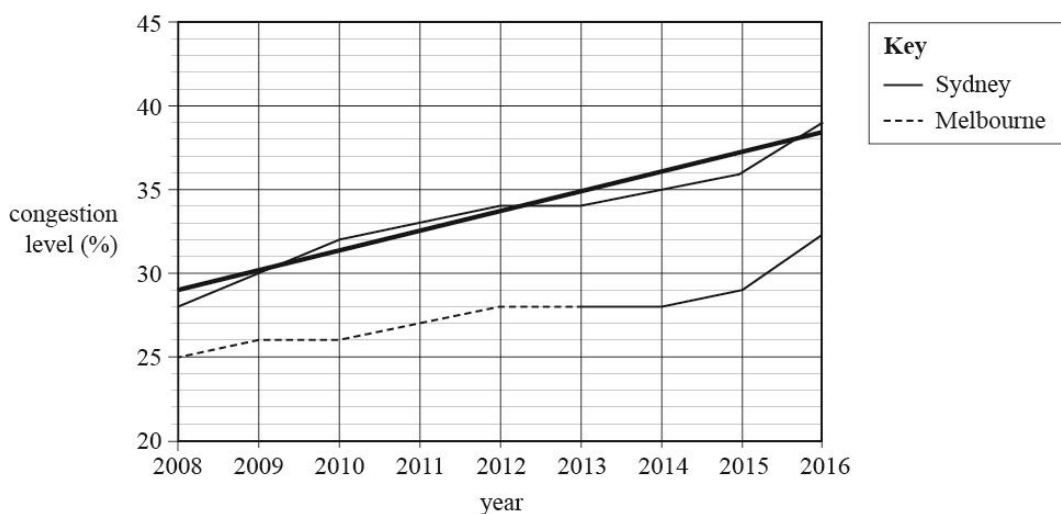
**Question 3a.**

Marks	0	1	Average
%	12	88	0.9



**Question 3bi.**

Marks	0	1	Average
%	49	51	0.5



A significant number of students did not answer this question. Those who answered correctly typically used a ruler and often also wrote down the coordinates of the two endpoints as (2008, 29.2) and (2016, 38.4).

**Question 3bii.**

Marks	0	1	Average
%	64	36	0.4

1.15% per year

This question was not well answered as many students did not realise that the slope of the least squares line was a measure of the average rate of increase. Some seemed to get confused with the R value used in financial mathematics and gave 15%.

**Question 3biii.**

Marks	0	1	Average
%	26	74	0.8

2020

**Question 3c.**

Marks	0	1	Average
%	34	66	0.7

-1515

This question was reasonably well answered; however, some gave -1514 or gave an answer rounded to four decimal places.

**Question 3d.**

Marks	0	1	2	Average
%	22	21	57	1.4

$$\text{congestion level} = \boxed{-1515} + \boxed{0.7667} \times \text{year}$$

Some students did not recognise the link with Question 3c. and wrote a different number in the first box. Some who correctly rounded in Question 3c. used -1514.7556 in Question 3d.

**Question 3e.**

Marks	0	1	2	Average
%	67	31	2	0.4

2008 Sydney value higher,  $29.2 > 24.5$ , and Sydney line slope greater,  $1.15 > 0.7667$

Only a small proportion of students answered the question entirely correctly.

Many students focused on the starting values for Sydney and Melbourne from the table rather than the least squares lines. Some gave gradient figures for Sydney and Melbourne but did not clearly say that Sydney's slope was greater.

Again some students seemed to get confused with the R value used in financial mathematics and described Melbourne's gradient of 0.7667 as a decrease of about 23%.

## Recursion and financial modelling

### Question 4a.

Marks	0	1	Average
%	4	96	1

\$12 000

### Question 4bi.

Marks	0	1	Average
%	11	89	0.9

$$V_1 = 1.0062 \times 12\,000$$

This question was answered well, although some students used methods other than simple recursion, which were not acceptable.

