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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Specialist Mathematics
Examination 2

Question and Answer Book

VCE (NHT) Examination – Friday 24 May 2024

- Reading time is **15 minutes**: 2.00 pm to 2.15 pm
- Writing time is **2 hours**: 2.15 pm to 4.15 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Sheet Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Section A (20 questions, 20 marks) 2–10
Section B (6 questions, 60 marks) 12–25

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Section A – Multiple-choice questions

Instructions for Section A

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1

Consider the following proof.

Prove that $\sqrt{15} + \sqrt{7} > \sqrt{19}$.

Assume $\sqrt{15} + \sqrt{7} \le \sqrt{19}$.

Then $\left(\sqrt{15} + \sqrt{7}\right)^2 \le 19$

$15 + 2\sqrt{105} + 7 \le 19$

$2\sqrt{105} \le -3$

hence $\sqrt{15} + \sqrt{7} > \sqrt{19}$.

This proof can be best described as a

A. direct proof.

B. proof by contrapositive.

C. proof by contradiction.

D. proof by counter-example.

E. proof by mathematical induction.

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Question 2

Consider the following algorithm to estimate the arc length of a certain curve.

```
define f(x)
return x^2
        sum ← 0
        a ← 1
        b ← 2
        h ← 0.5
        left ← a
        right ← a + h
while right ≤ b do
        arc ← √((f(right) − f(left))^2 + h^2)
        sum ← sum + arc
        left ← left + h
        right ← right + h
end while
print sum
```

Correct to two decimal places, the above algorithm will print the value

A. 1.35

B. 1.82

C. 2.38

D. 2.96

E. 3.17

Question 3

The graph of $y = \frac{1}{1 - x^2} - a$, where $a \in R$, has two x-intercepts when

A. $a < 1$

B. $a \geq -1$

C. $-1 \leq a \leq 1$

D. $a < 0$ or $a \geq 1$

E. $a < 0$ or $a > 1$

Question 4

The solutions of $\frac{1+5\sin(x)\cos(x)}{\cos^2(x)} - 7 = 0$ can be found by solving

A. $(\tan(x)-2)(\tan(x)+3)=0$

B. $(\tan(x)-1)(\tan(x)+7)=0$

C. $(\tan(x)-3)(\tan(x)-2)=0$

D. $(\tan(x)-1)(\tan(x)+6)=0$

E. $(\tan(x)+1)(\tan(x)+6)=0$

Question 5

For $z \in C$ and $a \in R$, the discriminant of the quadratic equation $az^2 - aiz - 5 = 0$ is 36.
The possible solutions for z to $az^2 - aiz - 5 = 0$ are

A. $\frac{1}{2}(\pm 3 + i)$ or $\frac{1}{6}(\pm 1 + 3i)$

B. $\frac{1}{6}(\pm 3 + i)$ or $\frac{1}{2}(\pm 1 + 3i)$

C. $\frac{1}{2}(\pm 3 - i)$ or $\frac{1}{6}(\pm 1 - 3i)$

D. $\frac{1}{6}(\pm 3 - i)$ or $\frac{1}{2}(\pm 1 + 3i)$

E. $\frac{1}{6}(\pm 3 - i)$ or $\frac{1}{2}(\pm 1 - 3i)$

Question 6

The equation $\text{Arg}(z + 2 - i) = \frac{\pi}{3}$ defines a ray in the complex plane. Identifying complex numbers $x + yi$ with the points (x, y), this ray lies on the line with equation $y = mx + c$.

The values of m and c are respectively

A. $\frac{1}{\sqrt{3}}$ and $1 + 2\sqrt{3}$

B. $\sqrt{3}$ and $1 - 2\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$ and $2 + \sqrt{3}$

D. $\sqrt{3}$ and $1 + 2\sqrt{3}$

E. $\sqrt{3}$ and $2 + \sqrt{3}$

Question 7

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 1 + 2y$ with the initial condition $f(0) = 1$.

What is the approximation for $f(1)$ using Euler's method, starting at $x = 0$ with a step size of 0.5?

