VCE Further Mathematics 2016–2018

Written examinations 1 and 2 – End of year

Examination specifications

Overall conditions

There will be two end-of-year examinations for VCE Further Mathematics – examination 1 and examination 2.

The examinations will be sat at a time and date to be set annually by the Victorian Curriculum and Assessment Authority (VCAA). VCAA examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.

Both examinations will have 15 minutes reading time and 1 hour and 30 minutes writing time.

For both examinations, students are permitted to bring into the examination room an approved technology with numerical, graphical, symbolic, financial and statistical functionality, as specified in the *VCAA Bulletin* and the *VCE Exams Navigator*. One bound reference may be brought into the examination room. This may be a textbook (which may be annotated), a securely bound lecture pad, a permanently bound student-constructed set of notes without fold-outs or an exercise book. Specifications for the bound reference are published annually in the *VCE Exams Navigator*.

A formula sheet will be provided with both examinations.

The examinations will be marked by a panel appointed by the VCAA.

The examinations will each contribute 33 per cent to the study score.

Content

The *VCE Mathematics Study Design 2016–2018* ('Further Mathematics Units 3 and 4') is the document for the development of the examination. All outcomes in 'Further Mathematics Units 3 and 4' will be examined.

All content from the areas of study, and the key knowledge and skills that underpin the outcomes in Units 3 and 4, are examinable.

Examination 1 will cover both Areas of study 1 and 2. The examination is designed to assess students' knowledge of mathematical concepts, models and techniques, and their ability to reason, interpret and apply this knowledge in a range of contexts.

Examination 2 will cover both Areas of study 1 and 2. The examination is designed to assess students' ability to select and apply mathematical facts, concepts, models and techniques to solve extended application problems in a range of contexts.

Format

Examination 1

The examination will be in the form of a multiple-choice question book.

The examination will consist of two sections.

Section A will consist of 24 multiple-choice questions derived from the core component of the course. Of these 24 questions,16 will be on data analysis and 8 will be on recursion and financial modelling. All questions will be compulsory. Section A will be worth a total of 24 marks.

Section B will consist of eight multiple-choice questions on each of the four modules in Unit 4. Students must answer questions on **two** modules. Section B will be worth a total of 16 marks.

The total marks for the examination will be 40.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

All answers are to be recorded on the answer sheet provided for multiple-choice questions.

Examination 2

The examination will be in the form of a question and answer book.

The examination will consist of two sections.

Section A will consist of short-answer and extended-answer questions, including multi-stage questions of increasing complexity. Questions will be derived from the core component of the course. Of these, 24 marks will be allocated to data analysis and 12 marks will be allocated to recursion and financial modelling. All questions will be compulsory. Section A will be worth a total of 36 marks.

Section B will consist of short-answer and extended-answer questions, including multi-stage questions of increasing complexity. Questions will be derived from each of the four modules in Unit 4. Each module will contain questions that total 12 marks. Students must answer questions on **two** modules. Section B will be worth a total of 24 marks.

The total marks for the examination will be 60.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

Answers are to be recorded in the spaces provided in the question and answer book.

Approved materials and equipment

The list below applies to both examinations 1 and 2:

- normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)
- an approved technology with numerical, graphical, symbolic, financial and statistical functionality
- one scientific calculator
- one bound reference

Relevant references

The following publications should be referred to in relation to the VCE Further Mathematics examinations:

- *VCE Mathematics Study Design 2016–2018* (‘Further Mathematics Units 3 and 4’)
- *VCE Further Mathematics – Advice for teachers 2016–2018* (includes assessment advice)
- *VCE Exams Navigator*
- *VCAA Bulletin*

Advice

During the 2016–2018 accreditation period for VCE Further Mathematics, examinations will be prepared according to the examination specifications above. Each examination will conform to these specifications and will test a representative sample of the key knowledge and skills from all outcomes in Units 3 and 4.

The following sample examinations provide an indication of the types of questions teachers and students can expect until the current accreditation period is over.

Answers to multiple-choice questions are provided at the end of examination 1.

Answers to other questions are not provided.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

FURTHER MATHEMATICS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 32 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 4 – September 2016

THIS PAGE IS BLANK

SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1

The following stem plot shows the areas, in square kilometres, of 27 suburbs of a large city.

key: 1|6 = 1.6 km^2

1	5	6	7	8				
2	1	2	4	5	6	8	9	9
3	0	1	1	2	2	8	9	
4	0	4	7					
5	0	1						
6	1	9						
7								
8	4							

The median area of these suburbs, in square kilometres, is

A. 3.0
B. 3.1
C. 3.5
D. 30.1
E. 30.5

Question 2

The time spent by shoppers at a hardware store on a Saturday is approximately normally distributed with a mean of 31 minutes and a standard deviation of 6 minutes.

If 2850 shoppers are expected to visit the store on a Saturday, the number of shoppers who are expected to spend between 25 and 37 minutes in the store is closest to

A. 16
B. 68
C. 460
D. 1900
E. 2400

Use the following information to answer Questions 3–6.

The following table shows the data collected from a random sample of seven drivers drawn from the population of all drivers who used a supermarket car park on one day. The variables in the table are:

- *distance* – the distance that each driver travelled to the supermarket from their home
- *sex* – the sex of the driver (female, male)
- *number of children* – the number of children in the car
- *type of car* – the type of car (sedan, wagon, other)
- *postcode* – the postcode of the driver's home.

***Distance* (km)**	***Sex* (F = female, M = male)**	***Number of children***	***Type of car* (1 = sedan, 2 = wagon, 3 = other)**	***Postcode***
4.2	F	2	1	8148
0.8	M	3	2	8147
3.9	F	3	2	8146
5.6	F	1	3	8245
0.9	M	1	3	8148
1.7	F	2	2	8147
2.5	M	2	2	8145

Question 3

The mean, $\bar{x}$, and the standard deviation, s_x, of the variable, distance, for these drivers are closest to

A. $\bar{x} = 2.5$ $s_x = 3.3$

B. $\bar{x} = 2.8$ $s_x = 1.7$

C. $\bar{x} = 2.8$ $s_x = 1.8$

D. $\bar{x} = 2.9$ $s_x = 1.7$

E. $\bar{x} = 3.3$ $s_x = 2.5$

Question 4

The number of discrete numerical variables in this data set is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 5

The number of ordinal variables in this data set is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 6

The number of female drivers with three children in the car is

A. 0

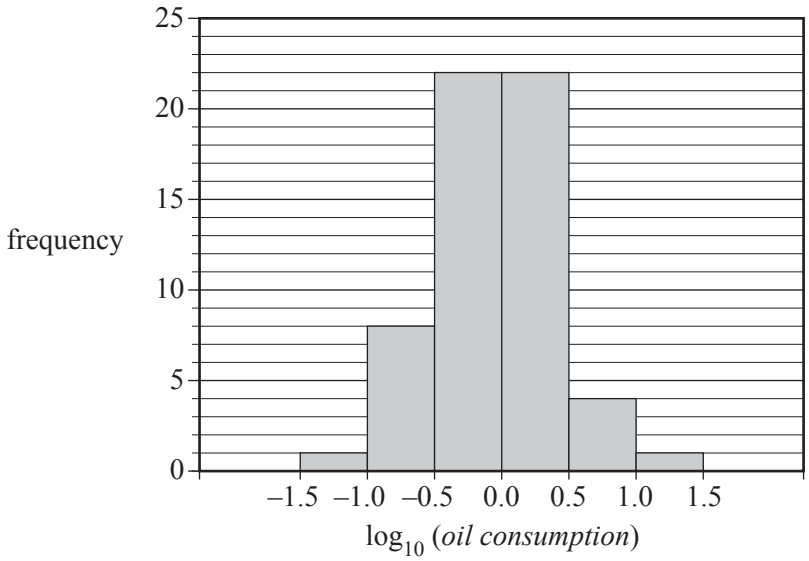
B. 1

C. 2

D. 3

E. 4

Question 7



The histogram above displays the distribution of the annual per capita *oil consumption* (tonnes) for 58 countries plotted on a **log** scale.

The percentage of countries with an annual per capita *oil consumption* of more than 10 tonnes is closest to

A. 1%

B. 2%

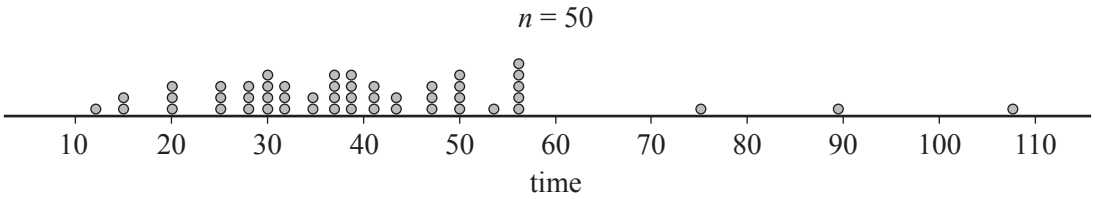
C. 27%

D. 57%

E. 98%

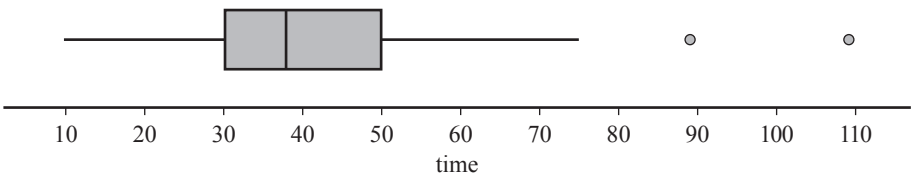
Question 8

The dot plot below shows the distribution of the time, in minutes, that 50 people spent waiting to get help from a call centre.

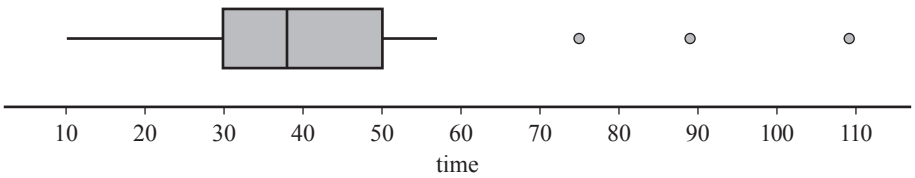


Which one of the following boxplots best represents the data?

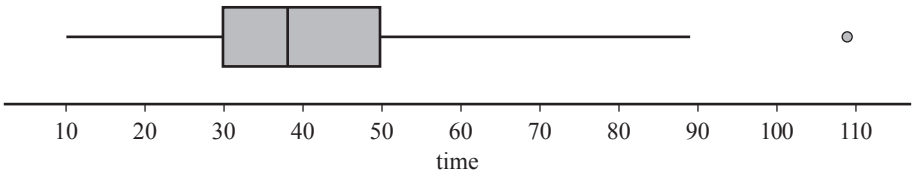
A.



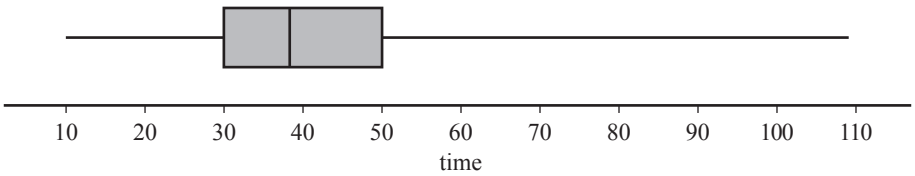
B.



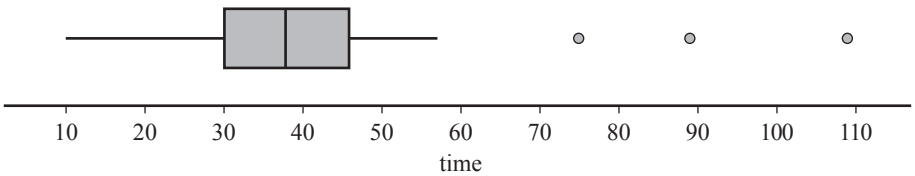
C.



D.