### Question 4bii.

Marks	0	1	Average
%	18	82	0.8

4 months

### Question 4ci.

Marks	0	1	Average
%	22	78	0.8

$$\text{balance} = \boxed{12\,000} \times \boxed{1.0062}^n$$

A common error was for students to write the two numbers in the incorrect boxes.

### Question 4cii.

Marks	0	1	Average
%	35	65	0.7

36

### Question 5a.

Marks	0	1	Average
%	45	55	0.6

15%

This question was not well answered. Some students gave the R value of 0.85.

**Question 5b.**

Marks	0	1	2	Average
%	53	13	34	<b>0.8</b>

\$5484.23

Some students obtained one mark for correctly finding the value after three years.

Many depreciated by 8.6% for the entire eight years.

**Question 6a.**

Marks	0	1	Average
%	59	41	<b>0.4</b>

4.68%

A common error was 0.18%, which did not account for the fortnightly payments. Perpetuities was an area that requires improvement for many students.

**Question 6bi.**

Marks	0	1	Average
%	52	48	<b>0.5</b>

\$2800

**Question 6bii.**

Marks	0	1	2	Average
%	80	9	11	<b>0.3</b>

\$20 488.89

Balance start Year 2: 480 242.25

Balance end Year 2: 467 131.14

Balance reduction 13 111.11

Payments in Year 2 =  $12 \times 2800 = 33\,600$

Interest =  $33\,600.00 - 13\,111.11 = 20\,488.89$

An answer of \$20 488.88, based on an unrounded value from Question 6bi., was also awarded full marks.

Many students did not attempt this question. Many found the future value after two years of \$467 131 but could not proceed further.

A few students tried unsuccessfully to use the annuities formula.



## Section B – Modules

### Module 1 – Matrices

#### Question 1a.

Marks	0	1	Average
%	1	99	1

\$2.87

#### Question 1b.

Marks	0	1	Average
%	6	94	1

$3 \times 1$

#### Question 1c.

Marks	0	1	Average
%	34	66	0.7

$[1 \ 0 \ 2]$

Some students gave the correct numbers but in a column matrix.

#### Question 2a.

Marks	0	1	Average
%	38	62	0.6

$\begin{bmatrix} 2184 \\ 1782 \\ 1666 \end{bmatrix}$

Some students gave only one or two of the correct elements.

#### Question 2b.

Marks	0	1	Average
%	60	40	0.4

$\begin{bmatrix} 1.04 & 0 & 0 \\ 0 & 0.99 & 0 \\ 0 & 0 & 0.98 \end{bmatrix}$

Most students recognised that a  $3 \times 3$  matrix was required. Numbers such as 0.04,  $-0.01$  and  $-0.02$  were often included in students' answers.

**Question 3a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	52	48	

70 km

Many students gave 390 as their answer, indicating that they had not read the question fully.

Some gave 10% or 0.1 rather than finding the length.

**Question 3b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	23	77	

$$\boxed{0.2} \times 700 + \boxed{0.1} \times 400 + \boxed{0} \times 200 + \boxed{0.2} \times 1400 = 460$$

Some students listed the numbers in the G column rather than the row.

**Question 3c.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	18	82	

390 and 360

**Question 3d.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	83	17	

48.5%

This question was not answered well, with most students giving 97%, which did not take account of the 0.5 change.

**Question 3e.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	61	39	

56.7%

Some students left the answer as 1532.1 and did not convert to the required percentage.

**Question 4a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	61	39	

30

Many students did not recognise that the 2700 km of highway remained constant throughout.

**Question 4b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	77	23	<b>0.3</b>

457 km

Students who found  $k$  correctly in part a. were often able to answer this question correctly. Some wrote the correct matrix for 2022 only and did not extract the number 457 as their answer.

**Module 2 – Networks and decision mathematics****Question 1ai.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	10	90	<b>0.9</b>

9

**Question 1aii.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	23	77	<b>0.8</b>

13

Some gave an answer of 14 by not allowing for the 1 against the flow.

**Question 1b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	68	32	<b>0.3</b>

7

This question was not answered well. The answer of 9 from part a. was often repeated.