A. 1.5

B. 2.5

C. 3.5

D. 4.5

E. 5.5

Question 8

Using the substitution $u^2 = x + 1$, the definite integral $\int_3^8 \left(\frac{1}{x\sqrt{x+1}}\right) dx$ can be expressed as

A. $\frac{1}{2}\int_2^3 \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du$

B. $\int_2^3 \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du$

C. $\int_2^3 \left(\frac{1}{u+1} - \frac{1}{u-1}\right) du$

D. $\int_3^8 \left(\frac{1}{u+1} - \frac{1}{u-1}\right) du$

E. $\int_2^3 \left(\frac{1}{u-1} + \frac{1}{u+1} - \frac{2}{u}\right) du$

Question 9

The length of the curve given by $x = 3\left(1 - \tan^2(t)\right)$, $y = 4\sec^2(t)$ where $t \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$, is given by

A. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 10\tan(t)\sec^2(t)\, dt$

B. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 100\tan^2(t)\sec^4(t)\, dt$

C. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\tan(t)\sec^2(t)\, dt$

D. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 28\tan^2(t)\sec^4(t)\, dt$

E. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{2}\sqrt{\tan(t)}\sec^2(t)\, dt$

Question 10

Consider the surface of revolution formed by rotating the curve $y = \frac{1}{x},\ 1 \le x \le a$ about the x-axis.

The area of this surface is given by

A. $2\pi \int_1^a x\sqrt{1+x^2}\,dx$

B. $2\pi \int_1^a \frac{\sqrt{1+x^4}}{x}\,dx$

C. $2\pi \int_1^a \frac{\sqrt{1+x^4}}{x^3}\,dx$

D. $2\pi \int_1^a \left(1+\frac{1}{x^2}\right)dx$

E. $2\pi \int_1^a \left(\frac{1+x^4}{x^5}\right)dx$

Question 11

If $I_n = \int_1^e x^2 \left(\log_e x\right)^n dx$ where $n \in N$, then for $n \ge 2$, I_n is equal to

A. $-\frac{n}{3} I_{n-1}$

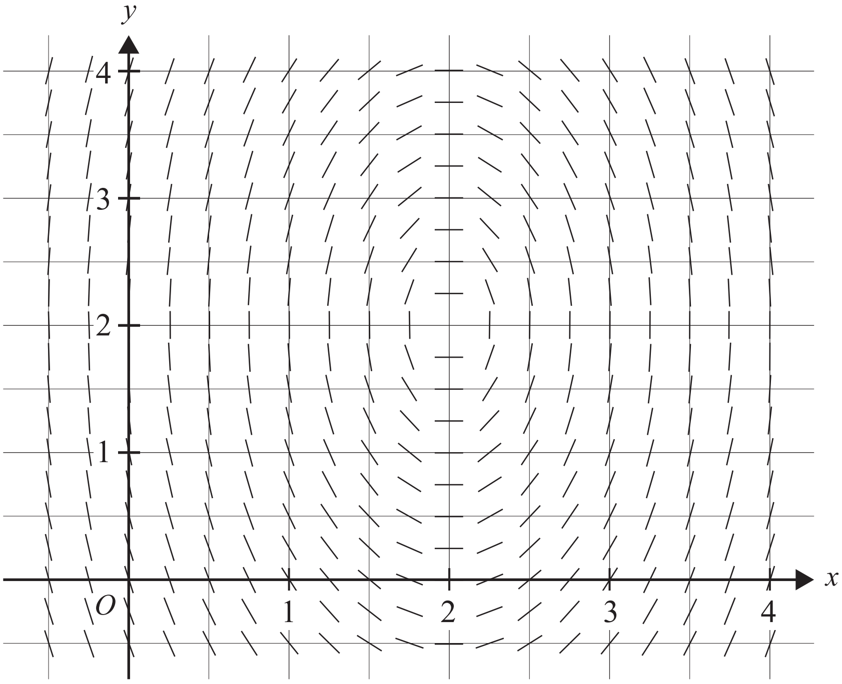
B. $\frac{x^3 \left(\log_e x\right)^n}{3} - \frac{n}{3} I_{n-1}$

C. $\frac{e^3}{3} - \frac{1}{3} I_{n-1}$

D. $\frac{e^3}{3} - \frac{n}{3} I_{n-1}$

E. $\frac{e^3}{3} - \frac{1}{3} - \frac{n}{3} I_{n-1}$

Question 12



The direction field shown above best represents the differential equation

A. $\frac{dy}{dx} = (x-2)^2 + \frac{(y-2)^2}{4}$

B. $\frac{dy}{dx} = \frac{8-2x}{y-2}$

C. $\frac{dy}{dx} = \frac{(x-2)^2}{4} + (y-2)^2$

D. $\frac{dy}{dx} = x^2 + \frac{y^2}{2}$

E. $\frac{dy}{dx} = \frac{8-4x}{y-2}$

Question 13

A racing car is travelling on a straight horizontal track at a velocity of 80 m s^{-1} when the brakes are applied at time $t = 0$ seconds. From time $t = 0$ to the moment the car stops, the acceleration of the car is given by $a(t) = -(6t^2 + t) \text{ m s}^{-2}$.

