2024 VCE
Specialist Mathematics 2

Marking guidelines and sample responses

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2024 VCE Specialist Mathematics Examination 2

Marking guidelines and sample responses

Marking guidelines will indicate the initial criteria that will be used to award marks.

This report provides sample responses, or an indication of what responses may have included.

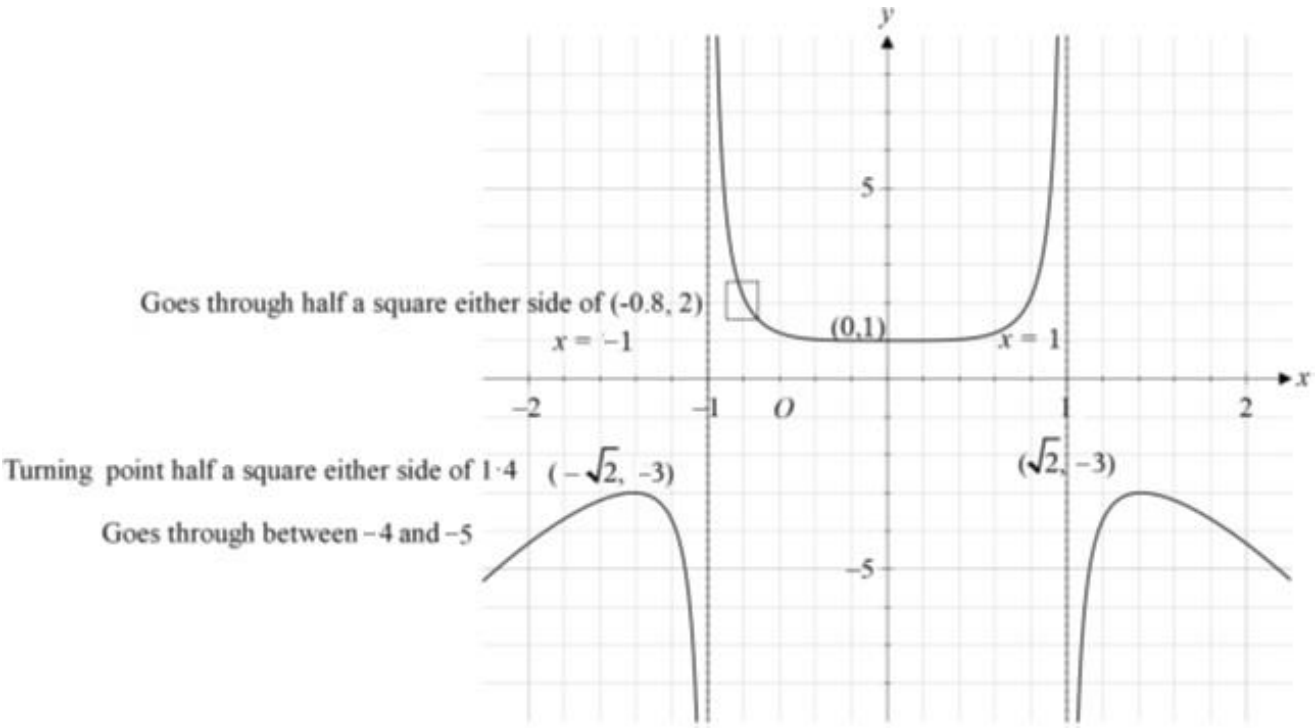
Section A

Question	Answer
1	C
2	A
3	C
4	A
5	C
6	D
7	C
8	C
9	A
10	A
11	B
12	C
13	B
14	B
15	A
16	A
17	B
18	D
19	A
20	B

Section B

Question 1a

Answer	1 mark
Answer	1 mark
Answer	1 mark



Three branches drawn correctly:

- graph central region is ‘flatish’
- graph should be within half-a-grid square $(-0.8, 2)$ and $(0.8, 2)$
- two turning points below x-axis should be on the line $y = -3$ and half-a-grid square either side of ± 1.4
- graph should exit left and right sides of the grid between $y = -4$ and $y = -5$.

Two asymptotes are drawn and labelled.

Three stationary points are labelled with the correct coordinates.

Question 1bi

Method	1 mark
Answer	1 mark

Attempt to solve for x^2 to use in $\pi\int_1^6 x^2 dy$

$\pi\int_1^6 \frac{1-y+\sqrt{(y-1)^2-4(y-1)}}{2}dy$ or $\pi\int_1^6 \frac{1-y+\sqrt{y^2+2y-3}}{2}dy$

Question 1bii

Answer	1 mark

11.2

Question 1c

Answer	1 mark

$b=-1$

Question 1di

Answer	1 mark

$b \leq -1$

Question 1dii

Answer	1 mark

$b \geq 0$

Question 1diii

Answer	1 mark

$-1 < b < 0$

Question 2a

Method	1 mark
Answer	1 mark

$(x-1)^2+(y-2)^2=(x-4)^2+y^2$ (or equivalent)

OR

(1, 2) & (4, 0) grad is $-\frac{2}{3}$, grad = $\frac{3}{2}$, $y=\frac{3}{2}x+c$

midpoint $\left(\frac{5}{2},1\right), 1=\frac{3}{2}\times\frac{5}{2}+c, c=-\frac{11}{4}$

$y=\frac{3}{2}x-\frac{11}{4}$

Question 2b

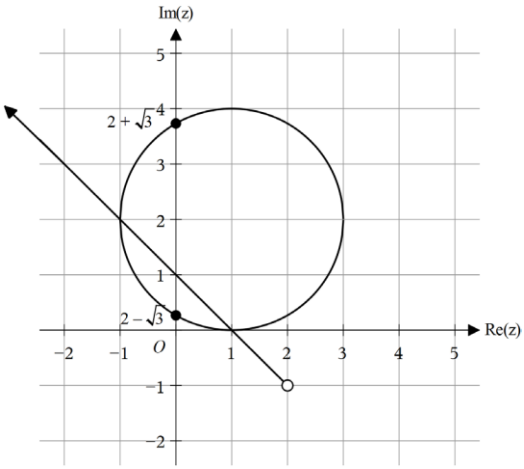
Method	1 mark
Answer	1 mark

midpoint is $\left(\frac{5}{2},1\right)$, radius is $\frac{\sqrt{(-3)^2+2^2}}{2}=\frac{\sqrt{13}}{2}$

So $\left|z-\left(\frac{5}{2}+i\right)\right|=\frac{\sqrt{13}}{2}$

Question 2c

Answer	1 mark
Answer	1 mark



Circle consistent with their equation.

Correct y-intercept values as shown above, can be co-ordinates.

Accept imaginary intercepts of $\left(2\pm\sqrt{3}\right)i$.

Question 2di

Answer	1 mark

Ray as shown in Answer 2c. (Origin point at (2,−1) must be open.)

Question 2dii

Answer	1 mark

$\text{Arg}(z-2+i)=\frac{3\pi}{4}$ OR $\text{Arg}(z-(2-i))=\frac{3\pi}{4}$

Question 2e

Method	1 mark
Answer	1 mark

$A=\frac{1}{2}\times 2^2\times\left(\frac{\pi}{2}-\sin\left(\frac{\pi}{2}\right)\right)$ or other method shown

$A=\pi-2$

Question 3a

Answer	1 mark

$0.05 = \frac{1}{20}$ (cubic metres per day) when $t = 2$

Question 3b

Method	1 mark
Answer	1 mark
Answer	1 mark

$$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$$

$$\frac{dr}{dt} = \frac{8t}{240 + 5t^4} \times \frac{1}{2\pi r \times 0.001}$$

$$\frac{dr}{dt} = \frac{8 \times 4}{240 + 5 \times 4^4} \times \frac{1}{2\pi \times 6.54 \times 0.001} = 0.51$$

Question 3ci

Answer	1 mark

$$\frac{4}{\sqrt{5}} \int \frac{1}{240 + u^2} \, du$$

Question 3cii

Answer	1 mark

$$\frac{1}{5\sqrt{3}} \arctan\left(\frac{t^2}{4\sqrt{3}}\right)$$

Question 3d

Answer	1 mark
Consequential	1 mark

Max Volume = $\frac{\sqrt{3}}{15} \times \frac{\pi}{2}$

$$\frac{\sqrt{3}}{15} \times \frac{\pi}{2} = \pi r^2 \times 0.001, \text{ so max } \pi r^2 = \frac{\sqrt{3}}{15} \times \frac{\pi}{2} \times 1000 \approx 181.38$$

Question 3e

Method	1 mark
Answer	1 mark

$$\frac{\sqrt{3}}{15} \arctan\left(\frac{\sqrt{3}(T+5)^2}{12}\right) - 0.05 \times T = 0$$

$T = 3.4$

Question 4a

Answer	1 mark

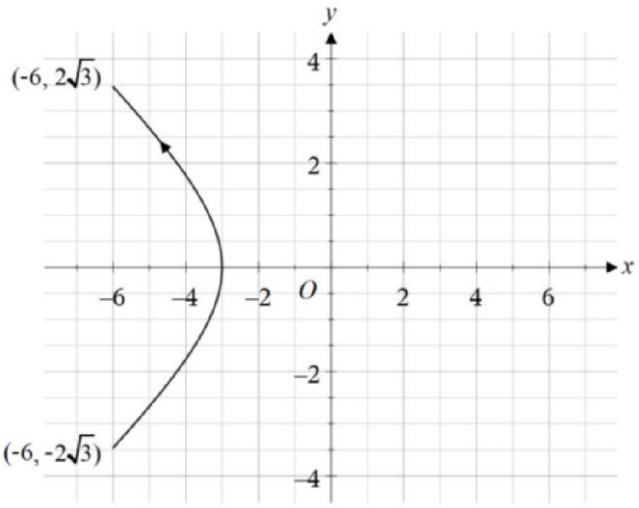
$x = 3\sec t,\ y = 2\tan t,\ \frac{x}{3} = \sec t,\ \frac{y}{2} = \tan t$

$1 + \tan^2 t = \sec^2 t$ or $\sec^2 t - \tan^2 t = 1$

$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{2}\right)^2 = 1,\ \frac{x^2}{9} - \frac{y^2}{4} = 1$

Question 4b

Answer	1 mark
Answer	1 mark



Correct path shown.

Arrow showing direction and coordinates of endpoints labelled.

Note that $2\sqrt{3} < 3.5$ so endpoints must not be below -3.5 or above 3.5, curve should have a vertical tangent at the x-intercept and it should be a hyperbolic shape with no sharp points.

Question 4ci

Answer	1 mark

$13\sec^4 t - 9\sec^2 t$

Question 4cii

Answer	1 mark

π (minutes)

Question 4ciii

Answer	1 mark

2

Question 4civ

Answer	1 mark

$(-3, 0)$

Question 4di

Method	1 mark

$$L = \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \sqrt{\left(3\sec t \tan t\right)^2 + 4\left(\sec t\right)^4}\, dt$$

Question 4dii

Answer	1 mark

$L = 9.4$

Question 4e

Method	1 mark
Answer	1 mark

$$D^2 = \left(2 - 3t - 3\sec t\right)^2 + \left(4t - 1 - 2\tan t\right)^2 + \left(6 - t\right)^2$$

$D = 11.1$

Question 5a

Answer	1 mark

$\underset{\sim}{r}(t) = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k} + t\left(\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}\right),\ t \in R$

OR

$\underset{\sim}{r}(t) = 2\underset{\sim}{i} - 5\underset{\sim}{j} - \underset{\sim}{k} + t\left(\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}\right),\ t \in R$

OR

either of these with direction vector reversed.

Other possibilities exist.