**Question 2a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	4	96	<b>1</b>

F

**Question 2b.**

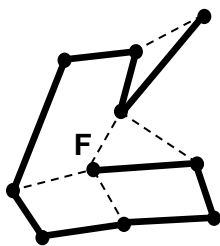
<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	46	54	<b>0.6</b>

2

Some students gave a definition of an Eulerian trail rather than stating how many extra edges were required.

**Question 2c.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	38	62	



This is one example of numerous possible paths that begin at F, have 9 edges and include all other vertices.

A reasonable number of students answered this question correctly; however, some paths returned to F, creating a circuit.

**Question 3a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	36	64	

10

The most common incorrect answer given by students was 8, which indicated that they did not recognise that the longest path to I was required.

**Question 3b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	24	76	

*B-E-G-H-J*

**Question 3c.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	55	45	

A and C

This question was not answered well, with many combinations of incorrect answers given by students.

**Question 3d.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	75	25	

end of activity

E

to the start of activity

J

This question was not answered well. Some students could identify  $E$  as the starting point of the required edge but could not identify  $J$  as the end point.

**Question 4a.**

Marks	0	1	Average
%	69	31	0.3

2

An Eulerian circuit was required; therefore, two extra edges were needed to make all vertices of even degree. Common incorrect answers were 3 and 4.

**Question 4b.**

Marks	0	1	Average
%	86	14	0.2

108 km

The extra two edges ( $PQ$  and  $ST$ ) needed to be added to the sum of all distances (86) on the graph.

This question was not well answered, with many different incorrect responses given, including 109 and 113.

A number of students incorrectly calculated the minimum distance to visit all the towns (Hamiltonian) rather than check all the roads (Eulerian).

**Module 3 – Geometry and measurement****Question 1a.**

Marks	0	1	Average
%	14	86	0.9

6.8 cm

**Question 1b.**

Marks	0	1	Average
%	25	75	0.8

508.4 cm<sup>2</sup>**Question 1ci.**

Marks	0	1	Average
%	13	87	0.9

$$V = \frac{4}{3}\pi \times 3.4^3$$

Some students used a power of 2 rather than 3. The correct calculation needed to involve  $\rho$  and not an approximate value such as 3.14.

**Question 1cii.**

Marks	0	1	Average
%	18	83	0.8

$$V = \pi \times 3.4^2 \times 20.4$$

**Question 1ciii.**

Marks	0	1	Average
%	26	74	0.8

$$247 \text{ cm}^3$$

Some students allowed for only one tennis ball and gave 576 as their answer.

**Question 2a.**

Marks	0	1	Average
%	71	29	0.3

13 hours 30 minutes

This question was not answered well. Common incorrect answers included 9 hours 30 minutes where time difference was not considered, and 5 hours 30 minutes where the time difference was subtracted. 13.3 hours was not an equivalent form of 13:30 hours and could not be accepted.

**Question 2bi.**

Marks	0	1	Average
%	48	52	0.5

$$r = 6400 \times \cos(11^\circ)$$

Equivalent forms were accepted.

**Question 2bii.**

Marks	0	1	Average
%	67	33	0.4

1754 km

Many students used 6400 rather than 6282 in the calculation, giving 1787 as the final answer.

**Question 3a.**

Marks	0	1	Average
%	27	73	0.8

18.8 m

**Question 3b.**

Marks	0	1	Average
%	40	60	0.6

19 m

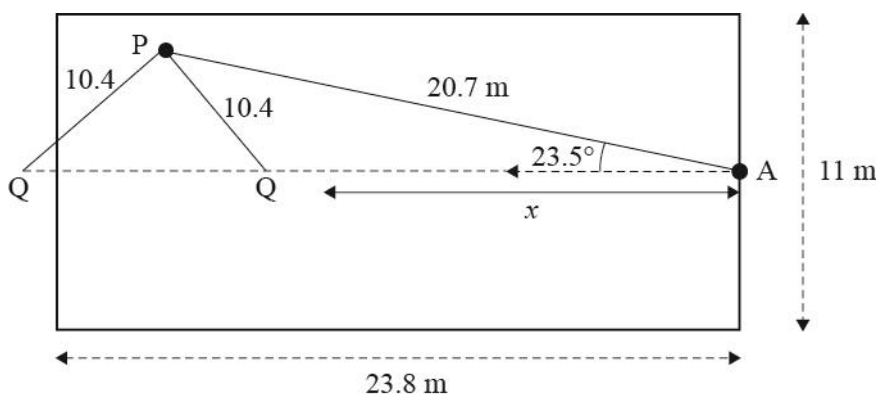
A consequential mark was available to those who used their incorrect result for part a.