During this time period, the distance the car travels, correct to the nearest metre, is

A. 188

B. 199

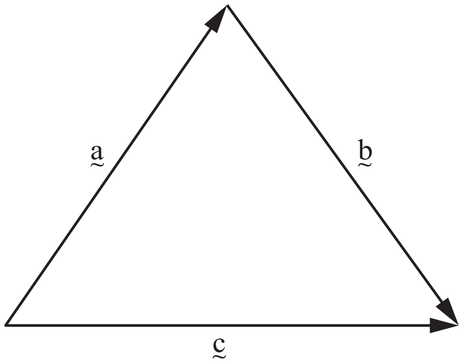
C. 213

D. 260

E. 267

Question 14

An equilateral triangle is spanned by the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, as shown below.



The scalar product $\underset{\sim}{a} \cdot \underset{\sim}{b}$ is equivalent to

A. $\frac{1}{2}|\underset{\sim}{a}|^2$

B. $-\frac{1}{2}|\underset{\sim}{a}|^2$

C. $|\underset{\sim}{a}|^2$

D. $\frac{\sqrt{3}}{2}|\underset{\sim}{a}|^2$

E. $-\frac{\sqrt{3}}{2}|\underset{\sim}{a}|^2$

Question 15

A normal vector to the plane that contains the points $(2, -1, -1)$, $(3, 1, 1)$ and $(-1, -1, 2)$ is

A. $2\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$

B. $-2\underset{\sim}{i} - 7\underset{\sim}{j} - 10\underset{\sim}{k}$

C. $-2\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$

D. $2\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$

E. $2\underset{\sim}{i} + 7\underset{\sim}{j} + 10\underset{\sim}{k}$

Question 16

The position vector $\underset{\sim}{r}(t)$ of a particle at time t is given by $\underset{\sim}{r}(t) = e^{-4t}\underset{\sim}{i} + e^{t}\underset{\sim}{j},\ t \geq 0.$

The value of t when the particle's velocity is perpendicular to its acceleration is

A. $\frac{1}{5}\log_e(64)$

B. $\frac{1}{2}\log_e(2)$

C. $\frac{1}{6}\log_e(32)$

D. $\frac{3}{5}\log_e(2)$

E. $\frac{1}{4}\log_e(64)$

Question 17

The angle at which the line with equation $\underset{\sim}{r}(t) = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k} + t\left(\sqrt{2}\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right),\ t \in R$, intersects the plane with equation $\sqrt{2}\,x - y + z = 3$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{3\pi}{4}$

E. $\frac{2\pi}{3}$

Question 18

The shortest distance between the two parallel planes with equations $2x + 2y - z = 6$ and $-4x - 4y + 2z = -24$ is

A. 1

B. 2

C. 3

D. 4

E. 6

Question 19

When grown under the same conditions, two varieties of tomato plant have yields (the quantity of tomatoes produced) that are independent and approximately normally distributed.

Variety A has a mean yield of 6 kg per plant, with a standard deviation of 1.2 kg.

Variety B has a mean yield of 5 kg per plant, with a standard deviation of 1.5 kg.

The probability that a randomly selected tomato plant of variety B will have a higher yield than a randomly selected plant of variety A is closest to

A. 0.108

B. 0.133

C. 0.301

D. 0.393

E. 0.699

Question 20

A publisher prints a large number of notebooks and textbooks.

The thickness of the notebooks is known to have a variance of $0.04\ \text{cm}^2$, and the thickness of the textbooks is known to have a variance of $0.16\ \text{cm}^2$. The variance of the total thickness of a random sample of m notebooks and n textbooks is $4.36\ \text{cm}^2$, and the variance of the total thickness of a random sample of n notebooks and m textbooks is $2.44\ \text{cm}^2$.

