2024 VCE Mathematical Methods Examination 1

Marking guidelines and sample responses

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2024 VCE Mathematical Methods Examination 1

Marking guidelines and sample responses

Marking guidelines will indicate the initial criteria that will be used to award marks.

This report provides sample responses, or an indication of what responses may have included.

Question 1a

Answer	1 mark

$$\frac{dy}{dx} = e^x \cos(3x) - 3e^x \sin(3x) = e^x(\cos(3x) - 3\sin(3x))$$

Question 1b

Method	1 mark
Answer	1 mark

$$f'(x) = \frac{1}{x^3 - 3x + 2} \times (3x^2 - 3)$$

$$f'(3) = \frac{3(3)^2 - 3}{3^3 - 3(3) + 2}$$

$$f'(3) = \frac{24}{20} = \frac{6}{5}$$

Question 2

Method	1 mark
Method	1 mark
Answer	1 mark

Using gradients, we require same gradient but different y-intercept

$$y_1 = \frac{3k}{2}x - \frac{(k+4)}{2}$$

$$y_2 = \frac{-(k-4)x}{k} - 1$$

$$\frac{3k}{2} = -\frac{(k-4)}{k} \Rightarrow 3k^2 = -2(k-4) \Rightarrow 3k^2 + 2k - 8 = 0$$

$$\Rightarrow (3k-4)(k+2) = 0$$

$$\Rightarrow k = \frac{4}{3}, k = -2$$

Then check y-intercept

$$\frac{(k+4)}{2} \neq 1$$

$$k + 4 \neq 2$$

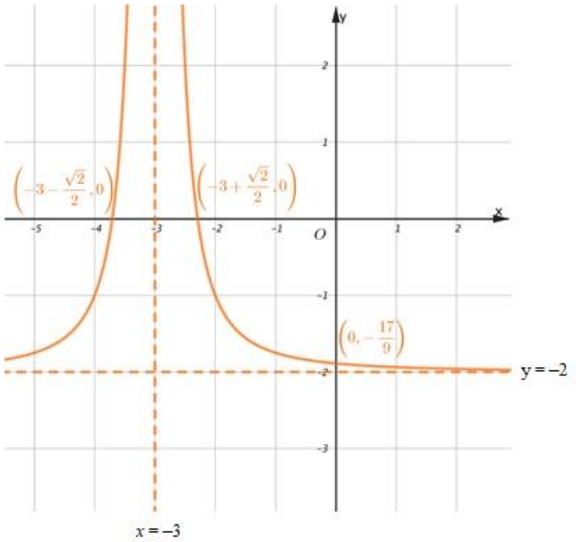
$$k \neq 2$$

So $k = \frac{4}{3}$

Question 3a

Answer	1 mark
Answer	1 mark
Answer	1 mark

Shape (truncus) – drawn with solid line with clear asymptotic behaviour



Asymptotes – labelled correctly and must include '$x =$' and '$y =$' and be dotted/dashed line

Intercepts – labelled and positioned between –4 and –2 correctly, symmetry of x-intercepts

Question 3b

Method	1 mark
Answer	1 mark

$$\int_0^{-2}\left(\frac{1}{(x+3)^2}-2\right)dx$$

$$=\left[-\frac{1}{x+3}-2x\right]_0^{-2}$$

$$=\left(-1-(-4)-\left(-\frac{1}{3}-0\right)\right)$$

$$=\left(-1+4+\frac{1}{3}\right)$$

$$=\frac{10}{3} \text{ or } 3\frac{1}{3} \text{ or } 3.\bar{3} \text{ (recurring decimal)}$$

Question 4a

Answer	1 mark

$$\text{sd}(X) = \frac{3}{5} \text{ or } \frac{6}{10} = 0.6$$

Question 4b

Method	1 mark
Answer	1 mark

$$\Pr(X < 2) = \binom{4}{0}\left(\frac{9}{10}\right)^0\left(\frac{1}{10}\right)^4 + \binom{4}{1}\left(\frac{9}{10}\right)^1\left(\frac{1}{10}\right)^3$$

$$= 1\cdot\frac{1}{10000} + 4\cdot\frac{9}{10}\cdot\frac{1}{1000}$$

$$= \frac{37}{10000} = 0.0037 = \frac{37}{10^4}$$

Question 5a

Answer	1 mark

600

Question 5b

Method	1 mark
Answer	1 mark

$$h'(t) = -\frac{3000}{(t+1)^2}$$

$$\therefore h_1'(t) = -\frac{1500}{(t+1)^2}$$

$h_1(t) = \dfrac{1500}{t+1} + 1500$ to have same population when last measured.

Transformations

Dilation of factor $\frac{1}{2}$ from the t-axis/in h direction/parallel to h-axis

Translation of 1500 units in the positive h direction/up/positive vertical direction

Atlernative

Translation of 3000 units in the positive h direction/up/positive vertical direction

Dilation factor of $\frac{1}{2}$ from the t-axis/in h direction/parallel to h-axis

Question 5ci

Method	1 mark
Answer	1 mark

$$\hat{p} = \frac{60}{100} = \frac{3}{5}$$

$$\left(\frac{3}{5} - 2\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{100}}, \frac{3}{5} + 2\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{100}} \right)$$

$$\left(\frac{30 - 2\sqrt{6}}{50}, \frac{30 + 2\sqrt{6}}{50} \right) = \left(\frac{3}{5} - \frac{2\sqrt{6}}{50}, \frac{3}{5} + \frac{2\sqrt{6}}{50} \right)$$

$$\left(\frac{15 - \sqrt{6}}{25}, \frac{15 + \sqrt{6}}{25} \right) = \left(\frac{3}{5} - \frac{\sqrt{6}}{25}, \frac{3}{5} + \frac{\sqrt{6}}{25} \right)$$

Question 5cii

Answer	1 mark

300

Question 6

Method	1 mark
Method	1 mark
Answer	1 mark
Answer	1 mark

$$\log_3\left((x-4)^2\right)+\log_3(x)=2$$

$$\log_3\left(x^3-8x^2+16x\right)=\log_3(9)$$

$$x^3-8x^2+16x=9$$

$$x^3-8x^2+16x-9=0$$

$$P(1)=1-8+16-9=0$$

$\therefore (x-1)$ is a factor

Factorise to get

$$(x-1)\left(x^2-7x+9\right)=0$$

From Quadratic formula

$x=1$ or $x=\dfrac{7-\sqrt{13}}{2}$ or $x=\dfrac{7+\sqrt{13}}{2}$

Check for $x>4$ (from initial expression)

$$\therefore x=\frac{7+\sqrt{13}}{2}$$

Question 7a

Method	1 mark
Method	1 mark
Answer	1 mark

$$A_{trapezium} = \frac{\pi - 0}{2\times 3}\left(f(0) + 2f\left(\frac{\pi}{3}\right) + 2f\left(\frac{2\pi}{3}\right) + f(\pi)\right)$$

$$= \frac{\pi}{6}\times\left(0 + 2\times\frac{\pi\sqrt{3}}{6} + 2\times\frac{2\pi\sqrt{3}}{6} + 0\right)$$

$$= \frac{\pi}{6}\times\frac{6\pi\sqrt{3}}{6}$$

$$= \frac{\sqrt{3}\pi^2}{6} \text{ or } \frac{6\sqrt{3}\pi^2}{36} = \frac{3\sqrt{3}\pi^2}{18} = \frac{\pi^2}{2\sqrt{3}}$$

x	$f(x)$
0	0
$\frac{\pi}{3}$	$\frac{\pi}{3}\times\frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{6}$
$\frac{2\pi}{3}$	$\frac{2\pi}{3}\times\frac{\sqrt{3}}{2} = \frac{2\pi\sqrt{3}}{6}$
π	0

Question 7bi

Answer	1 mark

$\sin(x) + x\cos(x)$

Question 7bii

Method	1 mark
Answer	1 mark

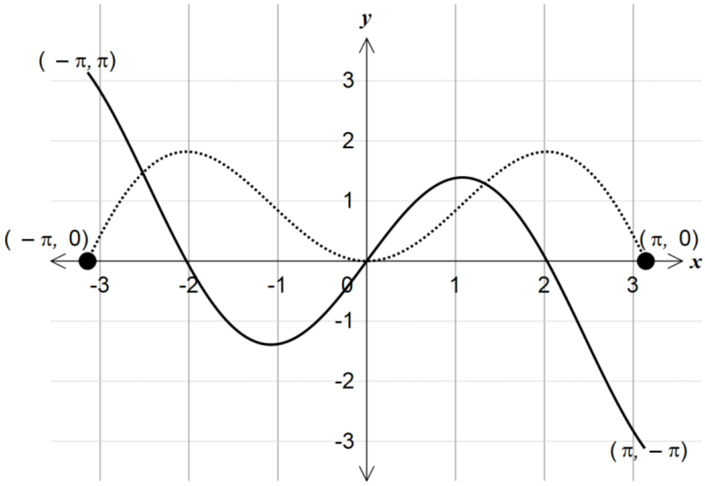
Range of $f'(x)\left[\frac{\sqrt{3}}{2}-\frac{\pi}{3},1\right]$

As $\frac{\sqrt{3}}{2}-\frac{\pi}{3}<0$ and $1>0$, ($f'(x)$ is continuous is implied.)

$\therefore f'(x)=0$ over the interval $x\in\left[\frac{\pi}{2},\frac{2\pi}{3}\right]$

Question 7c

Answer	1 mark
Answer	1 mark
Answer	1 mark



Shape

Endpoints

Five key points on graph (x-intercepts and turning points)

Question 8a

Answer	1 mark

$\left(0, m-\sqrt[3]{k}\right)$

Question 8b

Method	1 mark
Answer	1 mark

$$g'(x)=\frac{1}{3}(x-k)^{-\frac{2}{3}}$$

$$g'(0)=\frac{1}{3}(-k)^{-\frac{2}{3}}$$

Question 8c

Answer	1 mark

$k=m^3$

Question 8d

Method	1 mark
Method	1 mark
Answer	1 mark

Using derivative in terms of *k*

$g'(x) = g'(0)$

$\frac{1}{3}(x-k)^{\frac{-2}{3}} = \frac{1}{3}(-k)^{\frac{-2}{3}}$

$(x-k)^{\frac{-2}{3}} = (-k)^{\frac{-2}{3}}$

$(x-k)^{-2} = (-k)^{-2}$

$(x-k)^{2} = (-k)^{2}$

$x-k = \pm k$

$x = 0$ or $x = 2k$

Then find y value as above

$Q\left(2m^3, 2m\right)$