**Question 3ci.**

Marks	0	1	Average
%	75	25	<b>0.3</b>

52.5° and 127.5°

This involved the ambiguous case of the sine rule, solving  $\frac{\sin(Q)}{20.7} = \frac{\sin(23.5^\circ)}{10.4}$



This question was not answered well, with some students finding only the 52.5° solution.

**Question 3cii.**

Marks	0	1	Average
%	82	18	<b>0.2</b>

13 m

Students needed to choose the 127.5° angle and then solve for x.

$$\frac{x}{\sin(29^\circ)} = \frac{10.4}{\sin(23.5^\circ)}$$

Some students used the 52.5 angle to find 25 metres, which was outside the court.

## Module 4 – Graphs and relations

**Question 1a.**

Marks	0	1	Average
%	8	92	<b>0.9</b>

Day 2

**Question 1b.**

Marks	0	1	Average
%	22	78	<b>0.8</b>

Days 16 and 17

This question was well answered; however, some students did not give consecutive days.

**Question 2a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	47	53	

$$\frac{567}{20}$$

It was not sufficient to substitute in values or to give the equation and write 'solve'.

**Question 2b.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	12	88	

5.67 g

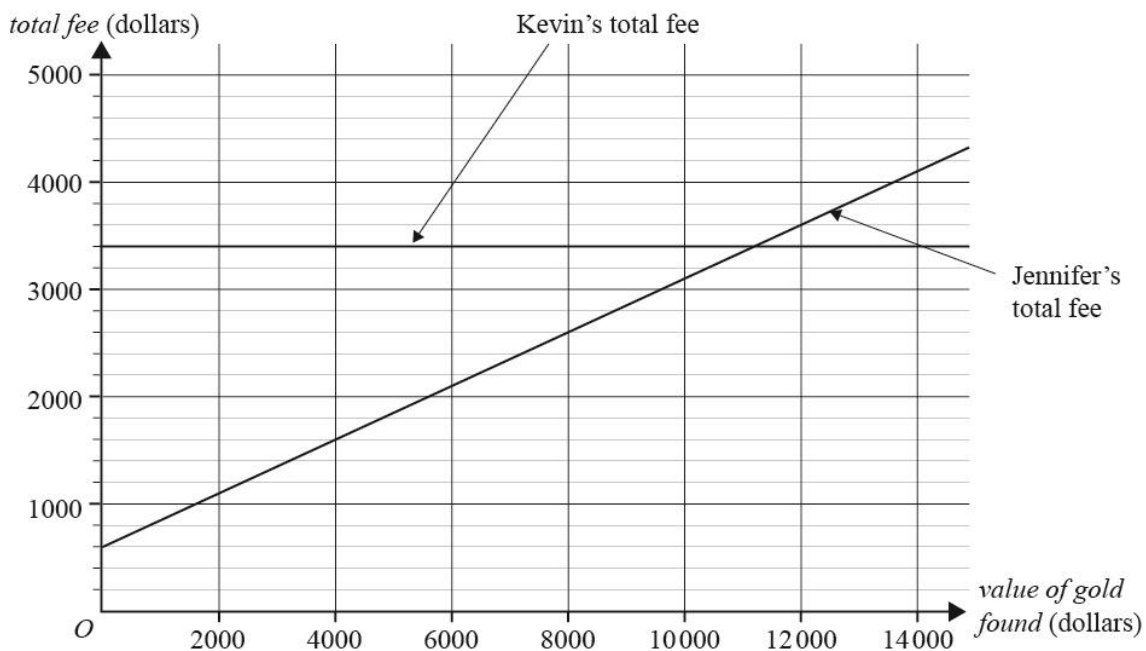
**Question 2c.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	33	67	

8 ounces

**Question 3a.**

<b>Marks</b>	<b>0</b>	<b>1</b>	<b>Average</b>
<b>%</b>	40	60	



Incorrect lines often began at the origin.



