2024 VCE Mathematical Methods Examination 2

Marking guidelines and sample responses





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Authorised and published by the Victorian Curriculum and Assessment Authority Level 7, 200 Victoria Parade East Melbourne VIC 3002

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2024 VCE Mathematical Methods Examination 2 Marking guidelines and sample responses

Marking guidelines will indicate the initial criteria that will be used to award marks.

This report provides sample responses, or an indication of what responses may have included.

Section A

Question	Answer
1	Α
2	D
3	С
4	В
5	D
6	В
7	С
8	Α
9	С
10	В

Question	Answer
11	В
12	Α
13	Α
14	В
15	Α
16	D
17	D
18	В
19	С
20	С

Section B

Question 1a

Answer	1 mark
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$$-1, -a, 2, 2a$$

Question 1bi

Answer	1 mark
Answer	1 mark

$$-2, -\frac{1}{2}, 0$$

Question 1bii

Consequential 1 m

$$R\setminus\left\{-2,-\frac{1}{2},0,1\right\}$$

Question 1ci

Answer	1 mark

$$2(x-2)(x+1)(2x-1)$$
 OR $4x^3-6x^2-6x+4$

Question 1cii

Answer	1 mark

$$\left(\frac{1}{2}, \frac{81}{16}\right)$$
 OR $\left(0.5, 5.0625\right)$

Question 1ciii

Answer 1 mark

$$(-1,\frac{1}{2})\cup(2,\infty)$$

Question 1civ

Method	1 mark
Answer	1 mark

Tangent lines are

$$y = -3\sqrt{3}x + \frac{3\sqrt{3}}{2} + \frac{27}{4}$$

$$y = 3\sqrt{3}x - \frac{3\sqrt{3}}{2} + \frac{27}{4}$$

These lines intersect at $\left(\frac{1}{2}, \frac{27}{4}\right)$

Question 1di

Translate $\frac{1}{2}$ units to the right and $\frac{17}{16} = 1.0625$ units up

Question 1dii

Answer	1 mark
Answer	1 mark

Dilate by a factor of $\frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$ from the y-axis

Translate $\frac{1}{2}$ units to the right

Translate $\frac{9}{4}$ units up

Question 2a

Answer	1 mark
Answer	1 mark

$$f'(t) = \begin{cases} 30 & 0 \le t < \frac{1}{3} \\ 0 & t > \frac{1}{3} \end{cases}$$

Question 2b

Answer	1 mark
--------	--------

20

Question 2ci

Answer	1 mark
--------	--------

 $60e^{-6t}$

Question 2cii

Answer	1 mark
--------	--------

0.299

Question 2d

Answer	1 mark
--------	--------

0.27

Question 2e

Answer	1 mark

0.12

Question 2fi

Method	1 mark

$$0.3 + Ae^{-10(0.4)} = 1.5 \text{ OR } 0.3 + Ae^{-4} = 1.5$$

$$Ae^{-4} = 1.2$$

$$A = 1.2e^4$$

Question 2fii

Answer	1 mark
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 $\frac{1}{3}$

Question 2fiii

Method	1 mark
Answer	1 mark

$$\int_{0.4}^{T} \left(0.3 + Ae^{-10t} \right) dt = 0.4$$

1.33

Question 3ai

Method	1 mark
Method	1 mark
Answer	1 mark

$$p(2) = 2500$$
, $p(11) = 4400$, $p'(2) = 0$, $p'(11) = 0$
 $a = -5.21$, $b = 101.65$, $c = -344.03$, $d = 2823.18$

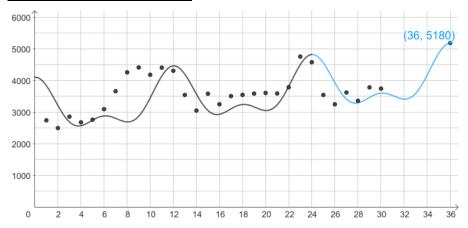
Question 3aii

Answer	1 mark
Answer	1 mark

$$h = 12$$
, $k = 350$

Question 3bi

Answer	1 mark
Answer	1 mark



Question 3bii

Answer 1	mark
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360

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Question 3biii

Answer	1 mark	
$-400\pi \sin\left(\frac{\pi t}{3}\right)$	$350\pi\sin\left(\frac{\pi t}{6}\right)$	+ 30
3	3	-+30

