2024 VCE Specialist Mathematics Examination 1

Marking guidelines and sample responses

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Acknowledgement

The Victorian Curriculum and Assessment Authority proudly acknowledges and pays respect to Victoria's Aboriginal and Torres Strait Islander communities and their rich and enduring cultures.

We acknowledge Aboriginal and Torres Strait Islander people as Australia's first peoples and as the Traditional Owners and custodians of the lands and waters on which we rely. We pay respect to Elders past and present of the lands where we conduct our work and recognise their ongoing contributions as the first educators on the land now known as Victoria.

Authorised and published by the Victorian Curriculum and Assessment Authority
Level 7, 200 Victoria Parade
East Melbourne VIC 3002

Contact us if you need this information in an accessible format, for example, large print or audio.

Telephone (03) 9032 1635 or email vcaa.publications@education.vic.gov.au

2024 VCE Specialist Mathematics Examination 1

Marking guidelines and sample responses

Marking guidelines will indicate the initial criteria that will be used to award marks.

This report provides sample responses, or an indication of what responses may have included.

Question 1a

Answer	1 mark

$$f\left(-\frac{2i}{3}\right)=\frac{8}{9}i-\frac{8}{9}i-2i+2i=0$$

Question 1b

Method	1 mark
Answer	1 mark

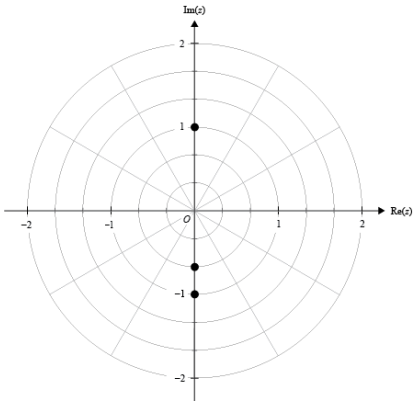
$3z(z^2+1)+2i(z^2+1)=0$

$(z^2+1)(3z+2i)=0$

$z=-\frac{2i}{3},\pm i$

Question 1c

Answer	1 mark



$z=-\frac{2i}{3},\pm i$

Question 2

Method	1 mark
Method	1 mark
Answer	1 mark

Let $x = 2k+1$ where $k \in Z$.

$$2x^3 - 3x - 7 = 2(2k+1)^2 + 3(2k+1) - 7$$

$$= 2(4k^2 + 4k + 1) - 6k - 3 - 7$$

$$= 8k^2 + 2k - 8$$

$$= 2(4k^2 + k - 4) \text{ which is even}$$

Question 3a

Answer	1 mark

$$\frac{x^2-2x+1}{x^2+2x+1}=\frac{x^2+2x+1}{x^2+2x+1}+\frac{-4x}{x^2+2x+1}$$

$$=1+\frac{-4(x+1)+4}{(x+1)^2}$$

$$=1-\frac{4}{x+1}+\frac{4}{(x+1)^2}$$

$A=1, B=-4, C=4$

Question 3b

Method	1 mark
Answer	1 mark

$$f'(x)=\frac{4}{(x+1)^2}-\frac{8}{(x+1)^3}=0$$

$$\frac{4}{(x+1)^2}=\frac{8}{(x+1)^3}$$

$x=1$

Turning point $(1, 0)$

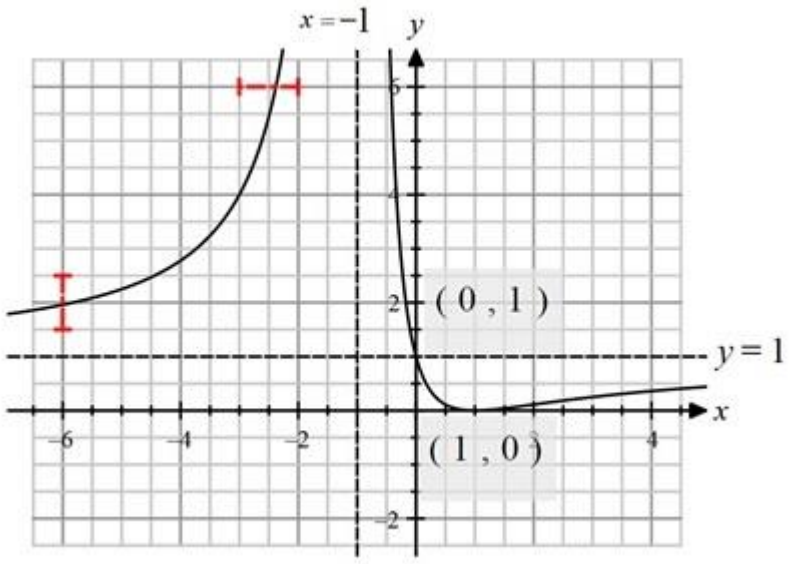
Question 3c

Answer	1 mark
Answer	1 mark
Answer	1 mark

Asymptotes and intercepts must be labelled with equations and coordinates (must have a curve that passes through (0,1) and (1, 0)).