**Question 3b.**

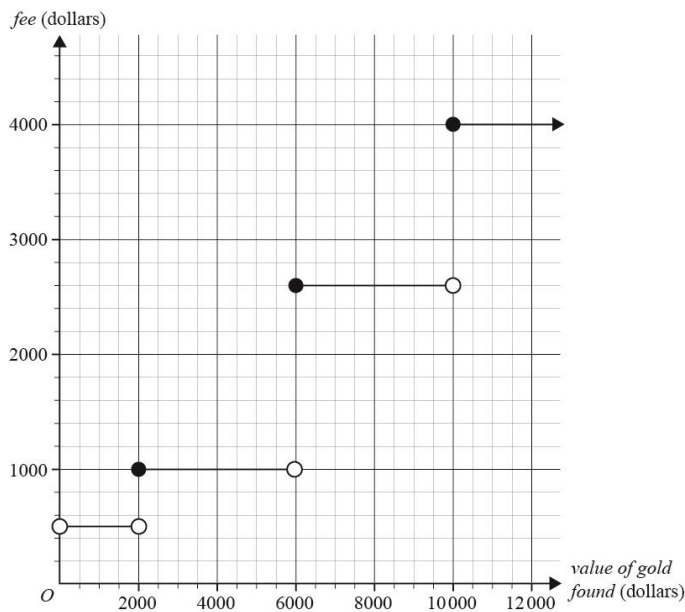
Marks	0	1	Average
%	59	41	<b>0.4</b>

\$11 200

Many students tried to read off the graph and gave an incorrect answer of \$11 000.

**Question 3c.**

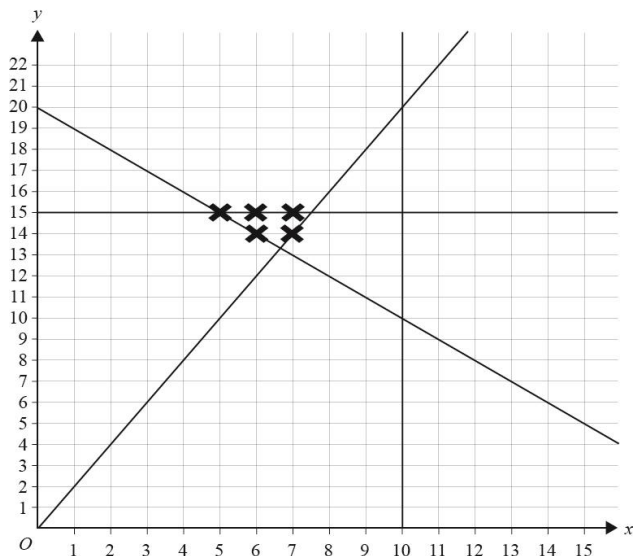
Marks	0	1	2	Average
%	13	15	73	<b>1.6</b>



Generally the two line segments were drawn in the right positions with mistakes tending to be with the open and closed circles at the endpoints.

**Question 4a.**

Marks	0	1	Average
%	70	30	0.3



This question was not answered well, with some students shading a region rather than marking five points.

Some students incorrectly thought the required points must always be the intersection points of boundary lines.

**Question 4b.**

Marks	0	1	Average
%	71	29	0.3

\$2560

Maximum cost at (7, 15)

$$C = 130 \times 7 + 110 \times 15 = 2560$$

This question was not answered well. Most students were able to generate a cost function and substitute into it but did not have the correct integer values.

**Question 4c.**

Marks	0	1	Average
%	78	22	0.2

\$2740

Minimum cost at (6, 14)

$$C = 130 \times 6 + 140 \times 14 = 2740$$

This question was not answered well. Students who scored well were able to slide a line with a gradient of  $-\frac{13}{14}$  to the first of the five integer points.