The values of m and n are respectively

A. $9, 25$

B. $6, 10$

C. $5, 3$

D. $25, 9$

E. $3, 5$

End of Section A

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Section B

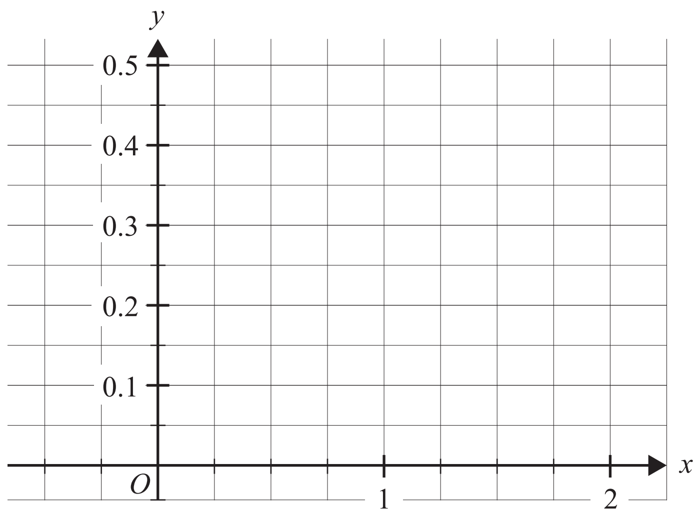
Instructions for Section B

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1 (10 marks)

Consider the function $f:[0, 2] \to R,\ f(x) = 1 - \dfrac{4}{x^2 + 4}$

a. Sketch the graph of $y = f(x)$ on the axes below, and label the point of inflection with its coordinates. 3 marks



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b. The region bounded by the y-axis, the graph of $y = f(x)$ and the line $y = 0.5$ is rotated about the y-axis to form a solid of revolution.

i. Write down a definite integral in terms of y that gives the volume of the solid of revolution. 1 mark

ii. Find the volume of the solid. 1 mark

c. The solid of revolution is used to model an open fishpond where lengths are measured in metres. A waterproof membrane of negligible thickness is to be applied to the interior surface of the pond.

Find the number of square metres of membrane needed. Give your answer correct to two decimal places. 2 marks

d. The pond, which is initially empty, is filled with water flowing in at a rate of 0.1 cubic metres per minute.

At what rate is the water level rising when the depth is 0.25 m? Give your answer in metres per minute, correct to three decimal places. 3 marks

Question 2 (11 marks)

a. Given $(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$ for $n \in Z^+$, show, using appropriate trigonometric identities, that
$(\cos(\theta) + i\sin(\theta))^{n+1} = \cos((n+1)\theta) + i\sin((n+1)\theta)$ for $n \in Z^+$ 2 marks

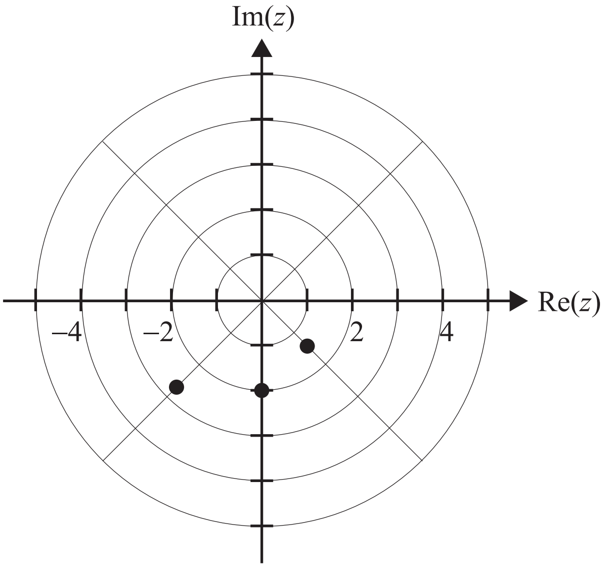
Let $z_n = (1 - i)^n$, $n \in Z^+$

b. Express z_n in polar form. 1 mark

c. Express z_3 in the form $a + bi$, where $a, b \in R$. 1 mark

The points representing z_1, z_2 and z_3 are shown on the Argand diagram below.

d. Plot the point that represents z_4. 1 mark



e. For what values of n will $\text{Arg}(z_n) = 0$ where $z_n = (1 - i)^n,\ n \in Z^+$? 2 marks

Let L be the line defined by $L: \{z : |z| = |z - (a + bi)|,\ z \in C \text{ and } a, b \in R\}$.