Question 5b

Method	1 mark
Method	1 mark
Answer	1 mark

A vector from $A(1, -2, 3)$ to a point on the line, say $(2, 1, -3)$ is $\underset{\sim}{i} + 3\underset{\sim}{j} - 6\underset{\sim}{k}$.

Must use a point on the line for method mark.

Scalar resolute : $\left(\underset{\sim}{i} + 3\underset{\sim}{j} - 6\underset{\sim}{k}\right).\dfrac{\left(-\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}\right)}{\sqrt{6}}\ \left(= -\dfrac{1}{\sqrt{6}}\right)$ must use unit direction

Distance = $\dfrac{5\sqrt{66}}{6}$

OR

$\text{norm}([2 - t - 1, 1 + 2t + 2, -3 + t - 3])$ vector from A to line

$= \sqrt{2(3t^2 - t + 23)}$

minimising this function gives $\left(t = \dfrac{1}{6}\right)$ dist $= \dfrac{5\sqrt{66}}{6}$.

Question 5c

Method	1 mark
Method	1 mark
Answer	1 mark

$\overrightarrow{AB} = \underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}$, $\overrightarrow{BC} = -2\underset{\sim}{i} + 7\underset{\sim}{j} - 4\underset{\sim}{k}$, $\overrightarrow{AC} = -\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}$ find two of these vectors in plane

$\overrightarrow{AB} \times \overrightarrow{AC} = 40\underset{\sim}{i} + 12\underset{\sim}{j} + \underset{\sim}{k}$ using cross product to find a normal

$40x + 12y + z = 19$

Question 5di

Answer	1 mark

$(6,0,0)$, $(0,-4,0)$, $(0,0,3)$

Question 5dii

Method	1 mark
Answer	1 mark

$\left(-6\underset{\sim}{i} - 4\underset{\sim}{j}\right) \times \left(-6\underset{\sim}{i} + 3\underset{\sim}{k}\right) = -12\underset{\sim}{i} + 18\underset{\sim}{j} - 24\underset{\sim}{k}$

cross product between any two spanning vectors

$\frac{1}{2} \times \left|-12\underset{\sim}{i} + 18\underset{\sim}{j} - 24\underset{\sim}{k}\right| = 3\sqrt{29}$

Cosine Rule – might find other angles and use half base × height

$5^2 = 52 + 45 - 2 \times \sqrt{52} \times \sqrt{45} \times \cos\theta$, $\cos\theta = \frac{36}{\sqrt{2340}} \Rightarrow \sin\theta = \frac{\sqrt{1044}}{\sqrt{2340}}$

$$A = \frac{1}{2} \times \sqrt{45} \times \sqrt{52} \times \frac{\sqrt{1044}}{\sqrt{2340}}, \quad A = \frac{1}{2} \times \sqrt{1044} = \sqrt{261} = 3\sqrt{29}$$

Question 6a

Answer	1 mark

$H_0: \mu = 1000$, $H_1: \mu < 1000$

Question 6bi

Answer	1 mark

$\bar{X} \sim \text{N}\left(1000, \frac{4.2^2}{9}\right)$, $\Pr\left(\bar{X} < 997.5\right) = 0.0371$ so $p = 0.037$

Question 6bii

Consequential	1 mark

$p < 0.05$, so pause machine

Question 6c

Method	1 mark
Answer	1 mark

Lower 5% tail cut off for $\bar{x}$ is 997.697 – critical $\bar{x}$

$\Pr\left(\bar{x} > 997.697 \mid \mu = 997\right) = 0.31$

Question 6d

Answer	1 mark

$a = 996.7$, $b = 1003.3$

Question 6e

Answer	1 mark

$1005 \pm 1.96 \times \frac{4}{\sqrt{50}}$ gives $(1003.9, 1006.1)$

Question 6f

Answer	1 mark

$40 \times 0.95 = 38$

Question 6g

Answer	1 mark

$|\mu - \bar{x}| = 1.96 \times \frac{\sigma}{\sqrt{n}}$, $1 = 1.96 \times \frac{4}{\sqrt{n}}$,

$n = 61.4656$ so take $n = 62$