Shape LHS (top $\in[-3,-2]$, lower left $\in[1.5,2.5]$, correct asymptotic behaviour)

Shape RHS (correct TP and intercept, asymptotic behaviour, must have first answer)



Question 4a

Method	1 mark
Answer	1 mark

$$\cos\theta = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \frac{-9}{3\sqrt{2}\times 3} = -\frac{1}{\sqrt{2}}$$

$$\theta = \frac{3\pi}{4} \text{ or } \theta = 135^\circ$$

Question 4b

Method	1 mark
Answer	1 mark

$$\underset{\sim}{a}\times\underset{\sim}{c} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0 & 3 & 3 \\ n & 2 & 1 \end{vmatrix} = -3\underset{\sim}{i} + 3n\underset{\sim}{j} - 3n\underset{\sim}{k}$$

$$|\underset{\sim}{a}\times\underset{\sim}{c}| = \sqrt{18n^2+9}$$

$$\underset{\sim}{a}\cdot\underset{\sim}{c} = 9$$

$$18n^2 + 9 = 81$$

$$n^2 = 4$$

$$n = \pm 2$$

Question 5

Method	1 mark
Answer	1 mark
Answer	1 mark

$$V = \pi\int_1^{\frac{k}{2}} k - \frac{1}{x^2}\,dx$$

$$= \pi\left[kx + \frac{1}{x}\right]_1^{\frac{k}{2}}$$

$$= \pi\left(\frac{1}{2}k^2 + \frac{2}{k} - k - 1\right)$$

$$\pi\left(\frac{1}{2}k^2 + \frac{2}{k} - k - 1\right) = \frac{7\pi}{2}$$

$$k^3 - 2k^2 - 9k + 4 = 0$$

Question 6a

Answer	1 mark

$\mathrm{E}(W_1 + W_2 + W_3) = 4.5$

$\mathrm{Var}(W_1 + W_2 + W_3)$

$= 0.3^2 + 0.4^2 + 0.5^2 = 0.5$

Question 6b

Method	1 mark
Answer	1 mark

$\mathrm{Var}(10W_1 + 20W_2 + 15W_3)$

$= 100\,\mathrm{Var}(W_1) + 400\,\mathrm{Var}(W_2) + 225\,\mathrm{Var}(W_3)$

$= 100 \times 0.09 + 400 \times 0.16 + 225 \times 0.25$

$= 9 + 64 + 56.25$

$= 129.25$ *or* $\frac{517}{4}$ *or* $73 + \frac{225}{4}$

Question 6c

Answer	1 mark
Answer	1 mark

Let $X = W_1 - W_2$

$\mathrm{E}(W_1 - W_2) = -0.5$

$\mathrm{Var}(W_1 - W_2) = 0.3^2 + 0.4^2 = 0.25$

$\Pr(X > 0) = \Pr\left(Z > \frac{0 + 0.5}{0.5}\right)$

$= \Pr(Z > 1)$

$= 0.16$

Question 7

Method	1 mark
Method	1 mark
Answer	1 mark
Answer	1 mark

$$2y\frac{dy}{dx}=\frac{-x}{\sqrt{x^2+1}},\ 2y\,dy=-\frac{x}{\sqrt{x^2+1}}dx$$

$$\int 2y\,dy=\int\frac{-x}{\sqrt{x^2+1}}dx$$

$$y^2=-\sqrt{x^2+1}+c$$

$$y(0)=-2\Rightarrow c=5$$

$$y^2=5-\sqrt{x^2+1}$$

$$y=-\sqrt{5-\sqrt{x^2+1}}$$

Question 8a

Method	1 mark
Answer	1 mark

$$2xy^2 + 2x^2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2xy^2 - y}{2x^2y + x}$$

$$= -\frac{y(2xy+1)}{x(2xy+1)}$$

$$= -\frac{y}{x} \text{ as } 2xy + 1 \neq 0$$

Question 8b

Method	1 mark
Answer	1 mark

$$y = x$$

$$x^4 + x^2 = 2$$

$$\left(x^2 + 2\right)\left(x^2 - 1\right) = 0$$

$$x = \pm 1$$

Therefore, the points are (1, 1) and (−1, −1).

Question 9a

Answer	1 mark

$v \geq 44$ to be fined.

At $x = 0$, $v^2 = 1600 + \frac{672}{\pi}\arccos\left(\frac{0}{20}\right) = 1936 = 44^2$

So yes.

Question 9b

Method	1 mark
Answer	1 mark
Answer	1 mark

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{2}\frac{d}{dx}\left(1600 + \frac{672}{\pi}\arccos\left(\frac{x}{20}\right)\right)$$

$$a = \frac{-336}{\pi\sqrt{400 - x^2}}$$

When $x = 12$, $a = \frac{-21}{\pi}$

Question 10

Consequential	1 mark
Answer	1 mark
Answer	1 mark

Let $\underset{\sim}{a}_1 = \underset{\sim}{i} + m\underset{\sim}{k}, \underset{\sim}{d}_1 = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$

$\underset{\sim}{a}_2 = 2\underset{\sim}{i} - \underset{\sim}{k}$ *and* $\underset{\sim}{d}_2 = -\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

$$\underset{\sim}{\hat{n}} = \frac{\underset{\sim}{d}_1 \times \underset{\sim}{d}_2}{\left|\underset{\sim}{d}_1 \times \underset{\sim}{d}_2\right|} = \frac{\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}}{\sqrt{35}}$$

$$\text{Distance} = \left|\left(\underset{\sim}{a}_2 - \underset{\sim}{a}_1\right) \cdot \underset{\sim}{\hat{n}}\right| = \left|\frac{-4 - 5m}{\sqrt{35}}\right|$$

$$\left|\frac{-4 - 5m}{\sqrt{35}}\right| = \frac{14}{\sqrt{35}}$$

$$-4 - 5m = 14 \Rightarrow m = -\frac{18}{5}$$

$$4 + 5m = 14 \Rightarrow m = 2$$