L passes through the points that represent $-2i$ and $1 - i$.

f. Find the values of a and b. 1 mark

Question 2 continues on the next page.

g. Find the area of the minor segment bounded by the circle $z\bar{z} = 10$ and the line L.

Give your answer correct to two decimal places. 3 marks

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Question 3 (10 marks)

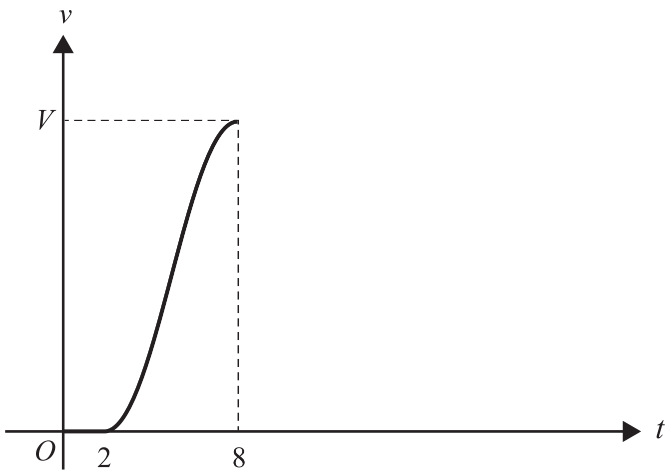
In a computer simulation, a speeding motorcycle, travelling along a straight, flat road at a constant velocity of 20 m s^{-1}, passes a stationary highway patrol car.

Two seconds later, the highway patrol car starts in pursuit of the motorcycle. The highway patrol car accelerates for six seconds and then continues at a constant velocity of $V \text{ m s}^{-1}$, until it draws level with the motorcycle.

The velocity of the highway patrol car while accelerating, in m s^{-1}, is given by

$v(t) = 12.5 - 12.5\cos\left(\frac{\pi}{6}(t-2)\right)$ for $2 \le t \le 8$

where t seconds is the time elapsed since the motorcycle passed the highway patrol car.



a. Find the value of V. 1 mark

b. **i.** Find the acceleration, $a(t)$, of the highway patrol car at time t for $2 < t < 8$. 1 mark

ii. Hence, find the maximum acceleration of the highway patrol car. Give your answer in m s^{-2}, correct to one decimal place. 1 mark

c. Sketch the velocity–time graph for the motorcycle on the axes on page 18 for $t \geq 0$. Clearly label any intercepts with the axes. 1 mark

d. Write down an expression for the number of metres travelled by the motorcycle t seconds after it passes the highway patrol car. 1 mark

e. How many metres will the highway patrol car travel to reach a speed of V at $t = 8$? 2 marks

f. For what value of t will the highway patrol car draw level with the motorcycle? 2 marks

g. How many metres will the highway patrol car need to travel to draw level with the motorcycle? 1 mark

Question 4 (10 marks)

The position vector, $\underset{\sim}{r}(t)$, from an origin, O, of a drone t seconds after leaving the ground is given by

$$\underset{\sim}{r}(t) = \left(50 + 25\sin\left(\frac{\pi t}{30}\right)\right)\underset{\sim}{i} + \left(50 + 25\cos\left(\frac{\pi t}{30}\right)\right)\underset{\sim}{j} + \frac{3t}{5}\underset{\sim}{k}, \; t \geq 0$$

where $\underset{\sim}{i}$ is a unit vector to the east, $\underset{\sim}{j}$ is a unit vector to the north and $\underset{\sim}{k}$ is a unit vector vertically up. Displacement components are measured in metres.

a. **i.** Show that it takes 50 seconds for the drone to gain an altitude of 30 m. 1 mark

ii. Find the angle of elevation from O of the drone when it is at an altitude of 30 m. Give your answer in degrees, correct to the nearest degree. 2 marks

b. After how many seconds will the drone first be directly above the point of take-off? 1 mark

c. Show that the velocity of the drone is always perpendicular to its acceleration. 2 marks

d. Find the constant speed of the drone, in m s^{-1}, correct to two decimal places. 1 mark

e. The top of a tree has the position vector $\underset{\sim}{r}_{\text{tree}} = 60\underset{\sim}{i} + 25\underset{\sim}{j} + 8\underset{\sim}{k}$.