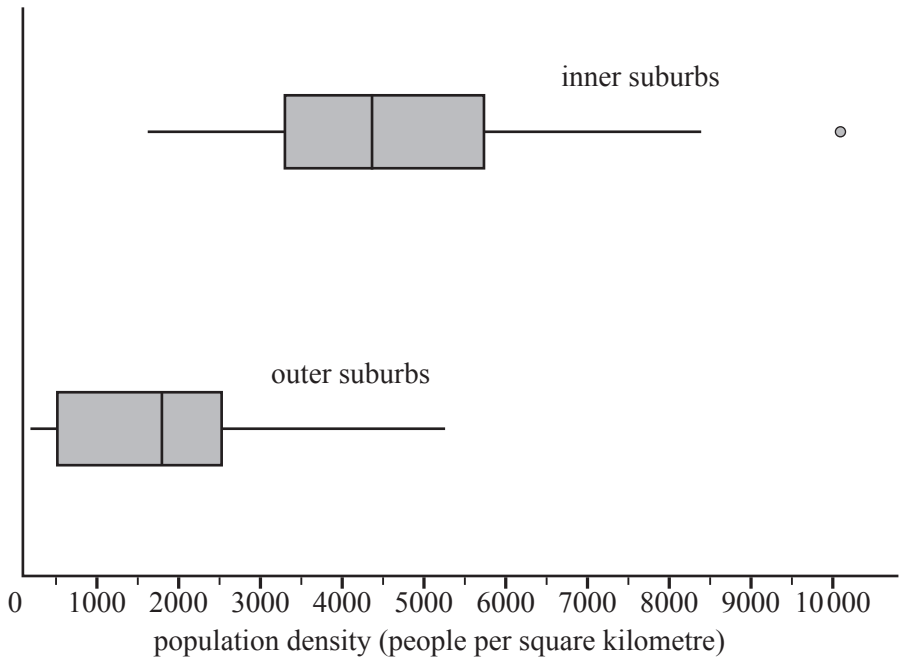


E.



Question 9

The parallel boxplots below summarise the distribution of population density, in people per square kilometre, for the inner suburbs and the outer suburbs of a large city.



Which one of the following statements is **not** true?

A. More than 50% of the outer suburbs have population densities below 2000 people per square kilometre.

B. More than 75% of the inner suburbs have population densities below 6000 people per square kilometre.

C. Population densities are more variable in the outer suburbs than in the inner suburbs.

D. The median population density of the inner suburbs is approximately 4400 people per square kilometre.

E. Population densities are, on average, higher in the inner suburbs than in the outer suburbs.

Question 10

A single back-to-back stem plot would be an appropriate graphical tool to investigate the association between a car's speed, in kilometres per hour, and the

A. driver's age, in years.

B. car's colour (white, red, grey, other).

C. car's fuel consumption, in kilometres per litre.

D. average distance travelled, in kilometres.

E. driver's sex (female, male).

Question 11

The equation of a least squares regression line is used to predict the fuel consumption, in kilometres per litre of fuel, from a car's weight, in kilograms.

This equation predicts that a car weighing 900 kg will travel 10.7 km per litre of fuel, while a car weighing 1700 kg will travel 6.7 km per litre of fuel.

The slope of this least squares regression line is closest to

A. –200.0

B. –0.005

C. –0.004

D. 0.005

E. 200.0

Question 12

A large study of secondary-school male students shows that there is a negative association between the time spent playing sport each week and the time spent playing computer games.

From this information, it can be concluded that

A. male students who spend a lot of time playing computer games do not play sport.

B. encouraging male students to spend less time playing sport will increase the time they spend playing computer games.

C. encouraging male students to spend more time playing sport will reduce the time they spend playing computer games.

D. male students who tend to spend more time playing sport tend to spend less time playing computer games.

E. male students who tend to spend more time playing sport tend to spend more time playing computer games.

Question 13

The seasonal index for heaters in winter is 1.25

To correct for seasonality, the actual heater sales in winter should be

A. reduced by 20%

B. increased by 20%

C. reduced by 25%

D. increased by 25%

E. reduced by 75%

Use the following information to answer Questions 14 and 15.

The seasonal indices for the first 11 months of the year for sales in a sporting equipment store are shown in the table below.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Seasonal index	1.23	0.96	1.12	1.08	0.89	0.98	0.86	0.76	0.76	0.95	1.12	

Question 14

The seasonal index for December is

A. 0.89

B. 0.97

C. 1.02

D. 1.23

E. 1.29

Question 15

In May, the store sold $213 956 worth of sporting equipment.

The deseasonalised value of these sales was closest to

A. $165 857

B. $190 420

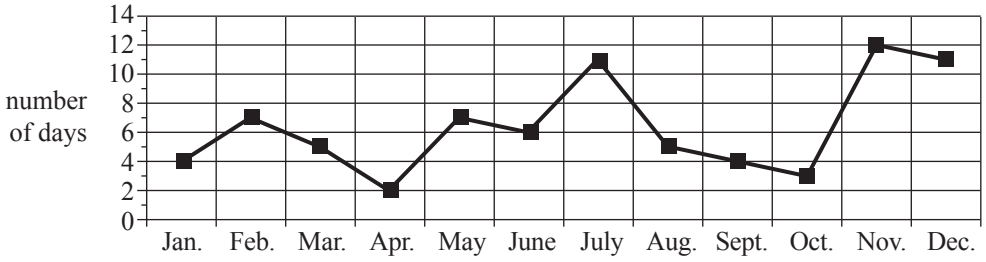
C. $209 677

D. $218 322

E. $240 400

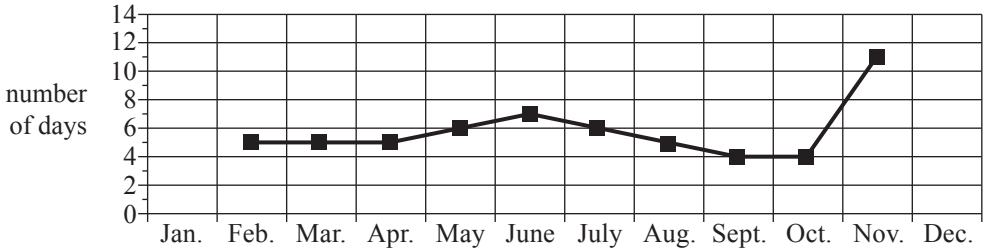
Question 16

The time series plot below shows the number of days that it rained in a town each month during 2011.

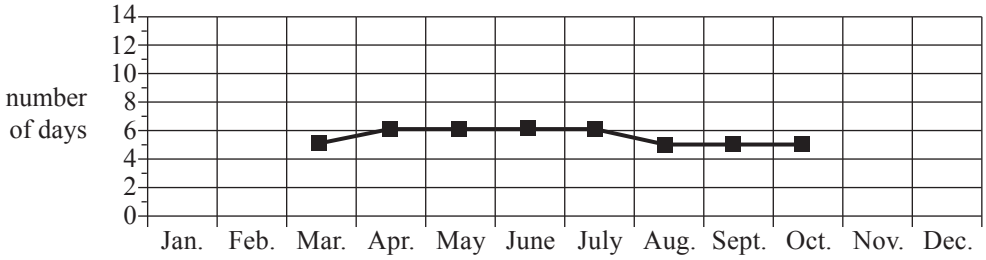


Using five-median smoothing, the smoothed time series plot will look most like

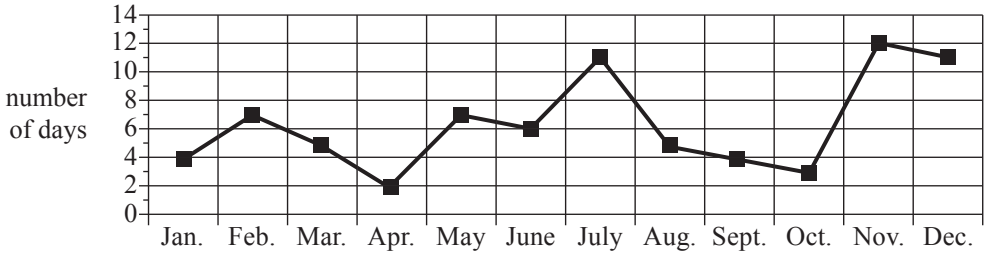
A.



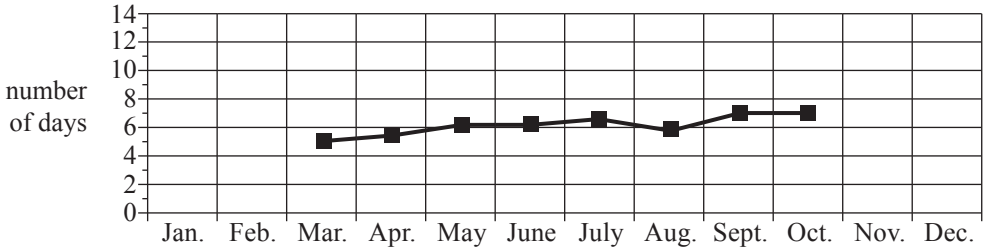
B.



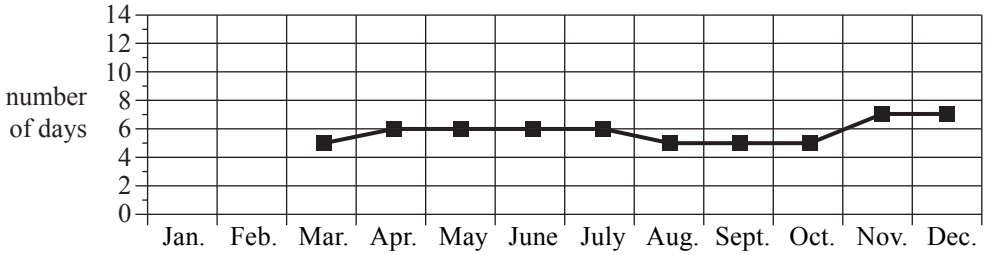
C.



D.



E.



Recursion and financial modelling

Question 17

$$P_0 = 2000,\ P_{n+1} = 1.5P_n - 500$$

The first three terms of a sequence generated by the recurrence relation above are

A. 500, 2500, 2000 …

B. 2000, 1500, 1000 …

C. 2000, 2500, 3000 …

D. 2000, 2500, 3250 …

E. 2000, 3000, 4500 …

Question 18

Which of the following recurrence relations will generate a sequence whose values decay geometrically?

A. $L_0 = 2000,\ L_{n+1} = L_n - 100$

B. $L_0 = 2000,\ L_{n+1} = L_n + 100$

C. $L_0 = 2000,\ L_{n+1} = 0.65L_n$

D. $L_0 = 2000,\ L_{n+1} = 6.5L_n$

E. $L_0 = 2000,\ L_{n+1} = 0.85L_n - 100$

Question 19

Eva has $1200 that she plans to invest for one year.

One company offers to pay her interest at the rate of 6.75% per annum compounding daily.

The effective annual interest rate for this investment would be closest to

A. 6.75%

B. 6.92%

C. 6.96%

D. 6.98%

E. 6.99%

Question 20

Rohan invests $15 000 at an annual interest rate of 9.6% compounding monthly.

Let V_n be the value of the investment after n months.

A recurrence relation that can be used to model this investment is

A. $V_0 = 15\,000,\ V_{n+1} = 0.96V_n$

B. $V_0 = 15\,000,\ V_{n+1} = 1.008V_n$

C. $V_0 = 15\,000,\ V_{n+1} = 1.08V_n$

D. $V_0 = 15\,000,\ V_{n+1} = 1.0096V_n$

E. $V_0 = 15\,000,\ V_{n+1} = 1.096V_n$

Use the following information to answer Questions 21–23.

Kim invests $400 000 in an annuity paying 3.2% interest per annum.

The annuity is designed to give her an annual payment of $47 372 for 10 years.

The amortisation table for this annuity is shown below.

Some of the information is missing.

Payment number (n)	Payment made	Interest earned	Reduction in principal	Balance of annuity
0	0	0.00	0.00	400 000.00
1	47 372.00	12 800.00	34 572.00	
2	47 372.00	11 693.70	35 678.30	329 749.70
3	47 372.00	10 551.99	36 820.01	292 929.69
4	47 372.00	9 373.75	37 998.25	254 931.44
5	47 372.00	8 157.81		215 717.24
6	47 372.00	6 902.95	40 469.05	175 248.19
7	47 372.00	5 607.94	41 764.06	133 484.14
8	47 372.00			90 383.63
9	47 372.00	2 892.28	44 479.72	45 903.90
10	47 372.00	1 468.92	45 903.08	0.83

Question 21

The balance of the annuity after one payment has been made is

A. $339 828.00

B. $352 628.00

C. $365 428.00

D. $387 200.00

E. $400 000.00

Question 22

The reduction in the principal of the annuity after payment number 5 is

A. $36 820.01

B. $37 998.25

C. $39 214.19

D. $40 469.05

E. $41 764.06

Question 23

The amount of payment number 8 that is the interest earned is closest to

A. $3799.82

B. $4074.67

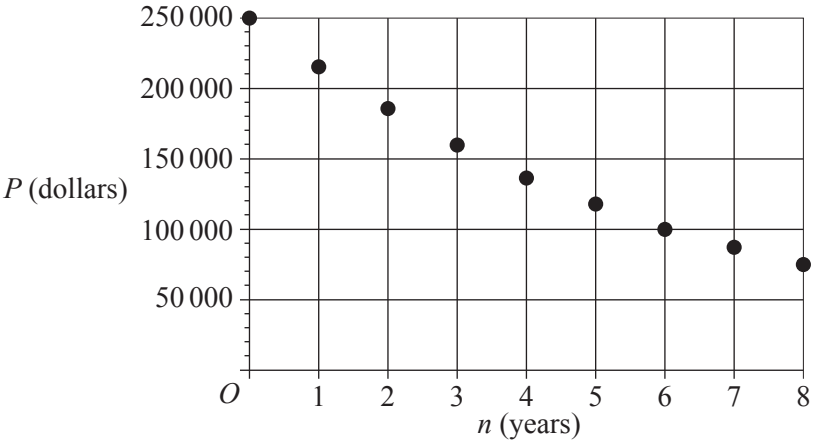
C. $4271.49

D. $4836.57

E. $5607.94

Question 24

The following graph shows the decreasing value of an asset over eight years.



Let P_n be the value of the asset after n years, in dollars.

A rule for evaluating P_n could be

A. $P_n = 250\,000 \times (1 + 0.14)^n$

B. $P_n = 250\,000 \times 1.14 \times n$

C. $P_n = 250\,000 \times (1 - 0.14) \times n$

D. $P_n = 250\,000 \times (0.14)^n$

E. $P_n = 250\,000 \times (1 - 0.14)^n$

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Matrix B, below, shows the number of photography (P), art (A) and cooking (C) books owned by Steven (S), Trevor (T), Ursula (U), Veronica (V) and William (W).

$$B = \begin{array}{c} \begin{array}{ccc} P & A & C \end{array} \\ \begin{bmatrix} 8 & 5 & 4 \\ 1 & 4 & 5 \\ 3 & 3 & 4 \\ 4 & 2 & 2 \\ 1 & 4 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ S \\ T \\ U \\ V \\ W \end{array}$$

The element in row i and column j of matrix B is b_{ij}.

The element b_{32} is the number of

A. art books owned by Trevor.

B. art books owned by Ursula.

C. art books owned by Veronica.

D. cooking books owned by Ursula.

E. cooking books owned by Trevor.

Question 2

The total cost of one ice-cream and three soft drinks at Catherine's shop is \$9.

The total cost of two ice-creams and five soft drinks is \$16.

Let x be the cost of an ice-cream and y be the cost of a soft drink.

The matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

D. $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

E. $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

Question 3

Consider the following four statements.

A permutation matrix is always:

I a square matrix

II a binary matrix

III a diagonal matrix

IV equal to the transpose of itself.

How many of the statements above are **true**?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

Four people, Ash (A), Binh (B), Con (C) and Dan (D), competed in a table tennis tournament.

In this tournament, each competitor played each of the other competitors once.

The results of the tournament are summarised in the matrix below.

A 1 in the matrix shows that the player named in that row defeated the player named in that column. For example, the 1 in row 3 shows that Con defeated Ash.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{winner} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

In the tournament, each competitor was given a ranking that was determined by calculating the sum of their one-step and two-step dominances.

The competitor with the highest sum is ranked number one (1). The competitor with the second-highest sum was ranked number two (2), and so on.

Using this method, the rankings of the competitors in this tournament were

A. Dan (1), Ash (2), Con (3), Binh (4).

B. Dan (1), Ash (2), Binh (3), Con (4).

C. Con (1), Dan (2), Ash (3), Binh (4).

D. Ash (1), Dan (2), Binh (3), Con (4).

E. Ash (1), Dan (2), Con (3), Binh (4).

Question 5

The matrix S_{n+1} is determined from the matrix S_n using the rule $S_{n+1} = T\,S_n - C$, where T, S_0 and C are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix},\ S_0 = \begin{bmatrix} 100 \\ 250 \end{bmatrix} \text{ and } C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Given this information, the matrix S_2 equals

A. $\begin{bmatrix} 100 \\ 250 \end{bmatrix}$

B. $\begin{bmatrix} 148 \\ 122 \end{bmatrix}$

C. $\begin{bmatrix} 170 \\ 140 \end{bmatrix}$

D. $\begin{bmatrix} 180 \\ 130 \end{bmatrix}$

E. $\begin{bmatrix} 190 \\ 160 \end{bmatrix}$

Question 6

A and B are square matrices such that $AB = BA = I$, where I is an identity matrix.

Which one of the following statements is **not** true?

A. $ABA = A$

B. $AB^2A = I$

C. B must equal A

D. B is the inverse of A

E. both A and B have inverses

Question 7

The order of matrix X is 3×2.

The element in row i and column j of matrix X is x_{ij} and it is determined by the rule

$$x_{ij} = i + j$$

The matrix X is

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

Question 8

A transition matrix, T, and a state matrix, S_2, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix S_1 is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$

B. $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$

C. $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

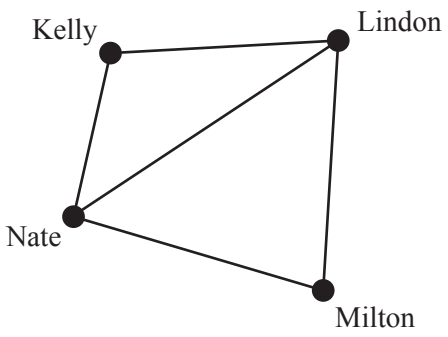
E. undefined

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below shows the roads connecting four towns: Kelly, Lindon, Milton and Nate.



A bus starts at Kelly, travels through Nate and Lindon, then stops when it reaches Milton.

The mathematical term for this route is

A. a loop.

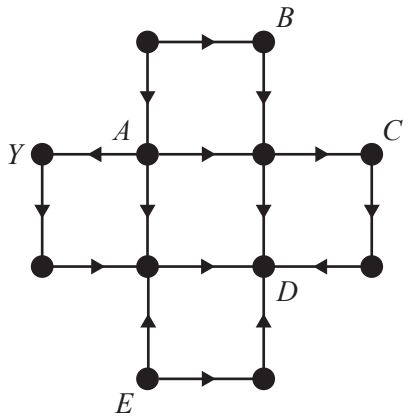
B. an Eulerian trail.

C. an Eulerian circuit.

D. a Hamiltonian path.

E. a Hamiltonian cycle.

Question 2



In the directed graph above, the only vertex with a label that can be reached from vertex *Y* is

A. vertex *A*.

B. vertex *B*.

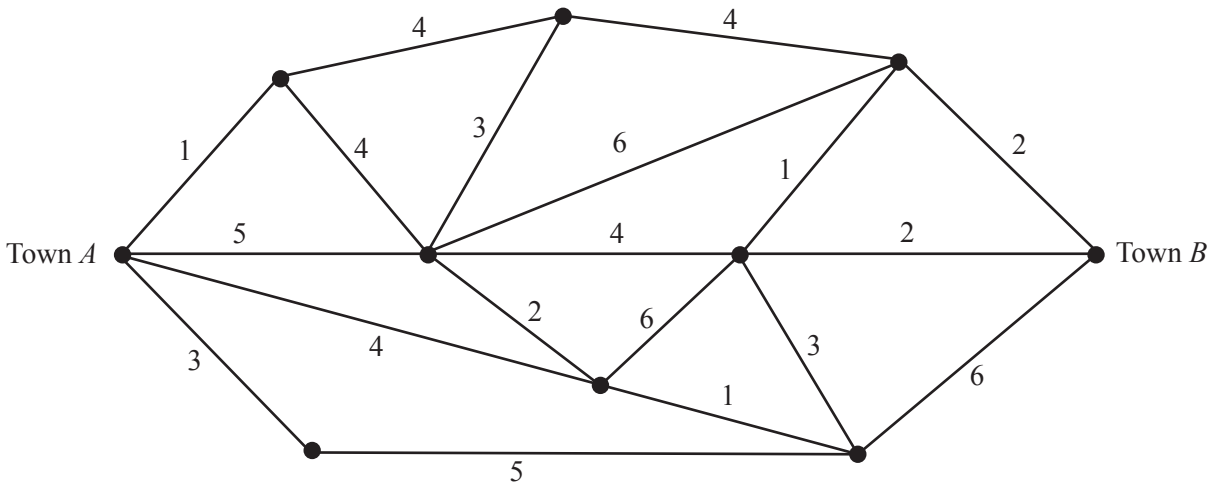
C. vertex *C*.

D. vertex *D*.

E. vertex *E*.

Question 3

The following network shows the distances, in kilometres, along a series of roads that connect Town *A* to Town *B*.



Using Dijkstra's algorithm, or otherwise, the shortest distance, in kilometres, from Town *A* to Town *B* is

A. 9

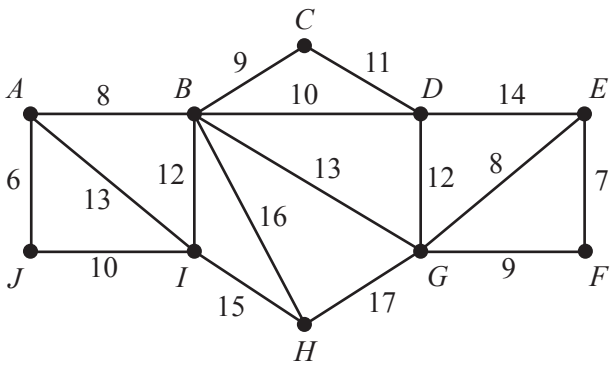
B. 10

C. 11

D. 12

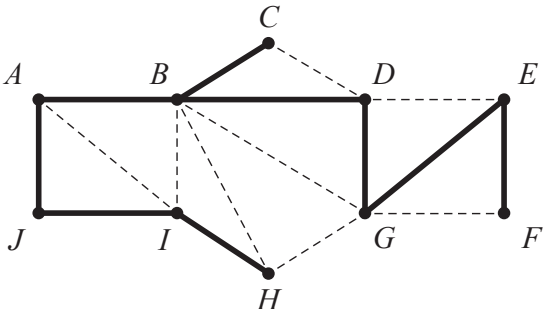
E. 13

Question 4

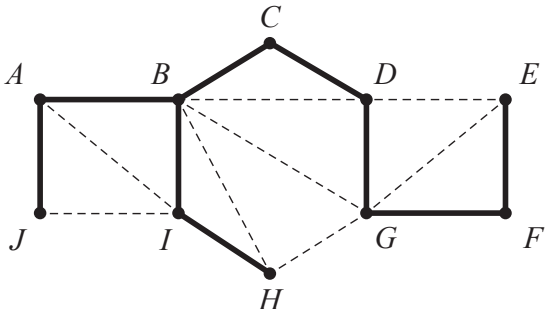


Which one of the following is the minimal spanning tree for the weighted graph shown above?

