2024
NHT

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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Mathematical Methods
Examination 1

Question and Answer Book

VCE (NHT) Examination – Tuesday 28 May 2024

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **1 hour**: 10.45 am to 11.45 am

Materials supplied

- Question and Answer Book of 12 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software) or notes of any kind into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given, unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = xe^{x^2+1}$.

Find and factorise $\frac{dy}{dx}$. 2 marks

b. Let $f(x) = \frac{x^3}{\log_e(x)}$.

Evaluate $f'(x)$ at $x = e$. 2 marks

Question 2 (5 marks)

Consider the simultaneous linear equations

$$ax + (2 - a)y = 3$$

$$x + ay = \frac{2a + 1}{2}$$

where $a \in R$ and $x, y \in R$.

a. Find the value of a for which there are infinitely many solutions. 3 marks

b. Find the values of a for which there is a unique solution. 1 mark

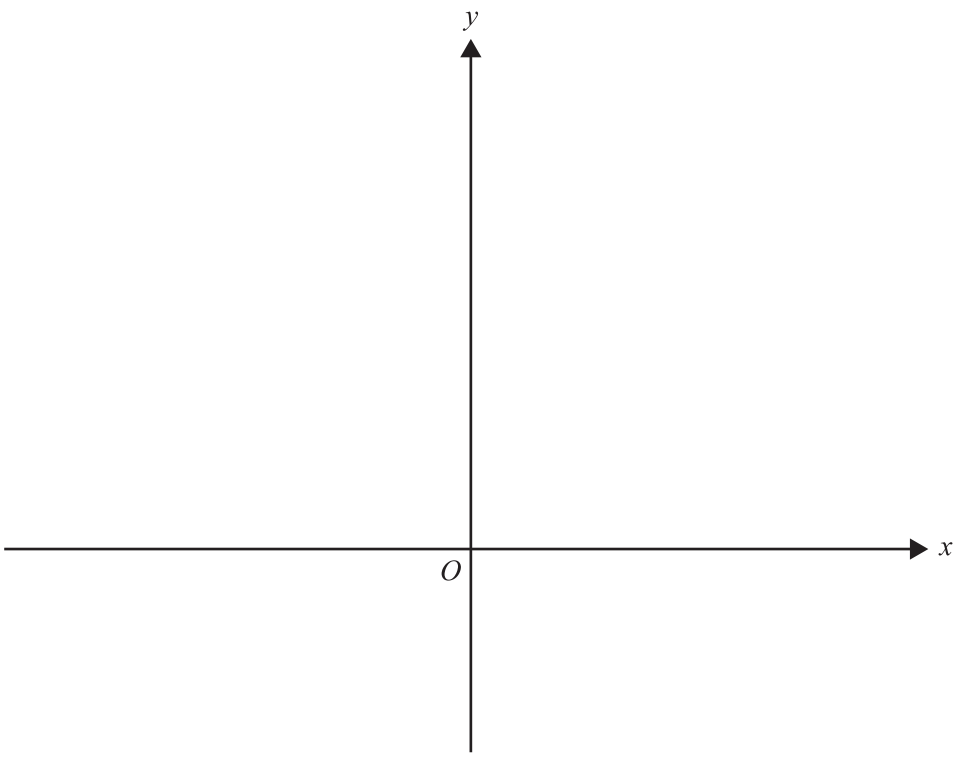
c. Find a value of a such that the lines meet at right angles. 1 mark

Question 3 (5 marks)

Let $f: D \to R,\ f(x) = 3\log_e(2 - x)$, where D is the domain for f.

a. State the maximal domain for $f(x)$. 1 mark

b. Sketch the graph of $y = f(x)$, labelling the asymptote with its equation and the axial intercepts with their coordinates. 3 marks



c. Find the values of x for which $0 \le f(x) \le 3$. 1 mark

Question 4 (3 marks)

Let $g:\left(\frac{3}{2},\infty\right)\to R,\ g(x)=\log_e(2x-3)$ and $h: R\to R,\ h(x)=e^{3x}+2$.

a. Show that $g \circ h$ is defined for all $x \in R$. 2 marks

b. Find the range of $g \circ h$, given the domain of $g \circ h$ is $x \in R$. 1 mark

Question 5 (4 marks)

In a nursery, 90% of the seeds will grow into seedlings. Let $\hat{P}$ be the random variable representing the sample proportion of seeds that will grow into seedlings for samples of size 100.