Question 3biv

Answer	1 mark
Answer	1 mark

Maximum instantaneous rate of change ≈ 725 (million dollars per month)

$$t = 10.2$$
, $t = 22.2$, $t = 34.2$

Question 4a

Answer	1 mark
$\int_{22}^{30} f(x) dx$	

Question 4bi

Answer	1 mark
--------	--------

18

Question 4bii

Method	1 mark
Answer	1 mark

Method 1

$$Var(X) = \int_0^{30} \frac{1}{67500} x^2 (30 - x) \cdot x^2 dx - 18^2$$

= 36

Method 2 (formula sheet)

$$Var(X) = \int_0^{30} \frac{1}{67500} x^2 (30 - x) \cdot (x - 18)^2 dx$$

= 36
$$sd(X) = 6$$

Question 4biii

Method	1 mark
Answer	1 mark

$$= \frac{\Pr(X > 23)}{\Pr(X > 18)} = \frac{\int_{23}^{30} f(x)dx}{\int_{18}^{30} f(x)dx} = \frac{0.23392...}{0.5248}$$

$$=0.446$$

Question 4ci

Method	1 mark
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$$0.234 \times 0.234 \times 0.5$$

$$=0.054756\times0.5$$

Question 4cii

Answer	1 mark
Answer	1 mark

0.700, 0.273

Question 4di

Method	1 mark
Answer	1 mark

$$Y \sim \text{Bi}(35, 0.234)$$

$$\Pr(\hat{P} > 2) = \Pr(Y > 7) = \Pr(Y \ge 8)$$
$$= 0.595$$

Question 4dii

Method	1 mark
Answer	1 mark

Method 1

$$E(\hat{P}) = 0.234$$
, $sd(\hat{P}) = \sqrt{\frac{0.234 \times (1 - 0.234)}{35}} \approx 0.072$

$$Pr(0.162... < \hat{P} < 0.305...) = Pr(6 \le Y \le 10) = 0.684$$

Method 2

$$E(Y) = 35 \times 0.234 = 8.19$$
, $sd(Y) = \sqrt{35 \times 0.234 \times (1 - 0.234)} \approx 2.505$

$$Pr(5.68529... < Y < 10.69470...) = Pr(6 \le Y \le 10) = 0.684$$

^{=0.027}

Question 4ei

Answer	1 mark
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(0.107, 0.293)

Question 4eii

Answer	1 mark
--------	--------

11, 39

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Question 5ai

Answer	1 mark
(0.9,1.0) OR (0.9,1)	

Question 5aii

Answer 1 ma	ırk
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$$[-1,1]$$

Question 5bi

Answer	1 mark
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$$2\cos(2x)\cos(\sin(2x))$$

Question 5bii

Method	1 mark
Method	1 mark

If
$$\cos(\sin(2x)) = 0$$

then
$$\sin(2x) = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$$

But $\sin(2x)$ has range [-1,1]

Hence no solutions

Question 5biii

$$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Question 5biv

Answer	1 mark
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$$\left[-\sin(1),\sin(1)\right]$$

Question 5ci

Answer	1 mark
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$$2\int_{0}^{\pi} (g \circ f)(x) - (f \circ g)(x) dx$$

OR

$$2\int_{0}^{\pi}\sin(2\sin(x))-\sin(\sin(2x))dx$$

OR

$$2\int_{\pi}^{2\pi} (f\circ g)(x) - (g\circ f)(x)dx$$

OR

$$2\int_{\pi}^{2\pi} \sin(\sin(2x)) - \sin(2\sin(x)) dx$$

Question 5cii

Answer 1 mark

4.97

Question 5d

Method	1 mark
Answer	1 mark

Require $ran(g) \subseteq dom(f_1)$

$$[-1,1]\not\in(0,2\pi)$$

$$(0,1] \subseteq (0,2\pi), 0 < \sin(2x) \le 1$$

$$\left(0,\frac{\pi}{2}\right) \cup \left(\pi,\frac{3\pi}{2}\right)$$