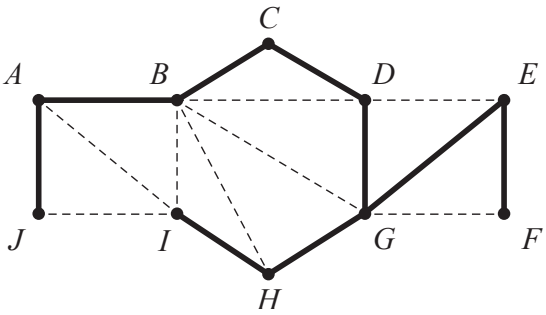
A.



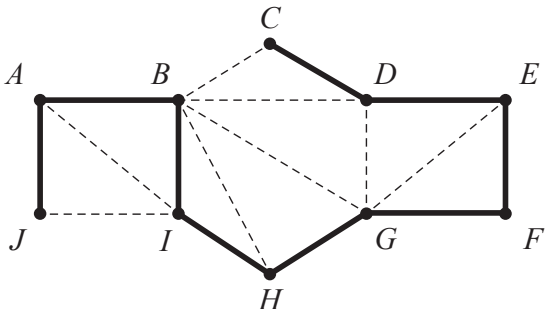
B.



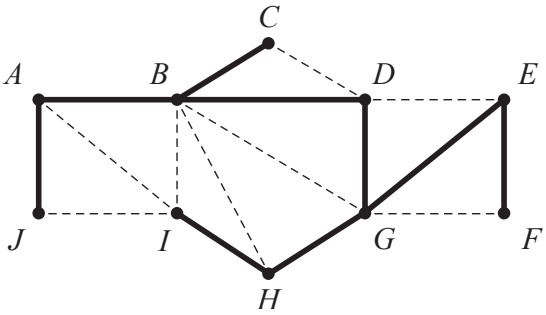
C.



D.

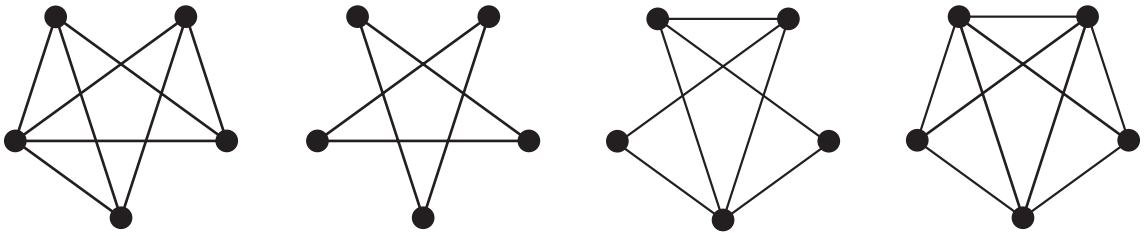


E.



Use the following information to answer Questions 5 and 6.

Consider the following four graphs.



Question 5

How many of the four graphs above have an Eulerian circuit?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 6

How many of the four graphs above are planar?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 7

Which one of the following statements about critical paths is **true**?

A. There can be only one critical path in a project.

B. A critical path always includes at least two activities.

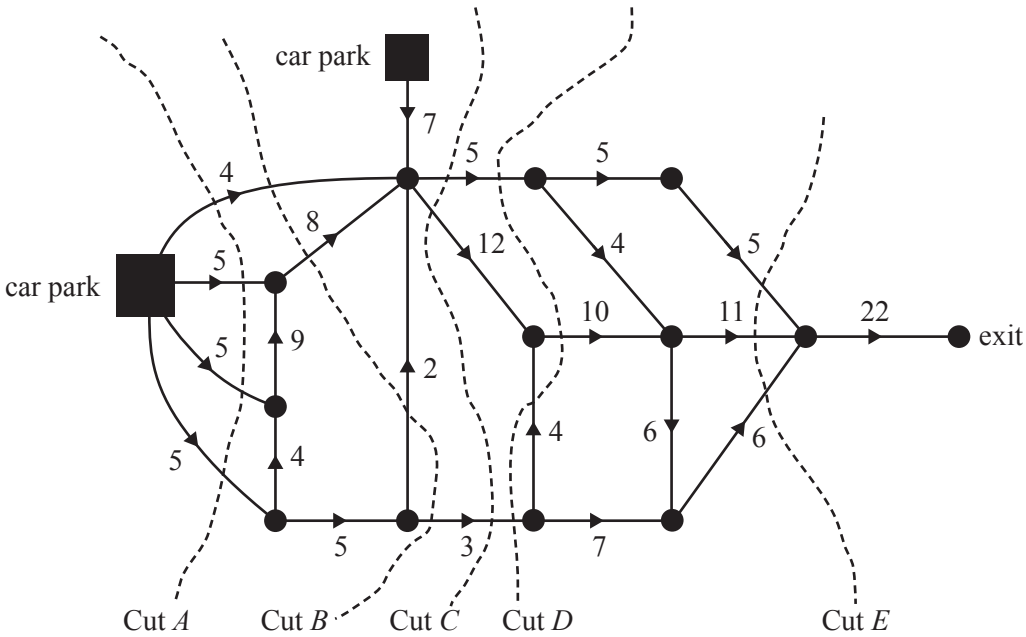
C. A critical path will always include the activity that takes the longest time to complete.

D. Reducing the time of any activity on a critical path for a project will always reduce the minimum completion time for the project.

E. If there are no other changes, increasing the time of any activity on a critical path will always increase the completion time of a project.

Question 8

A network of tracks connects two car parks in a festival venue to the exit, as shown in the directed graph below.



The arrows show the direction that cars can travel along each of the tracks and the numbers show each track's capacity in cars per minute.

Five cuts are drawn on the diagram.

The maximum number of cars per minute that will reach the exit is given by the capacity of

A. Cut *A*.

B. Cut *B*.

C. Cut *C*.

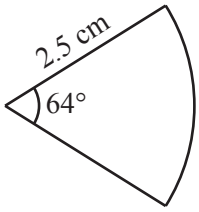
D. Cut *D*.

E. Cut *E*.

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



A sector of a circle of radius 2.5 cm subtends an angle of 64° at the centre of the circle.
The area of the sector, in square centimetres, is closest to

A. 2.8
B. 3.5
C. 7.0
D. 88.4
E. 110.5

Question 2

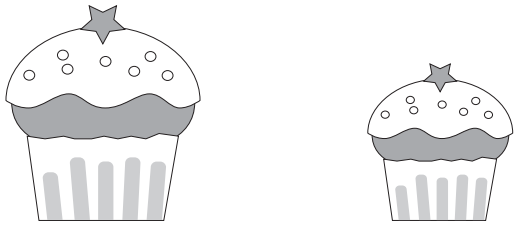
The city that is closest to the equator is

A. Athens, latitude 38.0° N
B. Belgrade, latitude 44.8° N
C. Kingston, latitude 45.3° S
D. Pretoria, latitude 25.7° S
E. Brisbane, latitude 27.5° S

Question 3

A cafe sells two sizes of cupcakes with a similar shape.

The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.



The price of a cupcake is proportional to its volume.

If the large cupcake costs \$5.40, then the small cupcake will cost

A. \$1.60

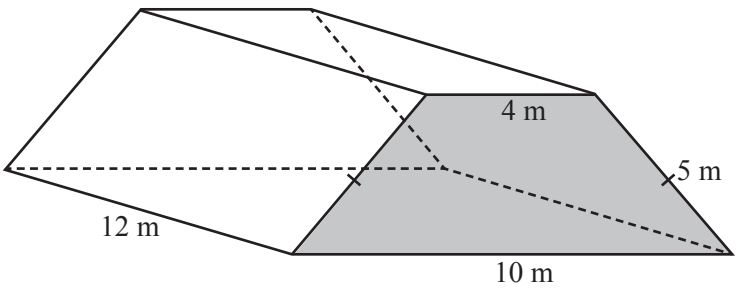
B. \$2.32

C. \$2.40

D. \$3.40

E. \$3.60

Question 4



A greenhouse is built in the shape of a trapezoidal prism, as shown in the diagram above.

The cross-section of the greenhouse (shaded) is an isosceles trapezium. The parallel sides of this trapezium are 4 m and 10 m respectively. The two equal sides are each 5 m.

The length of the greenhouse is 12 m.

The five exterior surfaces of the greenhouse, **not** including the base, are made of glass.

The total area of the glass surfaces of the greenhouse, in square metres, is

A. 196

B. 212

C. 224

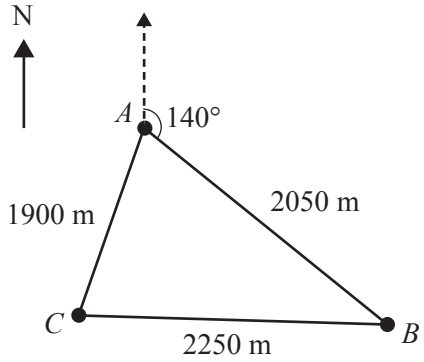
D. 344

E. 672

Use the following information to answer Questions 5 and 6.

Question 5

A cross-country race is run on a triangular course. The points A, B and C mark the corners of the course, as shown below.



The distance from A to B is 2050 m.

The distance from B to C is 2250 m.

The distance from A to C is 1900 m.

The bearing of B from A is 140°.

The bearing of C from A is closest to

A. 032°

B. 069°

C. 192°

D. 198°

E. 209°

Question 6

The area within the triangular course ABC, in square metres, can be calculated by evaluating

A. $\sqrt{3100 \times 1200 \times 1050 \times 850}$

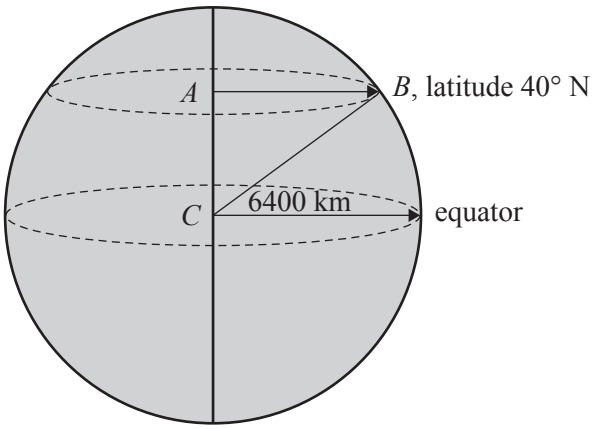
B. $\sqrt{3100 \times 2250 \times 2050 \times 1900}$

C. $\sqrt{6200 \times 4300 \times 4150 \times 3950}$

D. $\frac{1}{2} \times 2050 \times 2250 \times \sin(140°)$

E. $\frac{1}{2} \times 2050 \times 2250 \times \sin(40°)$

Question 7



Assume that the radius of Earth is 6400 km.

The diagram above shows a small circle of Earth, with centre at A, whose latitude is 40° N.

The radius of this small circle, in kilometres, is closest to

A. 4114

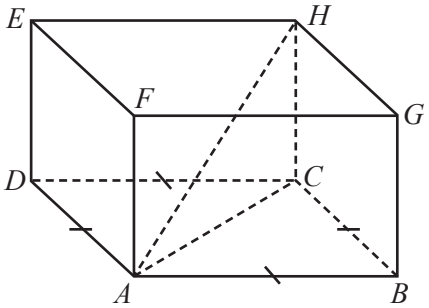
B. 4903

C. 5543

D. 6400

E. 7390

Question 8



A right rectangular prism with a square base, $ABCD$, is shown above.

The diagonal of the prism, AH, is 8 cm.

The height of the prism, HC, is 4 cm.

The volume of this rectangular prism, in cubic centimetres, is

A. 64

B. 96

C. 128

D. 192

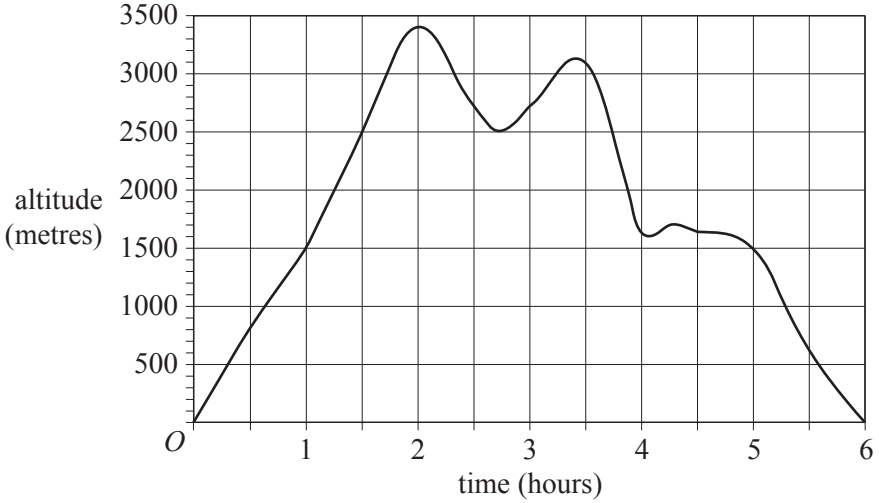
E. 256

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below shows the altitude, in metres, of a balloon over a six-hour flight.



Over the six-hour period, the length of time, in hours, where the altitude of the balloon was at least 1500 m is

A. 3

B. 4

C. 5

D. 6

E. 7

Question 2

The vertical line that passes through the point (3, 2) has the equation

A. $x + y = 5$

B. $xy = 6$

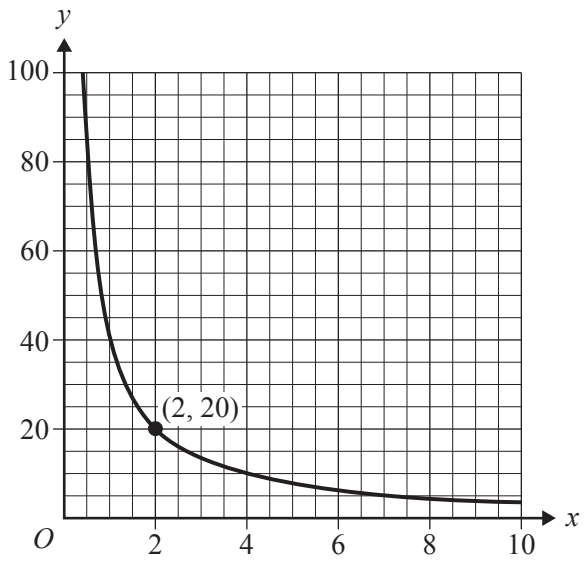
C. $3y = 2x$

D. $y = 2$

E. $x = 3$

Question 3

The point (2, 20) lies on the graph of $y = \frac{k}{x}$, as shown below.



The value of k is

A. 5

B. 10

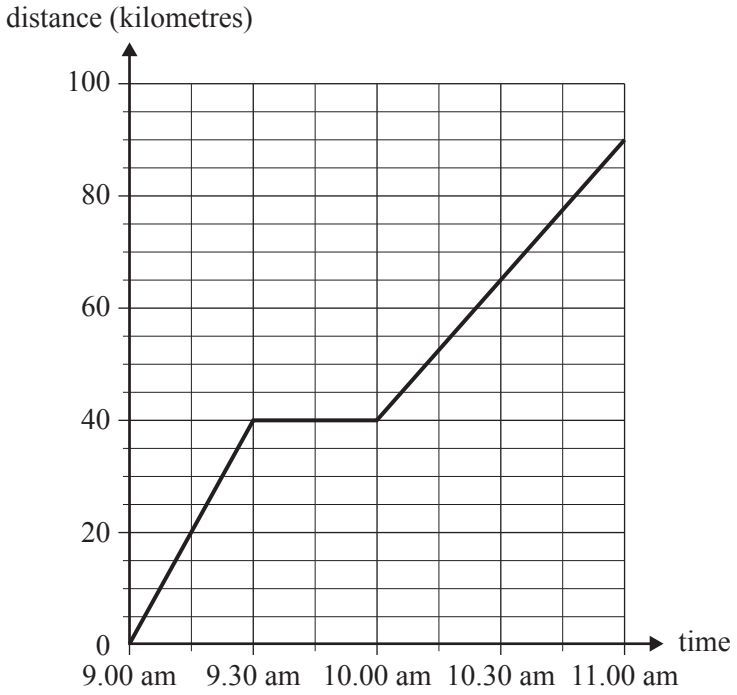
C. 20

D. 40

E. 80

Question 4

The distance–time graph below shows a train's journey between two towns.
During the journey, the train stopped for 30 minutes.



The average speed of the train, in kilometres per hour, for the journey is closest to

A. 45
B. 50
C. 60
D. 65
E. 80

Question 5

The Domestics Cleaning Company provides household cleaning services.
For two hours of cleaning, the cost is \$55.
For four hours of cleaning, the cost is \$94.
The rule for the cost of cleaning services is

$$cost = a + b \times hours$$

where a is a fixed charge, in dollars, and b is the charge per hour of cleaning, in dollars per hour.
Using this rule, the cost for five hours of cleaning is

A. \$19.50
B. \$97.50
C. \$99.50
D. \$113.50
E. \$121.50

Question 6

Which one of the following statements relating to the solution of linear programming problems is **true**?

A. Only one point can be a solution.

B. No point outside the feasible region can be a solution.

C. To have a solution, the feasible region must be bounded.

D. Only the corner points of a feasible region can be a solution.

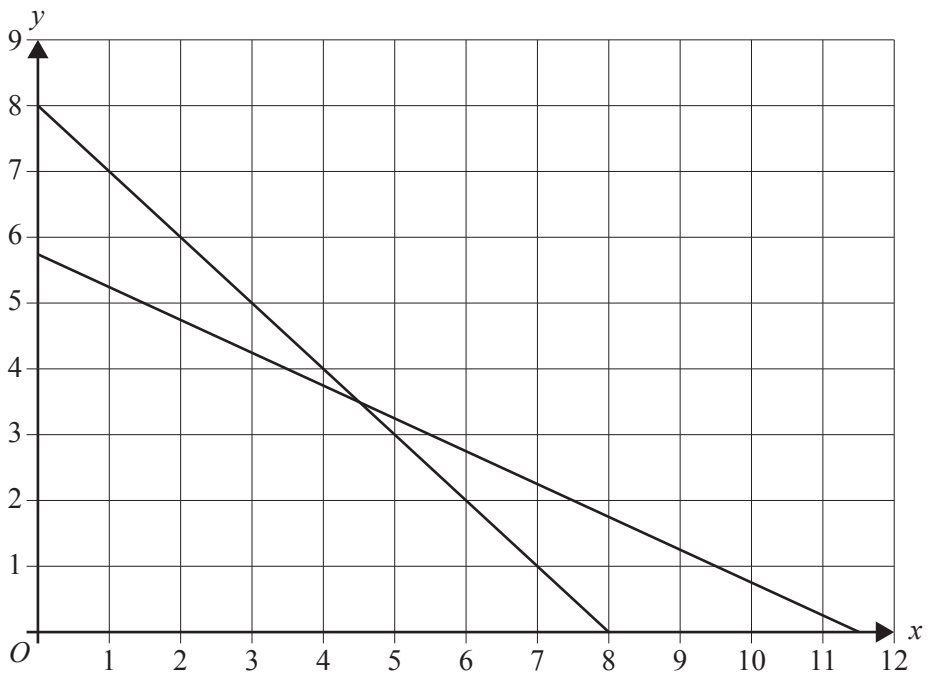
E. Only the corner points with integer coordinates can be a solution.

Question 7

The constraints of a linear programming problem are given by the following set of inequalities.

$$x + y \leq 8$$
$$2x + 4y \leq 23$$
$$x \geq 0$$
$$y \geq 0$$

The graph below shows the lines that represent the boundaries of these inequalities.



The coordinates of the points that define the boundaries of the feasible region for this linear programming problem are (0, 0), (0, 5.75), (4.5, 3.5) and (8, 0).

When maximising the objective function $Z = 2x + 3y$ for these constraints, the solution is found at the point (4.5, 3.5).

If only **integer solutions** are permitted for this problem, the solution will occur at

A. (1, 5)

B. (3, 4)

C. (4, 4)

D. (5, 3)

E. (6, 2)

Question 8

Xavier and Yvette share a job.

Yvette must work at least twice as many hours as Xavier.

They must work at least 40 hours each week, in total.

Xavier must work at least 10 hours each week.

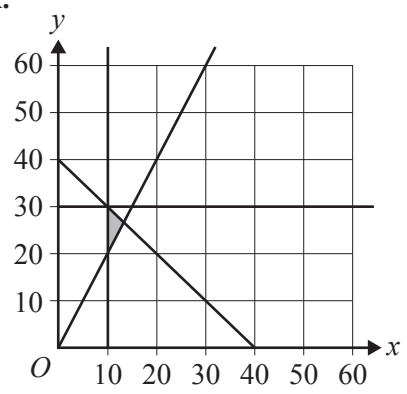
Yvette can only work for a maximum of 30 hours each week.

Let x represent the number of hours that Xavier works each week.

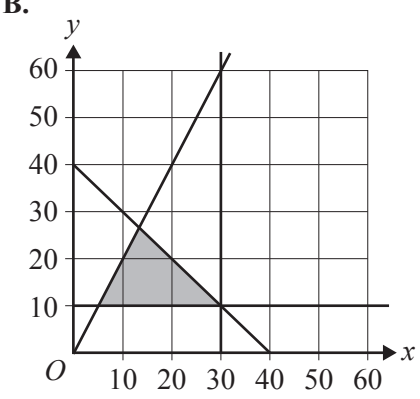
Let y represent the number of hours that Yvette works each week.

In which one of the following graphs does the shaded area show the feasible region defined by these conditions?

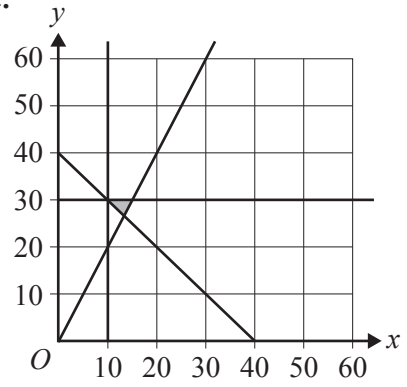
A.



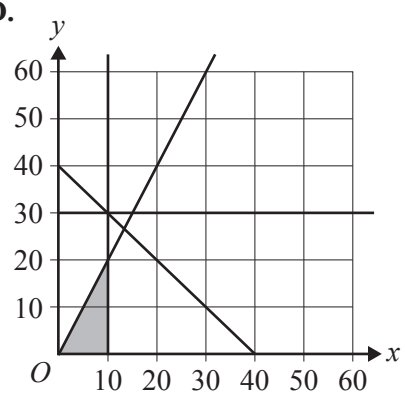
B.



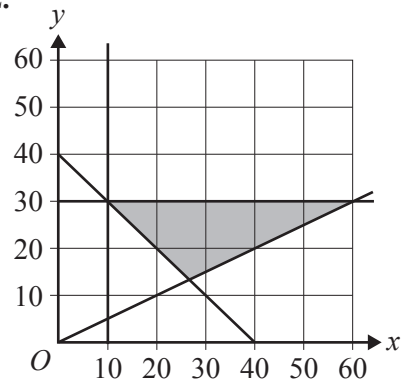
C.



D.



E.



END OF MULTIPLE-CHOICE QUESTION BOOK

Answers to multiple-choice questions

Section A – Core

Data analysis

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Answer	B	D	C	B	A	B	B	A	C	E	B	D	A	E	E	B

Recursion and financial modelling

Question	17	18	19	20	21	22	23	24
Answer	D	C	D	B	C	C	C	E

Section B – Modules

Module 1 – Matrices

Question	1	2	3	4	5	6	7	8
Answer	B	E	C	E	B	C	E	D

Module 2 – Networks and decision mathematics

Question	1	2	3	4	5	6	7	8
Answer	D	D	B	A	B	E	E	D

Module 3 – Geometry and measurement

Question	1	2	3	4	5	6	7	8
Answer	B	D	A	C	E	A	B	B

Module 4 – Graphs and relations

Question	1	2	3	4	5	6	7	8
Answer	B	E	D	A	D	B	D	C

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | | Letter | |

FURTHER MATHEMATICS

Written examination 2

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour 30 minutes)**

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 30 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 4 – September 2016

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

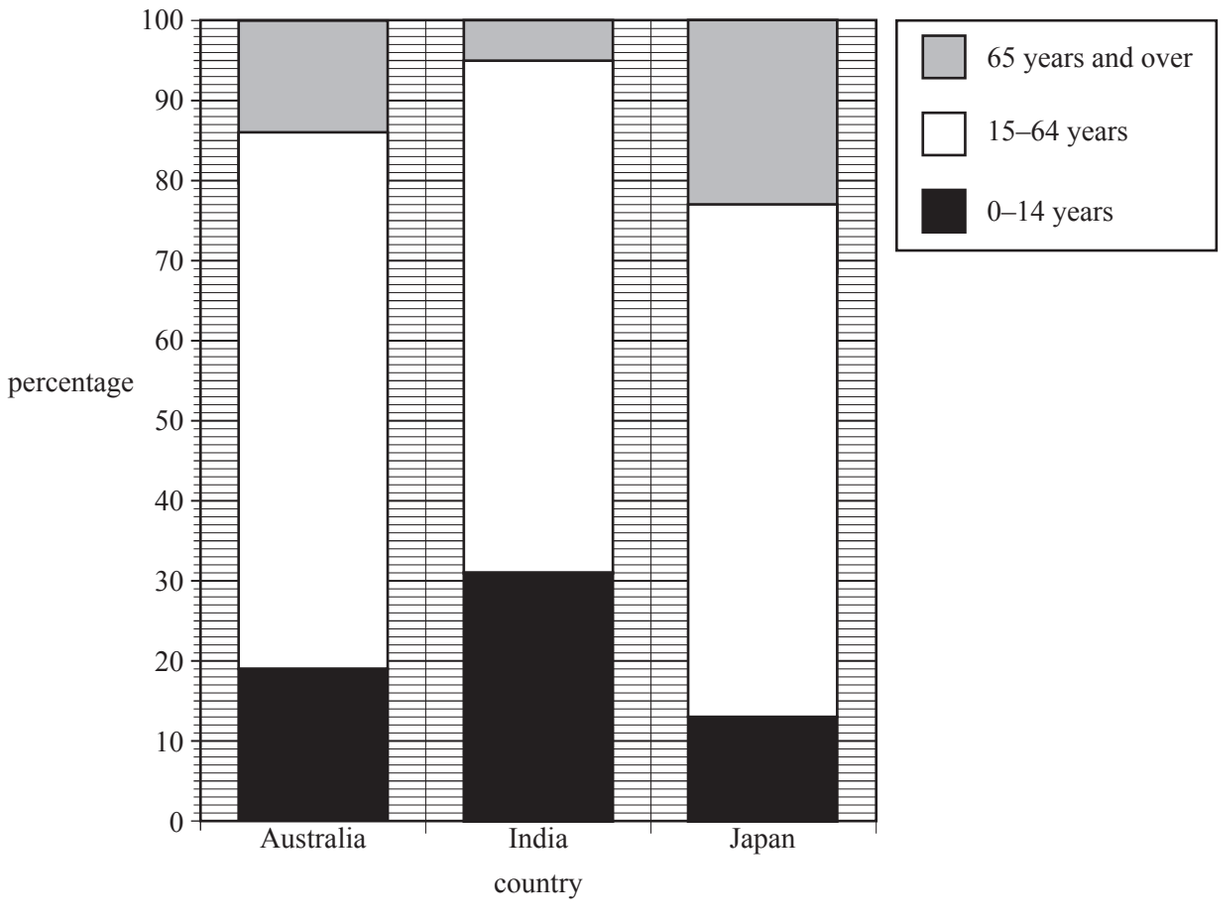
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



Source: Australian Bureau of Statistics, 3201.0 – *Population by Age and Sex, Australian States and Territories*, June 2010

DO NOT WRITE IN THIS AREA

a. Write down the percentage of people in Australia who were aged 0–14 years in 2010. 1 mark

b. In 2010, the population of Japan was 128 000 000.

How many people in Japan were aged 65 years and over in 2010? 1 mark

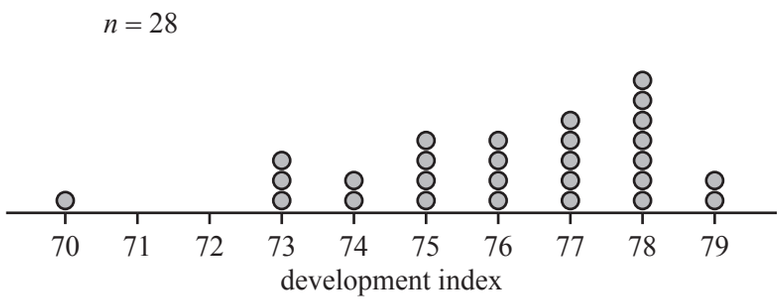
c. From the graph on page 2, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.

Explain why, quoting appropriate percentages to support your explanation. 1 mark

Question 2 (3 marks)

The development index for a country is a whole number between 0 and 100.

The dot plot below displays the values of the development indices for 28 countries.



a. Using the information in the dot plot, determine each of the following. 1 mark

The mode ☐ The range ☐

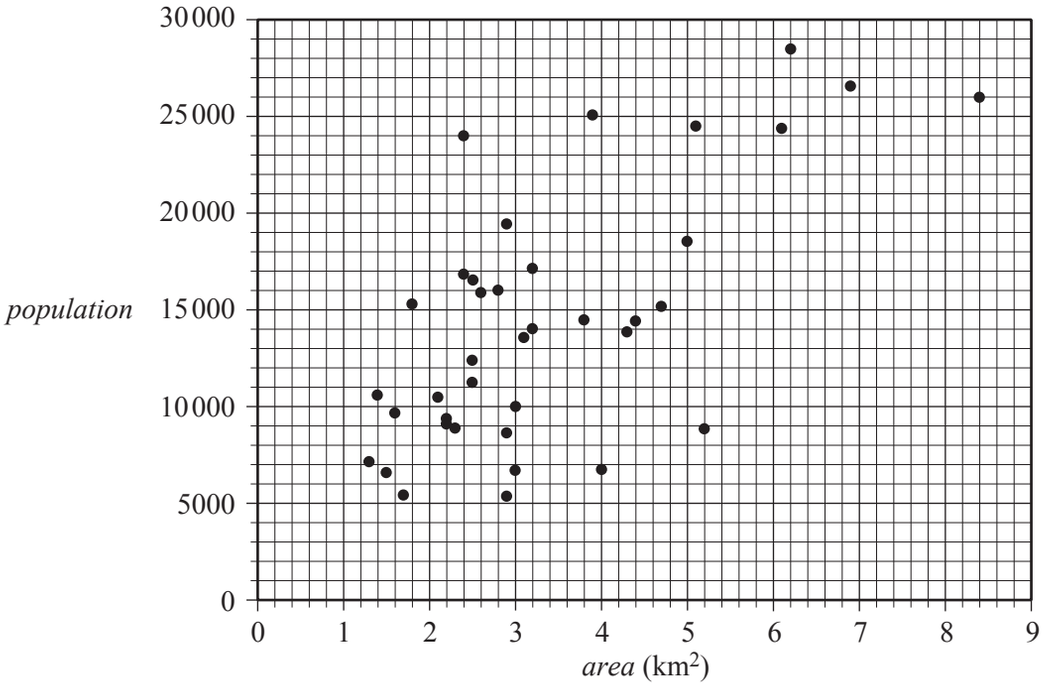
b. Write down an appropriate calculation and use it to explain why the country with a development index of 70 is an outlier for this group of countries. 2 marks

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is

$$population = 5330 + 2680 \times area$$

a. Write down the response variable. 1 mark

_____

b. Draw the least squares regression line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*. 2 marks

_____

_____

_____

d. Wiston is an inner suburb. It has an area of 4 km^2 and a population of 6690.

The correlation coefficient, r, is equal to 0.668

i. Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area. 1 mark

ii. What percentage of the variation in the population of the suburbs is explained by the variation in area?

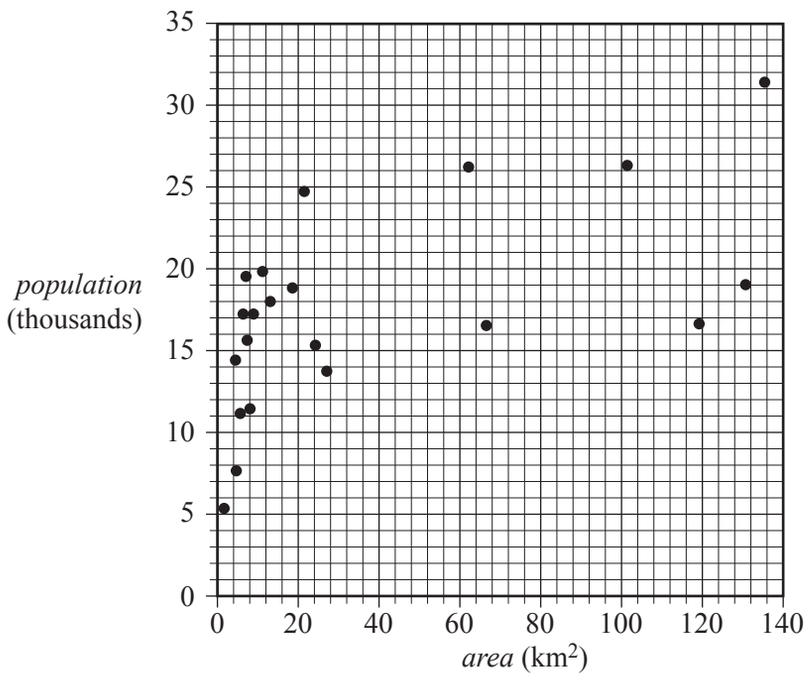
Round your answer to one decimal place. 1 mark

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 (3 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.



Area (km^2)	*Population* (thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

In the outer suburbs, the relationship between *population* and *area* is non-linear.

A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this least squares regression line in the boxes provided below.

Round your answers to two significant figures. 2 marks

population = [ ] + [ ] log (*area*)

b. Use the equation of the least squares regression line in **part a.** to predict the population of an outer suburb with an area of 90 km^2.

Round your answer to the nearest one thousand people. 1 mark

Question 5 (4 marks)

There is a negative association between the variables *population density*, in people per square kilometre, and *area*, in square kilometres, of 38 inner suburbs of the same city.

For this association, $r^2 = 0.141$

a. Write down the value of the correlation coefficient for this association between the variables *population density* and *area*.

Round your answer to three decimal places. 1 mark

b. The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

	Population density **(people per km^2)**	***Area*** **(km^2)**
Mean	4370	3.4
Standard deviation	1560	1.6

One of these suburbs has a population density of 3082 people per square kilometre.

i. Determine the standard z-score of this suburb's population density.

Round your answer to one decimal place. 1 mark

ii. Interpret the z-score of this suburb's population density with reference to the mean population density. 1 mark

iii. Assume the areas of these inner suburbs are approximately normally distributed.

How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?

Round your answer to the nearest whole number. 1 mark

DO NOT WRITE IN THIS AREA

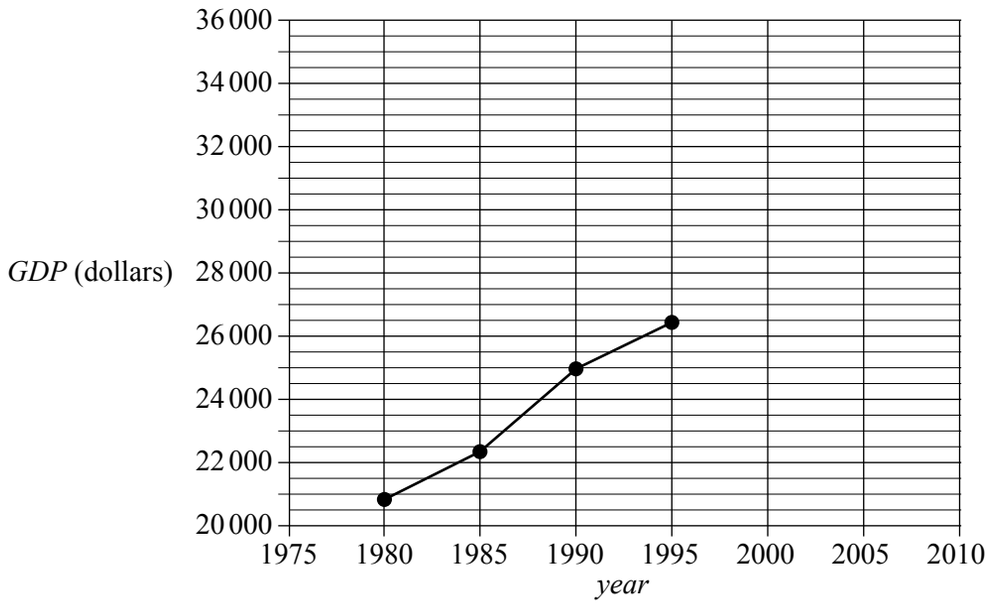
DO NOT WRITE IN THIS AREA

Question 6 (5 marks)

Table 1 shows the Australian gross domestic product (*GDP*) per person, in dollars, at five yearly intervals (*year*) for the period 1980 to 2005.

Table 1

Year	1980	1985	1990	1995	2000	2005
GDP	20 900	22 300	25 000	26 400	30 900	33 800



a. Complete the **time series plot above** by plotting the *GDP* for the years 2000 and 2005. 1 mark

(*Answer on the time series plot above*.)

b. Briefly describe the general trend in the data. 1 mark

_____

c. In Table 2, the variable year has been rescaled using 1980 = 0, 1985 = 5, and so on. The new variable is *time*.

Table 2

Year	1980	1985	1990	1995	2000	2005
Time	0	5	10	15	20	25
GDP	20 900	22 300	25 000	26 400	30 900	33 800

i. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the explanatory variable. 2 marks

ii. The least squares regression line in **part c.i.** above has been used to predict the *GDP* in 2010.

Explain why this prediction is unreliable. 1 mark

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Recursion and financial modelling

Question 7 (4 marks)

Hugo is a professional bike rider.

The value of his bike will be depreciated over time using the flat rate method of depreciation.

The value of Hugo's bike, in dollars, after n years, V_n, can be modelled using the recurrence relation below.

$$V_0 = 8400, \qquad V_{n+1} = V_n - 1200$$

a. Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000. 1 mark

Hugo will sell his bike when its value reduces to \$3600.

b. After how many years will Hugo sell his bike? 1 mark

The unit cost method can also be used to depreciate the value of Hugo's bike.

A rule for the value of the bike, in dollars, after travelling n kilometres is

$$V_n = 8400 - 0.25n$$

c. What is the depreciation of the bike per kilometre? 1 mark

After two years, the value of the bike when depreciated by the unit cost method will be the same as the value of the bike when depreciated by the flat rate method.

d. How many kilometres has the bike travelled after two years? 1 mark

Question 8 (5 marks)

Hugo won \$5000 in a road race. He deposited this money into a savings account.

The value of Hugo's savings after n months, S_n, can be modelled by the recurrence relation below.

$$S_0 = 5000, \qquad S_{n+1} = 1.004\, S_n$$

a. What is the annual interest rate (compounding monthly) for Hugo's savings account? 1 mark

b. What would be the value of Hugo's savings after 12 months? 1 mark

Using a different investment strategy, Hugo could deposit \$3000 into an account earning compound interest at the rate of 4.2% per annum, compounding monthly, and make additional payments of \$200 after every month.

Let T_n be the value of Hugo's investment after n months using this strategy.

The monthly interest rate for this account is 0.35%.

c. **i.** Write down a recurrence relation, in terms of T_{n+1} and T_n, that models the value of Hugo's investment using this strategy. 1 mark

ii. What is the total interest Hugo would have earned after six months? 2 marks

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 (3 marks)

Hugo needs to buy a new bike.

He borrowed $7500 to pay for the bike and will be charged interest at the rate of 5.76% per annum, compounding monthly.

Hugo will fully repay this loan with repayments of $430 each month.

a. How many repayments are required to fully repay this loan?
Round your answer to the nearest whole number. 1 mark

After the fifth repayment, Hugo increased his monthly repayment so that the loan was fully repaid with a further seven repayments (that is, 12 repayments in total).

b. **i.** What is the minimum value of Hugo's new monthly repayment? 1 mark

ii. What is the value of the final repayment required to ensure the loan is fully repaid after 12 repayments? 1 mark

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

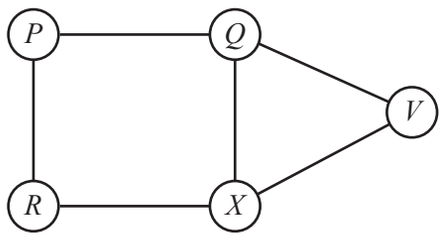
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Module 1 – Matrices

Question 1 (2 marks)

Five trout-breeding ponds, P, Q, R, X and V, are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

$$W = \begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ Q \\ R \\ X \\ V \end{array}$$

In matrix W:

- the 1 in row 2, column 1, for example, indicates that pond P is directly connected by a pipe to pond Q
- the 0 in row 5, column 1, for example, indicates that pond P is not directly connected by a pipe to pond V.

a. In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix W represent? 1 mark

The matrix W^2 is shown below.

$$W^2 = \begin{array}{c} \begin{array}{ccccc} P & Q & R & X & V \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 2 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ Q \\ R \\ X \\ V \end{array}$$

b. Matrix W^2 has a 2 in row 2 (Q), column 3 (R).

Explain what this number tells us about the pipe connections between Q and R. 1 mark

Question 2 (10 marks)

10 000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population.

In establishing this population:

- eggs (E) may die (D) or they may live and eventually become baby trout (B)
- baby trout (B) may die (D) or they may live and eventually become adult trout (A)
- adult trout (A) may die (D) or they may live for a period of time but will eventually die.

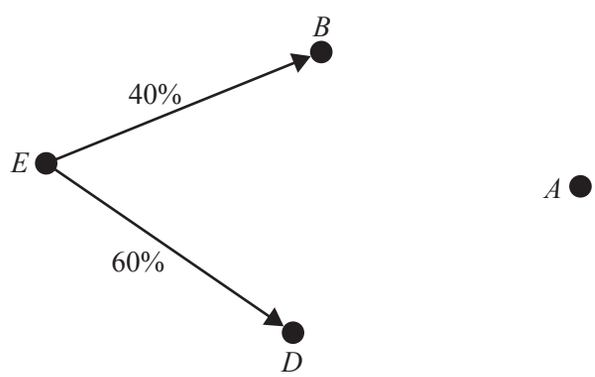
From year to year, this situation can be represented by the transition matrix T, where

$$T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} E & B & A & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ E \\ B \\ A \\ D \end{array} \textit{next year}$$

a. Use the information in the transition matrix T to

i. determine the number of eggs in this population that die in the first year 1 mark

ii. complete the transition diagram below, showing the relevant percentages. 2 marks



DO NOT WRITE IN THIS AREA

The initial state matrix for this trout population, S_0, can be written as

$$S_0 = \begin{array}{c} \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} E \\ B \\ A \\ D \end{array}$$

Let S_n represent the state matrix describing the trout population after n years.

b. Using the rule $S_{n+1} = T\,S_n$, determine

i. S_1 1 mark

ii. the number of adult trout predicted to be in the population after four years.
Round your answer to the nearest whole number of trout. 1 mark

c. The transition matrix T predicts that, in the long term, all of the eggs, baby trout and adult trout will die.

i. How many years will it take for all of the adult trout to die (that is, when the number of adult trout in the population is first predicted to be less than one)? 1 mark

ii. What is the largest number of adult trout that is predicted to be in the pond in any one year? 1 mark

d. Determine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at the end of each year, will ensure that the number of eggs, baby trout and adult trout in the population remains constant from year to year. 2 marks

The rule $S_{n+1} = T\,S_n$ that was used to describe the development of the trout in this pond does not take into account new eggs added to the population when the adult trout begin to breed.

To take breeding into account, assume that every year 50% of the adult trout each lay 500 eggs.

The matrix describing the population after n years, S_n, is now given by the new rule

$$S_{n+1} = T\,S_n + 500\,M\,S_n$$

where

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix},\ M = \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S_0 = \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

e. Use this new rule to determine S_2. 1 mark

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

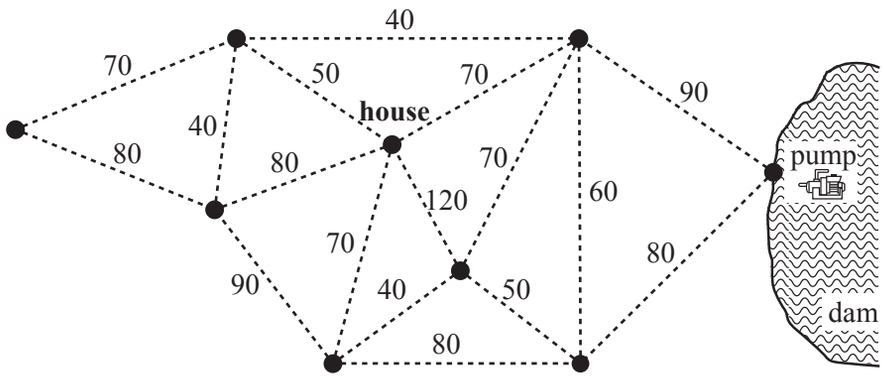
Module 2 – Networks and decision mathematics

Question 1 (6 marks)

Water will be pumped from a dam to eight locations on a farm.

The pump and the eight locations (including the house) are shown as vertices in the network diagram below.

The numbers on the edges joining the vertices give the shortest distances, in metres, between locations.



a. **i.** Determine the shortest distance between the house and the pump. 1 mark

ii. How many vertices on the network diagram have an odd degree? 1 mark

iii. The total length of all edges in the network is 1180 m.

A journey starts and finishes at the house and travels along every edge in the network.

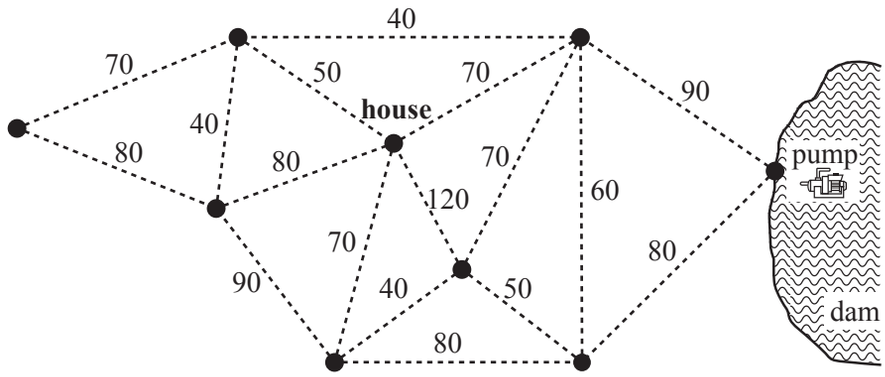
Determine the shortest distance travelled. 1 mark

iv. A Hamiltonian path, beginning at the house, is determined for this network.

How many edges does this path involve? 1 mark

The total length of pipe that supplies water from the pump to the eight locations on the farm is a minimum. This minimum length of pipe is laid along some of the edges in the network.

b. **i.** On the diagram below, draw the minimum length of pipe that is needed to supply water to all locations on the farm. 1 mark



ii. What is the mathematical term that is used to describe this minimum length of pipe in **part b.i.**? 1 mark

_____

DO NOT WRITE IN THIS AREA

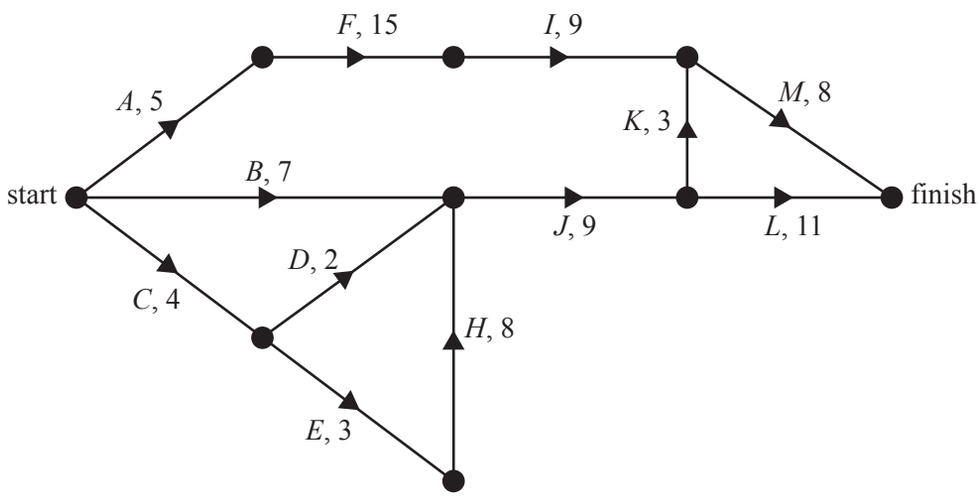
DO NOT WRITE IN THIS AREA

Question 2 (6 marks)

A project will be undertaken on the farm. This project involves the 13 activities shown in the table below. The duration, in hours, and predecessor(s) of each activity are also included in the table.

Activity	Duration (hours)	Predecessor(s)
A	5	–
B	7	–
C	4	–
D	2	*C*
E	3	*C*
F	15	*A*
G	4	*B*, *D*, *H*
H	8	*E*
I	9	*F*, *G*
J	9	*B*, *D*, *H*
K	3	*J*
L	11	*J*
M	8	*I*, *K*

Activity *G* is missing from the network diagram for this project, which is shown below.



a. Complete the **network diagram above** by inserting activity *G*. 1 mark

(*Answer on the network diagram above.*)

b. Determine the earliest starting time of activity *H*. 1 mark

c. Given that activity G is not on the critical path

i. write down the activities that are on the critical path in the order that they are completed 1 mark

ii. find the latest starting time for activity D. 1 mark

d. Consider the following statement:

'If just one of the activities in this project is crashed by one hour, then the minimum time to complete the entire project will be reduced by one hour.'

Explain the circumstances under which this statement will be true for this project. 1 mark

e. Assume activity F is crashed by two hours.

What will be the minimum completion time for the project? 1 mark

DO NOT WRITE IN THIS AREA

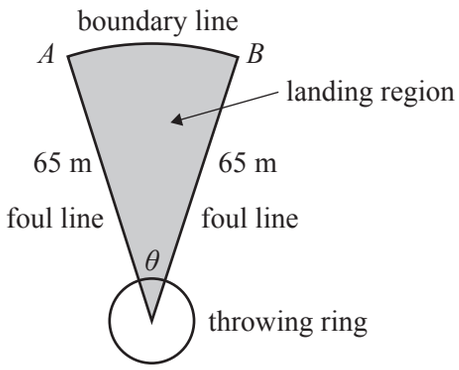
DO NOT WRITE IN THIS AREA

Module 3 – Geometry and measurement

Question 1 (3 marks)

One of the field events at athletics competitions is the discus.

The field markings for the discus event consist of a circular throwing ring, foul lines and the boundary line of the field, as shown in the diagram below. The shaded area on the diagram is the landing region for a discus throw.



The foul lines meet the boundary line at points A and B, 65 m from the centre of the throwing ring.

The angle θ is 34.92°.

a. What is the length of the boundary line from point A to point B?
Write your answer in metres, rounded to two decimal places. 1 mark

b. Calculate the area of the landing region.
Round your answer to the nearest square metre. 2 marks

Question 2 (5 marks)

Daniel lives in Mildura (34° S, 142° E). He will fly to Sydney (34° S, 151° E) and then fly on to Rome (42° N, 12° E) to compete in the discus event at an international athletics competition.

In this question, assume that the radius of Earth is 6400 km.

a. Find the shortest great circle distance to the South Pole from Mildura (34° S, 142° E).
Round your answer to the nearest kilometre. 1 mark

b. The flight from Mildura (34° S, 142° E) to Sydney (34° S, 151° E) travels along a small circle.

i. Find the radius of this small circle.
Round your answer to two decimal places. 1 mark

ii. Find the distance the plane travels between Mildura (34° S, 142° E) and Sydney (34° S, 151° E).
Round your answer to the nearest kilometre. 1 mark

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

c. How long after the sun rises in Sydney (34° S, 151° E) will the sun rise in Rome (42° N, 12° E)? Round your answer to the nearest minute. 1 mark

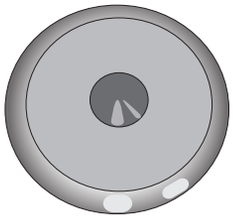
d. Daniel's flight to Rome leaves Sydney airport on Sunday, 6 March at 10.20 am, local time. The flight arrives in Rome on Monday, 7 March at 2.30 am. Assume the time difference between Sydney and Rome is 10 hours.

How long does the flight take to travel from Sydney to Rome?
Round your answer to the nearest minute. 1 mark

Question 3 (2 marks)

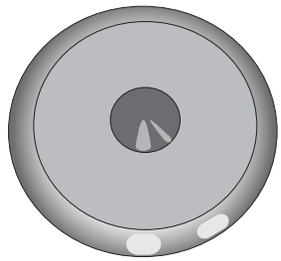
Daniel will compete in the intermediate division of the discus competition. Competitors in the intermediate division use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.

Intermediate discus



total surface area 500 cm^2

Senior discus



total surface area 720 cm^2

By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 (2 marks)

Daniel has qualified for the finals of the discus competition.

On his first throw, Daniel threw the discus to point A, a distance of 53.32 m on a bearing of 057°.

On his second throw from the same point, Daniel threw the discus a distance of 57.51 m.

The second throw landed at point B, on a bearing of 125°, measured from point A.

Determine the distance, in metres, between points A and B.

Round your answer to one decimal place.

Module 4 – Graphs and relations

Question 1 (8 marks)

Fastgrow and Booster are two tomato fertilisers that contain the nutrients nitrogen and phosphorus.

The amount of nitrogen and phosphorus in each kilogram of Fastgrow and Booster is shown in the table below.

	1 kg of Booster	**1 kg of Fastgrow**
Nitrogen	0.05 kg	0.05 kg
Phosphorus	0.02 kg	0.06 kg

a. How many kilograms of phosphorus are in 2 kg of Booster? 1 mark

b. If 100 kg of Booster and 400 kg of Fastgrow are mixed, how many kilograms of nitrogen would be in the mixture? 1 mark

Arthur is a farmer who grows tomatoes.

He mixes quantities of Booster and Fastgrow to make his own fertiliser.

Let x be the number of kilograms of Booster in Arthur's fertiliser.

Let y be the number of kilograms of Fastgrow in Arthur's fertiliser.

Inequalities 1 to 4 represent the nitrogen and phosphorus requirements of Arthur's tomato field.

Inequality 1	$x \geq 0$
Inequality 2	$y \geq 0$
Inequality 3 (nitrogen)	$0.05x + 0.05y \geq 200$
Inequality 4 (phosphorus)	$0.02x + 0.06y \geq 120$

Arthur's tomato field also requires at least 180 kg of the nutrient potassium.

Each kilogram of Booster contains 0.06 kg of potassium.

Each kilogram of Fastgrow contains 0.04 kg of potassium.

c. Inequality 5 represents the potassium requirements of Arthur's tomato field.

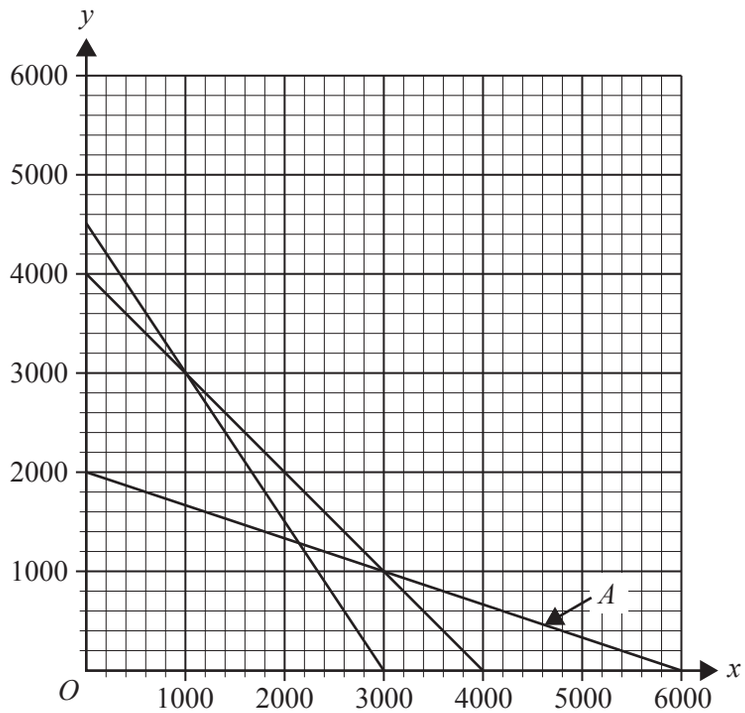
Write down Inequality 5 in terms of x and y. 1 mark

Inequality 5 (potassium) _____

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

The lines that represent the boundaries of Inequalities 3, 4 and 5 are shown in the graph below.



d. **i.** Using the graph above, write down the equation of line A. 1 mark

ii. On the **graph above**, shade the region that satisfies Inequalities 1 to 5. 1 mark

(*Answer on the graph above.*)

Arthur would like to use the least amount of his own fertiliser to meet the nutrient requirements of his tomato field and still satisfy Inequalities 1 to 5.

He will, therefore, minimise the total weight of fertiliser, W, where $W = x + y$.

The sliding-line method is to be used to determine the weight of Booster and Fastgrow fertilisers he will use.

e. **i.** Write down the gradient of the objective function $W = x + y$. 1 mark

ii. On the **graph above**, draw the line that passes through the point (0, 1000) using the gradient from **part e.i.** 1 mark

(*Answer on the graph above.*)

iii. Hence, show on the **graph above** the point(s) where the solution for minimum weight occurs. 1 mark

(*Answer on the graph above.*)

Question 2 (4 marks)

A shop owner bought 100 kg of Arthur's tomatoes to sell in her shop.

She bought the tomatoes for \$3.50 per kilogram.

The shop owner will offer a discount to her customers based on the number of kilograms of tomatoes they buy in one bag.

The revenue, in dollars, that the shop owner receives from selling the tomatoes is given by the piecewise defined relation below

$$\text{revenue} = \begin{cases} 5.4n & 0 < n \le 2 \\ 10.8 + 4(n-2) & 2 < n \le 10 \\ a + 2(n-10) & 10 < n < 100 \end{cases}$$

where n is the number of kilograms of tomatoes that a customer buys in one bag.

a. What is the revenue that the shop owner receives from selling 8 kg of tomatoes in one bag? 1 mark

b. A revenue of \$46.80 is received from selling 12 kg of tomatoes in one bag.

Show that a has the value 42.8 in the revenue equation above. 1 mark

c. Find the maximum number of kilograms of tomatoes that a customer can buy in one bag, so that the shop owner never makes a loss. 2 marks

END OF QUESTION AND ANSWER BOOK

DO NOT WRITE IN THIS AREA