Find the minimum distance from the drone to the top of the tree. Give your answer in metres, correct to one decimal place. 3 marks

Question 5 (10 marks)

Consider the three points $A\,(-1, 1, 2)$, $B\,(0, 3, 3)$ and $C\,(3, 2, -1)$, which lie in the plane P.

a. Show that the points A, B and C do not lie on a straight line. 2 marks

b. Find the Cartesian equation of the plane P. 2 marks

Now consider the plane Q, given by $2x - y + 3z = 1$, and the line L, given by
$\underset{\sim}{r}(t) = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k} + t\left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right), t \in R$.

c. Show that the line L does not intersect the plane Q. 3 marks

d. Find the shortest distance from the line L to the plane Q. 3 marks

Question 6 (9 marks)

A process to manufacture a particular type of vinegar involves the conversion of sugar. It is known that stopping the conversion process after four weeks results in the sugar concentration of the vinegar being approximately normally distributed, with a mean of 0.37 grams per litre and a standard deviation of 0.12 grams per litre. The vinegar is bottled for sale.

Bottles of the vinegar are selected at random and tested for their sugar concentration.

a. Write down the mean and standard deviation of the sampling distribution for the concentration of sugar in a random sample of nine bottles. 1 mark

b. What is the required minimum sample size so that there is 90% confidence that the true mean concentration of sugar is within ± 0.05 of the sample mean? 2 marks

A shorter manufacturing process for the vinegar is proposed, which only requires three weeks.

A random sample of 56 bottles containing the vinegar made by the shorter process has a mean sugar concentration of 0.41 grams per litre. A one-sided statistical test at the 1% level of significance will be conducted to determine whether the shorter process results in a higher sugar concentration than the original process. Assume that the population standard deviation is 0.12 grams per litre.

c. Write down suitable null and alternative hypotheses for the statistical test. 1 mark

d. **i.** Find the p value for this test correct to three decimal places. 1 mark

ii. Draw an appropriate conclusion about H_0 from the p value found in **part d.i**. Give a reason involving p for your conclusion. 1 mark

e. Find the largest mean concentration of sugar that could be observed from a sample of 56 bottles for the null hypothesis not to be rejected at the 1% level of significance. Give your answer correct to three decimal places. 1 mark

f. Assume that the true mean concentration of sugar from the shorter process is 0.40 grams per litre and the standard deviation is 0.12 grams per litre. A sample of 56 bottles is taken.

Find the probability of a Type II error for the one-sided test if the level of significance is changed to 5% for the test. Give your answer correct to one decimal place. 2 marks

End of examination questions

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Specialist Mathematics Examination 2

Formula Sheet

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2} bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3} Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$

You may keep this Formula Sheet.

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Data analysis, probability and statistics – continued

approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\mathrm{E}\left(\bar{X}\right) = \mu$
	variance	$\mathrm{Var}\left(\bar{X}\right) = \dfrac{\sigma^2}{n}$

Calculus

$\dfrac{d}{dx}\left(x^n\right) = n x^{n-1}$	$\int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c,\quad n \neq -1$
$\dfrac{d}{dx}\left(e^{ax}\right) = a e^{ax}$	$\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$	$\int \dfrac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}\left(\cot(ax)\right) = -a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax)\,dx = -\dfrac{1}{a}\cot(ax) + c$
$\dfrac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx = \dfrac{1}{a}\sec(ax) + c$
$\dfrac{d}{dx}\left(\mathrm{cosec}(ax)\right) = -a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\,dx = -\dfrac{1}{a}\mathrm{cosec}(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right) = \dfrac{a}{\sqrt{1-(ax)^2}}$	$\int \dfrac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right) = \dfrac{-a}{\sqrt{1-(ax)^2}}$	$\int \dfrac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right) = \dfrac{a}{1+(ax)^2}$	$\int \dfrac{a}{a^2 + x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \dfrac{1}{a(n+1)}(ax+b)^{n+1} + c,\quad n \neq -1$
	$\int \dfrac{1}{ax+b}\,dx = \dfrac{1}{a}\log_e\lvert ax+b\rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f\left(x_n, y_n\right)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

End of Formula Sheet