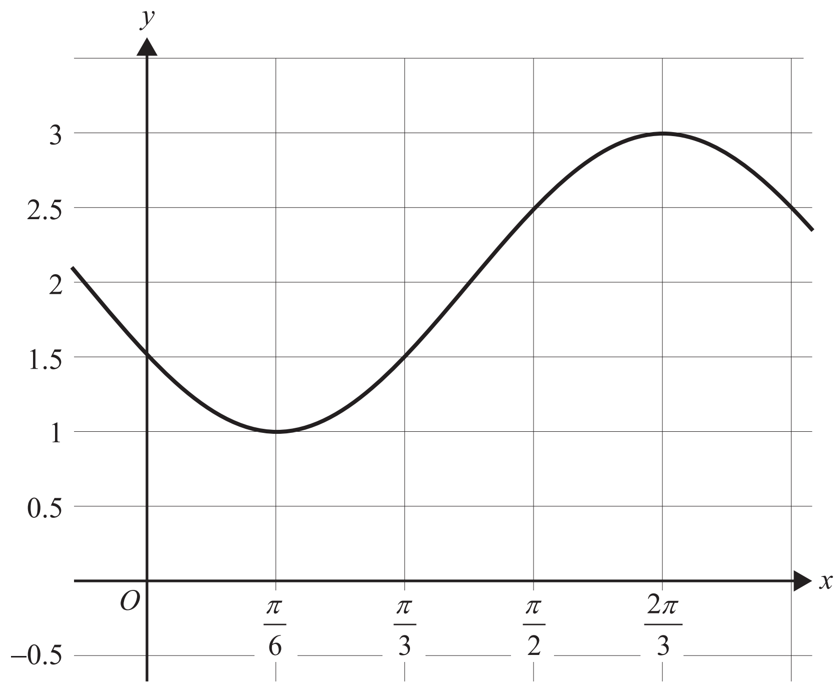
a. Find the mean and standard deviation of $\hat{P}$. 2 marks

b. Let Z be the standard normal random variable such that $Z \sim N(0, 1)$ and $\Pr(Z > -1) \approx 0.84$.

Using a normal approximation of $\hat{P}$, estimate the probability that in a sample of 100, more than 93 seeds will grow into seedlings. Give your answer correct to two decimal places. 2 marks

Question 6 (4 marks)

Part of the graph of $g(x) = \sin\left(2x - \frac{5\pi}{6}\right) + 2$ is shown below.



a. Using the trapezium rule approximation method and three trapeziums of equal width, estimate the area bounded by the graph of $y = g(x)$, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{2\pi}{3}$. 2 marks

b. Let $h : \left[\frac{\pi}{6}, \frac{2\pi}{3}\right] \to R, h(x) = kg(x)$, where $k \in R$.

Using calculus, find k, such that h is a probability density function. 2 marks

Question 7 (3 marks)

Let X be a discrete random variable with a probability mass function of $\Pr(X = x) = \frac{2}{3^x}$, where $x \in Z^+$.

a. Find $\Pr(X = 4)$. 1 mark

b. Find $\Pr(X < 4 \mid X \geq 2)$. 2 marks

Question 8 (4 marks)

Let $f:\left(-\frac{1}{3}, \infty\right) \rightarrow R,\ f(x)=\frac{1}{\sqrt{3x+1}}$.

a. Find an anti-derivative for $f(x)$. 1 mark

b. The average value of the function, f, over $0 \le x \le m$ is $\frac{1}{3}$.

Find the value of m. 3 marks

Question 9 (8 marks)

Let $f: (0, \infty) \rightarrow R, f(x) = (x-1)e^{-x}$.

a. Find $f'(x)$. 1 mark

b. State a sequence of transformations that will map $f(x)$ onto $f'(x)$. 2 marks

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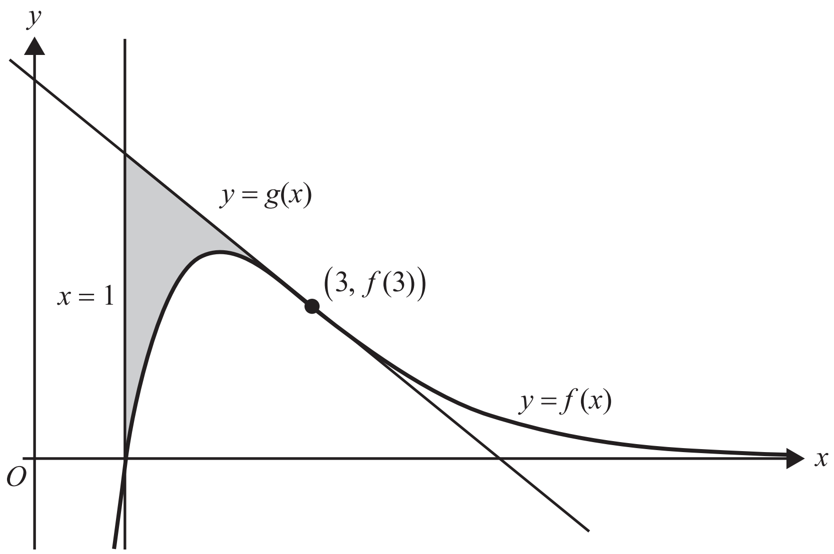
c. Show that $\frac{d}{dx}\left(-xe^{-x}\right) = f(x)$. 1 mark

d. Let $y = g(x)$ be the tangent to $y = f(x)$ at the point $(3, f(3))$.



Using $\frac{d}{dx}\left(-xe^{-x}\right) = f(x)$, or otherwise, determine the area of the region bounded by the lines $x = 1$, $y = g(x)$ and the graph of $y = f(x)$. 4 marks

End of examination questions

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Mathematical Methods
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Formula Sheet

You may keep this Formula Sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$\text{Area} \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet