Print exam correction: Question 5, part b, Question 6, formatting amended.

Write your **student number** in the boxes above. **Letter**

Mathematical Methods Examination 1

Question and Answer Book

VCE Examination – Wednesday 6 November 2024

- Reading time is **15 minutes**: 9.00 am to 9.15 am
- Writing time is **1 hour**: 9.15 am to 10.15 am

Materials supplied

- Question and Answer Book of 12 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software), or notes of any kind, into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = e^x \cos(3x)$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $f(x) = \log_e\left(x^3 - 3x + 2\right)$.

Find $f'(3)$. 2 marks

Do not write in this area.

Question 2 (3 marks)

Consider the simultaneous linear equations

$$3kx - 2y = k + 4$$
$$(k - 4)x + ky = -k$$

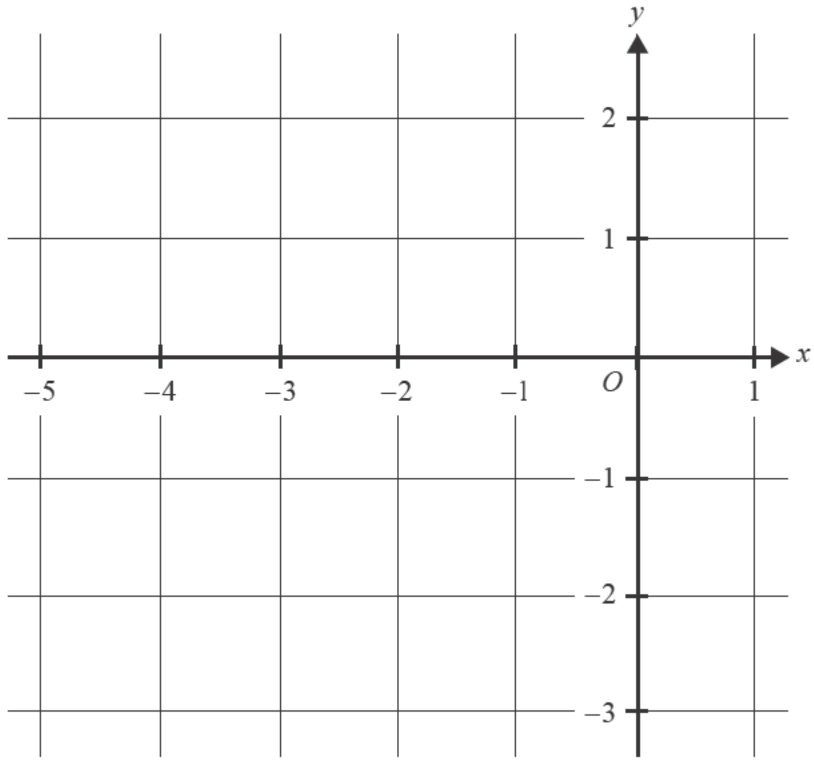
where $x, y \in R$ and k is a real constant.

Determine the value of k for which the system of equations has no real solution.

Question 3 (5 marks)

Let $g : R\backslash\{-3\} \to R,\ g(x) = \dfrac{1}{(x+3)^2} - 2.$

a. On the axes below, sketch the graph of $y = g(x)$, labelling all asymptotes with their equations and axis intercepts with their coordinates. 3 marks



b. Determine the area of the region bounded by the line $x = -2$, the x-axis, the y-axis and the graph of $y = g(x)$. 2 marks

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Question 4 (3 marks)

Let X be a binomial random variable where $X \sim \text{Bi}\left(4, \frac{9}{10}\right)$.

a. Find the standard deviation of X. 1 mark

b. Find $\Pr(X < 2)$. 2 marks

Question 5 (6 marks)

The function $h:[0,\infty)\to R,\ h(t)=\dfrac{3000}{t+1}$ models the population of a town after t years.

a. Use the model $h(t)$ to predict the population of the town after four years. 1 mark

b. A new function, h_1, models a population where $h_1(0)=h(0)$ but h_1 decreases at half the rate of h at any point in time.

State a sequence of two transformations that maps h to this new model h_1. 2 marks

c. In the town, 100 people were randomly selected and surveyed, with 60 people indicating that they were unhappy with the roads.

i. Determine an approximate 95% confidence interval for the proportion of people in the town who are unhappy with the roads. Use $z=2$ for this confidence interval. 2 marks

ii. A new sample of n people results in the same sample proportion.

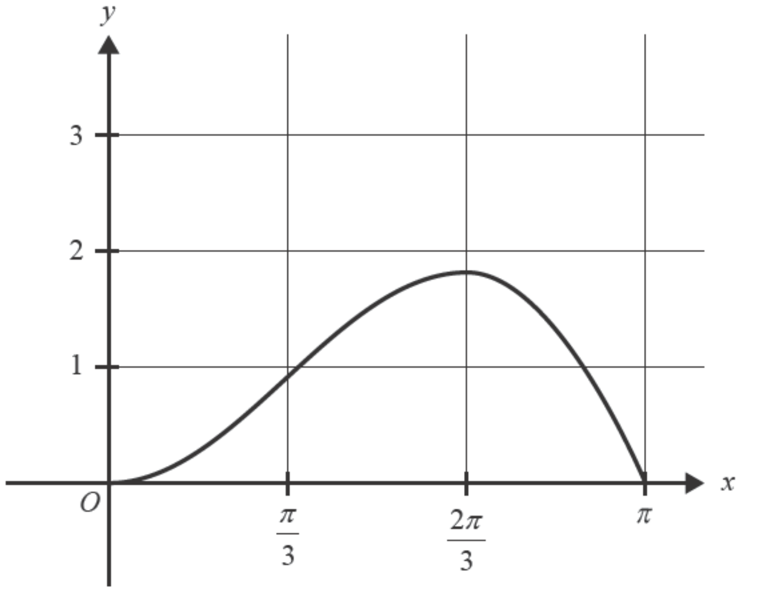
Find the smallest value of n to achieve a standard deviation of $\frac{\sqrt{2}}{50}$ for the sample proportion. 1 mark

Question 6 (4 marks)

Solve $2\log_3(x-4)+\log_3(x)=2$ for x.

Question 7 (9 marks)

Part of the graph of $f:[-\pi,\pi]\rightarrow R, f(x)=x\sin(x)$ is shown below.



a. Use the trapezium rule with a step size of $\frac{\pi}{3}$ to determine an approximation of the total area between the graph of $y=f(x)$ and the x-axis over the interval $x\in[0,\pi]$. 3 marks

b. **i.** Find $f'(x)$. 1 mark

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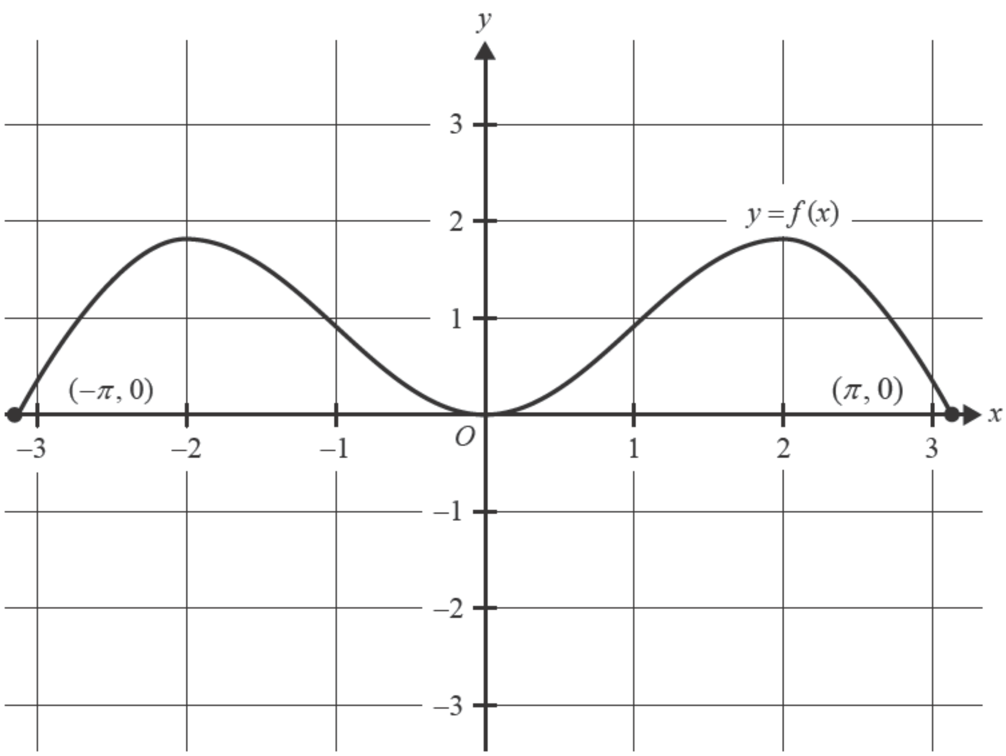
ii. Determine the range of $f'(x)$ over the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. 1 mark

iii. Hence, verify that $f(x)$ has a stationary point for $x \in \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. 1 mark

c. On the set of axes below, sketch the graph of $y = f'(x)$ on the domain $[-\pi, \pi]$, labelling the endpoints with their coordinates.

You may use the fact that the graph of $y = f'(x)$ has a local minimum at approximately $(-1.1, -1.4)$ and a local maximum at approximately $(1.1, 1.4)$. 3 marks



Question 8 (7 marks)

Let $g: R \to R, g(x) = \sqrt[3]{x-k} + m$, where $k \in R \setminus \{0\}$ and $m \in R$.

Let the point P be the y-intercept of the graph of $y = g(x)$.

a. Find the coordinates of P, in terms of k and m. 1 mark

b. Find the gradient of g at P, in terms of k. 2 marks

c. Given that the graph of $y = g(x)$ passes through the origin, express k in terms of m. 1 mark

d. Let the point Q be a point different from the point P, such that the gradient of g at points P and Q are equal.

Given that the graph of $y = g(x)$ passes through the origin, find the coordinates of Q in terms of m. 3 marks

End of examination questions

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Mathematical Methods Examination 1

2024 Formula Sheet

You may keep this Formula Sheet.

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet