2008 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

There were 25 746 students who sat the Further Mathematics examination 2 in 2008, compared with 25 644 students in 2007. The selection of modules by the students in 2007 and 2008 is shown in the table below.

MODULE	% 2007	% 2008
1 – Number patterns	42	37
2 – Geometry and trigonometry	84	86
3 – Graphs and relations	52	49
4 – Business-related mathematics	54	48
5 – Networks and decision mathematics	41	43
6 – Matrices	30	36

Overall, the Core section (Data analysis) and the six modules were of comparable difficulty and students performed well on the first questions for each module. It was encouraging to note that a smaller proportion of students than in previous years failed to attempt three modules.

Students must expect that their mathematical knowledge will continue to be examined in ways that require sensible use of calculator technology. This includes interpreting and communicating results, along with demonstrating an understanding of underlying mathematical ideas and the ability to appropriately formulate and analyse key components of the Further Mathematics study, as described in Outcomes 1–3.

In several questions throughout the paper, students were expected to explain their interpretation of information. Many students used this as an opportunity to express their opinions rather than make observations based on the mathematics. Students are expected to show understanding of explanations or assertions; this may be demonstrated by mathematics presented in a logical and clear manner or by directly relating a mathematical definition to data stated in the question.

It is not sufficient to say ‘As you can see, there are odd vertices.’ This could simply be a transcription from a definition in the book of notes without showing any reference to the data in the question. Responses such as this do not indicate that the student understands what vertices are, let alone what odd vertices are. A response that does indicate an understanding of vertices could be ‘Vertices at Q and P are of odd degree while all the other vertices are of even degree.’

There were also a number of questions throughout the paper that required students to ‘**show that…**’ a certain result is true. Each ‘show that’ question required students to give a clear demonstration of a mathematical process that produces the given numerical result. Steps should have been labelled and the calculation should then have followed. Mere substitution of the given number to demonstrate that it was a satisfactory result was not sufficient to ‘show that’ it was **the** result.

Another common purpose of a ‘show that’ question is to minimise consequential errors. For example, imagine there are three parts to a question, parts a., b. and c., and that the answer for part a. is required for parts b. and c. If the answer to part a. is then incorrect, then the correct answers to parts b. and c. are unlikely. A ‘show that’ question in part a. will give the correct outcome for that part even if the student is not able to do the mathematics. Regardless, this given number should be used in parts b. and c. and consequential marks will not apply in these parts, since an **incorrect** student answer to part a. should **not** be part of subsequent working.

When consequential marks are available, full working must be shown where a previous student-generated answer **must** be used. The mathematics must be evident, showing how the previous answer was used to generate the (possibly incorrect) answer to the current question.

There were many cases where students’ seemed to be unfamiliar with relevant mathematical terminology. A particular case was in Data analysis where the question required a **reciprocal** transformation. Many students did not seem to understand this term and inappropriately performed a **log** transformation instead.

Students and teachers could consider the development of a glossary inside the cover of their bound book. If this is developed through the year, terms that may be seen only occasionally could be readily accessed.

Alarmingly, there was some evidence that some students are learning methods that are no longer in the current study design, for instance, application of TVM calculator functionality can be used rather than the annuities formula.

Students should read the study design to ensure that their understanding covers all the aspects of the current design. They are also encouraged to read Assessment Reports from previous years as these can assist them to minimise preventable errors. Above all, they should read the questions in the examination papers. Failing to follow instructions reduces the likelihood of students being awarded full marks. An example is the provision of a matrix where a question specifically asks for a number. In such instances, the matrix cannot be accepted even though one of its elements may be the required number.

Rounding off continues to be an issue for some students, not necessarily because they do not know how to do it but because they did not read a question's instructions.

Formulaic approaches to some questions are still evident and continue to be of particular concern in Geometry and trigonometry and Business-related mathematics modules.

Areas of strength

Core
- completing a frequency table
- converting a fraction to a percentage
- reading scales where divisions represent two units
- determining whether a data point is an outlier

Number patterns
- arithmetic sequences

Geometry and trigonometry
- volume of a prism
- application of Pythagoras' theorem
- calculation of a compound area
- calculating an angle of elevation
- rounding numbers to the required number of decimal places

Graphs and relations
- reading data points from a graph
- writing a linear equation for a specific set of conditions
- explaining an inequality

Business-related mathematics
- interpreting a monthly bank account statement
- depreciating an amount by a percentage for one year

Networks and decision mathematics
- drawing a spanning tree but not necessarily a minimum spanning tree
- allocating tasks from a table simplified by the hungarian method of row and column reduction
- interpreting a directed graph that illustrates dominance

Matrices
- the order of a matrix
- multiplying two matrices
- identifying an element by its row and column designation
- writing a transition matrix from given data

Areas of weakness
Highlighting or underlining relevant data within questions throughout the year may help students to identify important information that must be extracted, used or answered.

Students' ability to read questions carefully was a significant issue this year. This was most evident in Number patterns – Question 3a. and 3c., Business-related mathematics – Question 3a. and 5c., Networks and decision mathematics – Question 2bi and 4a and Matrices – Questions 1bii., 1ci., 2a. and 4.

Students are encouraged to review their answer to each question since unreasonable answers are commonly seen, for example, a weekly allocation to homework of 1239.9 hours and another of -273.911 hours. A number of students seemed not to notice that, according to their calculation, a **plan** drawing of a concrete slab would itself be 3 metres long and 5 metres wide.

Core
- explaining an association between year level and arm span from parallel boxplots
- identifying the dependent variable out of two given associated variables
- interpreting the slope of a regression line in terms of the variables and the given situation
- understanding the term 'coefficients' as applied to the **equation** of a regression line
- understanding the term 'reciprocal' as applied to transforming a scatterplot
- writing the least squares regression line equation where a reciprocal transformation has been applied

Number patterns
- difference equations in general
- including the initial condition as a necessary part of a difference equation
- using the **sequence** facility of a calculator

Geometry and trigonometry
- determining a scale factor
- applying a scale factor to an area
- total surface area of a composite solid that does not fit one of the standard formulas a student may have in their bound book of notes
- showing full, clearly labelled and logical calculations for a 'Show that' question when explaining how a particular given answer would be achieved
- interpreting a three-dimensional diagram and related information

Graphs and relations
- calculating a **break even** point
- calculating a profit from the revenue and cost equations for a given number of competitors
- drawing the line $x = 120$
- accurately drawing the line $y = 1.5x$
- shading the feasible region for the conditions given in Question 3

Business-related mathematics
- applying an annual interest rate to a minimum monthly balance
- determining the annual flat rate of depreciation given the initial value and final book value after a given number of years
- using the TVM calculator facility to find the number of equal payments of $350 and recognising this must be a whole number
- finding the value of the final payment that will be made one month after all the equal payments have been made

Networks and decision mathematics
- finding a minimal spanning tree
- completing a network diagram from a road plan
- referring to the vertices of a particular diagram when explaining that a eulerian circuit cannot exist
- explaining the allocation of a task from a table reduced by the hungarian method

Matrices
- the difference between a **column** matrix and a **row** matrix
- interpreting a transition situation to determine the correct power to be applied to the transition matrix
- dealing with transitions that include an additional added column matrix at each transition

SPECIFIC INFORMATION

Section A

Core

Question 1
1a–b.

Marks	**0**	**1**	**2**	**Average**
%	1	9	90	**1.9**

1a.
10, 7, 8

1b.
32%

Question 2

Marks	**0**	**1**	**2**	**Average**
%	12	22	66	**1.6**

The percentaged segmented bar chart does support the opinion that lunch time activity (walked, sat or stood, ran) is associated with year level. For example, the percent that ran changed from around 78% to 40% to 10% from Years 6–8 and 8–10.

There are several ways of observing support from the bar chart. In general terms, an association is indicated by the percentage of girls undertaking a particular activity changing with the year level. Note that this change does not have to be a consistent increase or decrease, but it can be. For example, focussing on the activity 'sat or stood', the percentage of students who sat or stood changed from around 2% to 24% to 68% from Years 6–8 and 8–10. Or, focussing on the activity 'walked', the percentage of students who walked changed from around 20% to 36% to 22% from Years 6–8 and 8–10.

Some students referred to the changes in the percentages for an activity but then did not comment on whether the segmented bar chart **supported the opinion** that the association exists. Others forgot to quote relevant percentages as required to support their argument.

Question 3a–c.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	17	24	16	27	17	**2.1**

3a.
124, 148

3b.
- The median *arm span* increases with year level
- The IQR of *arm span* decreases with year level or the range of *arm span* decreases with year level

The majority of students erroneously said that the boxplots showed that *arm span* was increasing and explained that this is to be expected as the students are still in a growing stage. To compare the boxplots, one appropriate summary statistic for *arm span* (dependent variable) had to be compared across the year levels (independent variable). This could have been the median, the range or the IQR.

Some responses seemed to relate to students' own environment and suggested that *arm span* increased with age. The question did not state that a relationship between year level and age existed for this data and therefore this could not be assumed without some further information about the sample from which the data had been taken.

3c.
1.5 × IQR = 1.5 × 10 = 15
The lower fence is at Q1 – 15 = 160 – 15 = 145
An actual *arm span* of 140 is still lower than this and so is still an outlier.

Many students correctly calculated the lower boundary (fence) for Q1 – 1.5 × IQR and then said that 'it was therefore still an outlier' without any numerical comparison shown or suggested. As this does not explain what 'it' is, nor why 'it' is **still** an outlier, the actual value of 140 cm should be stated as still being lower than the calculated value of the fence. The incorrect value of 147 for the lower 'fence' was given by some students.

Question 4a–c.

Marks	0	1	2	3	4	Average
%	12	28	23	20	17	2

4a.
Height

The question required students to find 'a linear equation that allows *arm span* to be predicted from *height'* and so, for this equation, *arm span* (dependent variable) depends on height (independent variable).

4b.
arm span = –15.63 +1.09 × *height*

The variables *arm span* and *height* were required in the equation rather than *y* and *x*.

A common incorrect equation was *height* = 27.53 + 0.83 × *arm span,* found by choosing the incorrect independent variable.

Some students wrote the coefficients of their equation with fewer than two decimal places.

There seemed to be much confusion about the term 'coefficient' as many answers consisted of, or included, the values of pearson's correlation coefficient and the coefficient of determination. The question referred to 'writing an equation' with 'the coefficients written correct to two decimal places.' An equation (as in this question) is made up of variables (such as *height* or *arm span*) and coefficients (the values of *a* and *b* in the regression equation *a* + *bx* found from the calculator).

4c.
On average, *arm span* increases by 1.09 cm for each 1 cm increase in *height*

Question 5a–b.

Marks	0	1	2	3	Average
%	77	9	3	11	0.5

5a.

$$\text{Homework hours} = 5.12 + \frac{102.90}{TV\,hours}$$

The majority of students did not seem to understand the term 'reciprocal'.

Of those students who did the correct transformation on their calculator, few were then able to correctly write the equation with their correct coefficients. A common incorrect answer from this group failed to write *TV hours* as a denominator of a fraction. An equation written as 'homework hours = 5.12 + 102.90 (reciprocal) TV hours' was not accepted for full marks.

A common error involved interpreting 'reciprocal' transformation on a 'log' transformation.

The values of the correlation coefficient and coefficient of determination were given as common incorrect responses. Again, the real variable names were required in the equation rather than *x* and *y*.

5b.
13.7 hours

Some unreasonable answers were given for this question without any comment from students that the answer was absurd. For instance, an answer of 1239.9 hours of homework per week would be rather exhausting, and impossible with only 168 hours in a week. Equally, a negative number of hours would also present some difficulties for a student's real study program.

Module 1: Number patterns

Difference equations were a major component in this module this year and to gain high marks students needed to display a good understanding of their specification and application. This was not always apparent.

Question 1a–c.

Marks	0	1	2	3	Average
%	4	13	36	47	2.3

1a.
950

1b.
-150

The negative sign was sometimes omitted.

1c.
8 years

Question 2a–c.

Marks	0	1	2	3	4	5	Average
%	25	13	12	16	17	17	2.4

2ai.
1200

Substitution into a difference equation caused problems for some students as exemplified by answers such as $1.08 - 150 + 1250 = 1101.08$.

2aii.
883

The **sequence** facility of a calculator was useful in this question.

2bi.
8%

A common incorrect answer was 1.08.

2bii.
14 years

The **sequence** facility of a calculator was useful for this question.

2c.
100

8% × 1250 = 100

For the number of cows to remain the same each year, only the number by which the herd grows each year should be sold.

Question 3a–e.

Marks	0	1	2	3	4	5	Average
%	24	19	25	18	9	5	**1.9**

3a.
108

3b.
8 years (9 years was also accepted)

Solving $32 \times 1.5^{n} = 820$ produces $n = 7.9996$ and so $n = 8$

Some students argued that to exceed 820 deer, one should determine the time to reach 821 deer, and found this to be just over 8 years.

3c.
195

$D_1 = 1.5 \times 32 = 48$
$D_5 = 32 \times (1.5)^5 = 243$
$\therefore\ D_5 - D_1 = 243 - 48 = 195$

3d.
$D_n = 32 \times 1.5^{n}$

An acceptable alternative answer was $D_n = 48 \times 1.5^{n-1}$.

3e.
$D_n = 1.5\, D_{n-1}\quad D_0 = 32$

The initial value must be included in the specification of a difference equation.

The equation $D_{n+1} = 1.5\, D_n\quad D_1 = 32$ was not accepted as it did not answer the question asked.

Question 4

Marks	0	1	2	Average
%	79	4	17	**0.4**

12 years

The **sequence** facility of a calculator made this question relatively straightforward if the data and equations were entered and the resulting table was then interpreted correctly.

Generating a table and showing the last 2–3 lines on the table could be written as suitable ‘working out’ for this question.

Module 2: Geometry and trigonometry

Question 1a–b.

Marks	0	1	2	3	Average
%	9	47	30	14	**1.5**

1a.
12 m^3

1bi.
$\frac{1}{200}$

The scale factor for drawing the plan is the number by which we must multiply the actual dimension to determine the equivalent dimension on the plan. The ratio 1:200 is the scale but not the scale factor, which is $\frac{1}{200}$.

1bii.
15 cm^2

Plan area = slab surface area × $\left(\frac{1}{200}\right)^2$

Students who tended to answer this question poorly often did not consider squaring the scale factor for length. Many unreasonable answers had plans the size of an average living room.

A further example of a poorly answered 'show that' question was Question 1b. in Geometry, where students needed to show that $a = 1.6$. After an initial few lines of working that were generally quite well done, the calculation often concluded with:

$\therefore a^2 = 159.118$

$\therefore a = 1.6$

While it appears as though a square root has been taken for the last step, there is no explicit indication as to whether the student actually did this, since the last line was given in the question.

Question 2a–d.

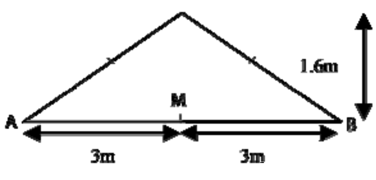
Marks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Average**
%	15	8	10	12	13	17	27	**3.6**

$OM = \sqrt{3.4^2 - 3^2}$

Setting out was often poor. Some students used trigonometry and were awarded the mark if their working was logical.

2b.
An example for the triangle shown is:

Area of the triangle = $A = \frac{1}{2}bh$



$= \frac{1}{2} \times 6 \times 1.6 = 4.8\ \text{m}^2$

An example of an unsatisfactory calculation was Area = 3 × 1.6 = 4.8. In this example, there is no explanation by the student where the numeral 3 came from.

Area = rectangle + triangle

= 2.2 × 6 + ½ × 6 × 1.6

= 18 m^2

The setting out of a 'show that' calculation should include some labels for the various sections of a calculator. The student must be clearly communicating all of the mathematical steps required to gain the given answer.

The following two example solutions do not meet the requirements as the two calculations do not show all the mathematical steps needed to find the area of 18 m²:

$13.2 + 4.8 = 18 \text{ m}^2$
No calculation is shown for either the rectangle or the triangle area.

Rectangle = $2.2 \times 6 = 13.2 \text{ m}^2$
Triangle = ½ *bh* = $\underline{4.8 \text{ m}^2}$
18 m^2
No substitution is shown for the triangle.

Some students obtained the answer by (correctly) using the more involved Heron's formula for calculating the area of the triangle AOB from its side lengths.

2c.
180 m^3

Despite the area of the end of the shed being given in Question 2b., some students recalculated it in this question and many then made an error. The area given in Question 2b. should have been used.

2di.
208 m^2

Area = floor + 2 ends + 2 sides + 2 ceiling surfaces
$= 6\times10 + 2\times18 + 2\times10\times2.2 + 2\times10\times3.4$
$= 60 + 36 + 44 + 68 = 208 \text{ m}^2$

Some students seemed to rely on their bound book of notes for formulas they do not understand.
An example of this was $SA = bh + bl + hl + l\sqrt{b^2 + h^2}$.

Several other students used a formula for the surface area of a rectangular prism TSA = 2(*lw*+*lh*+*wh*) and did not realise that this formula included a floor plus a rectangular ceiling. They then used another formula for the surface area of a triangular prism to account for the real ceiling and triangles at the ends, but again did not realise this also included another rectangular ceiling.

2dii.
13 litres

Question 3a–3b.

Marks	0	1	2	Average
%	27	31	42	1.2

3a.
42.7°

3b.
$NT^2 = 10^2 + 13^2 - 2 \ x \ 10 \ x \ 13 \ x \cos(65°)$
$\therefore NT^2 = 159.119$
$\therefore NT = \sqrt{159.119}$
$\therefore NT \approx 12.614 \approx 12.6$

Some students did not show that a square root had to be taken at the last step and simply went from
$\therefore NT^2 \approx 159.119$ to $\therefore NT = 12.6$, which was given in the question.

Question 3c–3e.

Marks	0	1	2	3	4	Average
%	49	27	15	3	6	0.9

3c.

69°

3d.
111°

180° - 69° = 111°

3e.
Nearest distance to shed = 13 sin 65º = 11.8 metres
$\therefore$ The tree will hit the shed as its height of 12 metres > 11.8 m

There is a section of the shed between C and N that is within 12 metres of the tree. Full marks were awarded for any correct calculation that identified any point within this section and then used this calculation to justify a written conclusion that the tree would hit.

Some students calculated the shortest distance correctly and said that the tree would hit the shed but gave no mathematical comparison between this distance and the tree height to justify their statement.

An answer of 'yes' without justification by calculation and comparison gained no marks.

Remarkably, most students seemed to have difficulty understanding the three-dimensional diagram. Only a minority understood that the tree did not have to fall along the line CT or the line NT. Many students incorrectly concluded that, as the tree was shorter than the distance CT or NT, it would not hit the shed.

Others used Pythagoras' theorem to incorrectly calculate the distance from the tree to a point halfway between C and N on the shed. The triangle involved in these calculations was **not** a right triangle and Pythagoras' theorem could not be used. A small number of students calculated the distance from the top of the tree to point C and used this 17.4 metre distance to justify their assertion that the tree would not hit.

Module 3: Graphs and relations

Question 1a–c.

Marks	0	1	2	3	4	Average
%	2	7	16	24	51	**3.2**

1a.
110

1b.
80

1ci.
maximum pulse rate = 220 − *age*

Incorrect answers included *maximum pulse rate* = *age* − 220 and *maximum pulse rate* = *age* + 220

The variables *maximum pulse rate* and *age* were required rather than *x* and/or *y*

1cii.
Between 120 and 150

Question 2a–c.

Marks	0	1	2	3	4	Average
%	9	7	20	22	41	**2.8**

2a.
$R = 35\,x$

An equation was required. An expression such as 35*x*, was not accepted.

2b.
C = 50 625 + 12.5x

An equation was required. An expression such as 50 625 + 12.5x, was not accepted.

2ci.
2250

$12.5x + 50\,625 = 35x$

$\therefore \quad 22.5x = 50\,625$

$\therefore \quad x = 2250$

Many students were unable to complete this question correctly.

A common incorrect answer was 4050 as the solution of an incomplete equation 50 625 = 12.5x which ignored the inclusion of the revenue equation.

2cii
$144 450

$P = 35x - (12.5x + 50\,625)$
$= 22.5x - 50\,625$
$= 22.5 \times 8670 - 50\,625$
$= 144\,450$

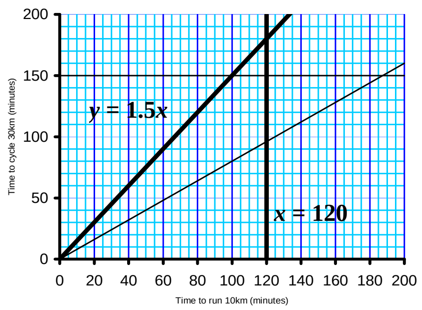
Question 3a–b.

Marks	0	1	2	3	4	Average
%	21	15	15	21	29	2.2

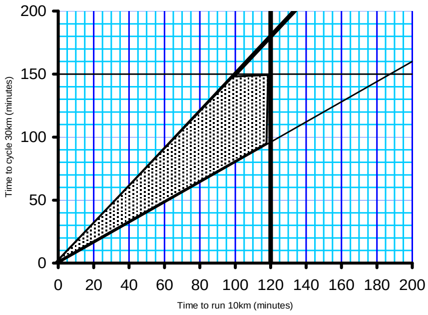
3a.
Must run 10 km in no more than 120 minutes

The correct answer included 120 minutes as an acceptable time to run 10 km. Therefore, and answer of ‘run 10 km in less than 120 minutes’ was not accepted.

3bi.



3bii.



Many students shaded the **excluded** section. This was acceptable as long as the correct feasible region was then identified, perhaps by a legend. This was essential as the question asked for the **feasible region** to be shaded, which can allowably be interpreted as 'identified.'

Several students drew the line $x = 120$ as a horizontal line and a few shaded in **above** the line $y = 150$

Question 3c–3dii.

Marks	0	1	2	3	Average
%	65	19	9	7	**0.6**

3c.
Between 80 and 150 minutes

3di.
54 minutes

Maximum cycle time (on $y = 1.5x$ line)

$x + y \le 90$ and $y = 1.5x$

$\therefore 2.5x \le 90$
$\therefore x \le 36$
$\therefore y \le 54$

3dii.
50 minutes

Maximum run time (on $y = 0.8x$ line)

$x + y \le 90$ and $y = 0.8x$
$\therefore 1.8x \le 90$
$\therefore x \le 50$

Module 4: Business-related mathematics

Although Australian currency has a five cent coin as its lowest denomination, financial calculations in Further Mathematics are expected to be correct to a given accuracy; for example, to the nearest cent or nearest dollar, as specified in the question or part of a question.

Sums of money correct to fractions of a cent are common in financial transactions. The price of petrol is given correct to the nearest tenth of a cent. The exchange rate for the Australian dollar is often quoted correct to the nearest hundredth of a cent. Students must carry out calculations to the nearest cent or an appropriate accuracy as specified in the question.

Questions 1–2

Marks	0	1	2	3	4	5	6	Average
%	4	31	15	17	9	11	13	**2.8**

1a.
$620

1b.
$15.30

$\frac{3}{12}\% \times 6120.86 = 15.30$

Some students did not divide the annual rate by 12 to get the monthly rate. Others did not use the **minimum** monthly balance.

2a.
$A = 3000 \times 1.041^4$
$A = \$3523.09$

Some students did not change the annual rate to a half-yearly rate. Others did not change the number of payments to four. Rounding off the answer to $3523.10 was not accepted.

2b.
$1137.40

$3000 \times 1.041^8 - 3000 = 1137.40$

The TVM facility of a calculator could readily be used in this question. Often, the principal amount was not subtracted to determine the interest amount.

Questions 3–4.

Marks	0	1	2	3	4	5	Average
%	28	16	19	14	12	11	2

3a.
$1377

$T_2 - T_3 = 17\,000 \times 0.9^2 - 17\,000 \times 0.9^3 = 1377$

Many students misread this question and found only depreciated value = $T_3 = 17\,000 \times 0.9^3 = \$12\,393$

3b.
9 years (8.4 years was also accepted)

$17\,000 \times 0.9^n = 7000$

$n = 8.42$

An answer of 8 years was not accepted as the car will not be completely paid off.

4a.
$900

$\frac{17\,000 - 3500}{15} = 900$

4b.
5.3%

$\frac{900}{17\,000} \times 100 = 5.294$

Question 5a–c.

Marks	0	1	2	3	4	Average
%	64	12	8	6	11	**0.9**

5a.
52

N = 52.4225
I = 9.4
PV = 15 000
AMT = -350
FV = 0
P/*Y* = 12

An answer of 52.4 equal monthly payments was inappropriate.

Some laborious and ultimately erroneous substitutions into the annuities formula were seen in some student responses to this question. The TVM calculator function is the recommended pathway to answering questions such as this. Knowledge of the annuities formula is **not** required in the current study design.

5b.
$147

N = 52
I = 9.4
PV = 15 000
AMT = -350
FV = 147.059924
P/Y = 12

This question was not well done by most students. Some students simply ignored having to repay interest and found 15 000 ÷ 350 = 42.85.

5c.
$388.30

N = 12
I = 9.4
PV = 15 000
AMT = -350
FV = **-12 086.6029**
P/*Y* = 12

That is, after 1 year, Michelle still owes $12 086.60, to be paid off in the remaining 36 payments at 9.7%.

This $12 086.60 then becomes the PV for the remaining three years as follows:
N = 36
I = 9.7
PV = 12 086.60
AMT = **-388.301**
FV = 0
P/*Y* = 12

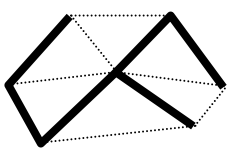
Most students were unable to complete this question. Many ignored the first year at 9.4% and simply calculated a four year loan at 9.7%.

Module 5: Networks and decision mathematics

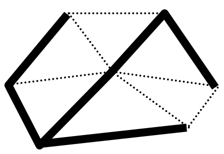
Questions 1-2

Marks	0	1	2	3	4	5	6	Average
%	10	13	18	23	19	14	3	**2.8**

1a.



or



Either of these two trees was accepted. A number of students drew circuits and these were not accepted.

1b.
16

1c.
Two, as shown in 1a. above.

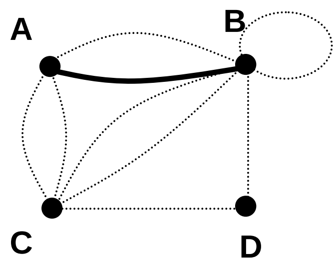
Question 2a.
7

The unnamed intersections are labelled as T_1 and T_2 from top to bottom.

The seven possible paths are: AT_1BD, AT_1T_2BD, AT_1T_2CD, AT_1BT_2CD, ACD, ACT_2BD, ACT_2T_1BD

Few students managed to find that there were seven paths.

2bi.



2bii.
To meet the requirements, there should be a eulerian circuit which can only exist if all vertices have an even degree. In this network, vertices C and B have odd degrees.

Generic definitions that did not specifically refer to the vertices in this network were not accepted. Examples of this were ‘a eulerian circuit requires all vertices of even degree’, ‘does not contain a eulerian circuit’ and ‘because all towns must have an even degree leading to it.’ None of these statements indicate that the student understands which, if any, of these vertices has an odd degree.

Question 3a–d.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	15	13	14	32	26	**2.5**

3a.
2, 0, 7, 5

3b.
She is the only child with a zero in the column for Concert 2.

The hungarian method attempts to reduce the table or matrix to give zeroes in columns. A single zero in a column with this method indicates that an allocation is appropriate.

Many students did not refer to the zero but said that the table at this stage showed that Tahlia ‘had to travel the least distance to get to Concert 2.’ This answer is too general as the same reasoning could be applied at any stage in the process and would often be wrong if no reduction had occurred.

3c.
3, 4, 2, **1** or **3**, **1**, 2, **4**

Either answer was accepted.

3d.
56

Question 4a–d.

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**Average**
%	14	13	19	24	23	6	**2.5**

4a.
Both of:
- Arnold and Edgar
- Barnaby and Cedric.

Both pairs were expected. One mark was awarded for each correct pair.

4b.
Edgar

4c.

Final order of dominance	**Lion**
1st	Darcy
2nd	Cedric
3rd	Barnaby
4th	Arnold
5th	Edgar

4d.
20

Each lion cub has a one-step dominance over two other lion cubs. These two other cubs each have a one-step dominance over two other cubs. Consequently, each lion cub has 2 × 2 = 4 two-step dominances over other cubs. Therefore, for five lion cubs, there are 5 × 4 = 20 two-step dominances in the group.

Module 6: Matrices

Question 1a–bii.

Marks	0	1	2	3	Average
%	8	12	51	30	2.1

1a.

1 × 5

1bi.

$$\begin{bmatrix} 23 & 57.5 & 80.5 & 207 & 92 \\ 18 & 45 & 63 & 162 & 72 \end{bmatrix}$$

1bii.

The number of students who are expected to get a D in Chemistry

Question 1c–2

Marks	0	1	2	3	4	5	6	7	Average
%	15	17	17	16	16	9	6	5	2.8

1ci.

$$\begin{matrix} B & C \end{matrix}$$
$$\begin{bmatrix} 110 & 95 \end{bmatrix}$$

Despite a row matrix being required, many students wrote a column matrix instead. The question expected students to label the matrix as shown.

1cii.

$$\begin{bmatrix} 110 & 95 \end{bmatrix}\begin{bmatrix} 460 \\ 360 \end{bmatrix} = \begin{bmatrix} 84800 \end{bmatrix}$$

2ai.

$$\begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix}$$

$$S_2 = T\ S_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix}$$

A common mistake resulted from students misreading the question. The *state matrix* S_1 was given and so a single application of the transition matrix to find S_2 was required. Many students renamed the state matrix S_1 as S_0 and then squared the transition matrix.

2aii.

421

$$S_5 = T^4 S_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}^4 \begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 421.46 \\ 154.54 \end{bmatrix}$$

2b.

$S_n = T^{n-1} \times S_1$

The initial state matrix defined as S_1 in the question was required here.

2c.
Lecture 8

2d.
384

When two different calculations involving $S_n = T^{n-1} \times S_1$ produce the same result, the long-term state matrix has been produced. High powers for n , such as n = 50 and 51, could be used.

$T^{50}S_1 = \begin{bmatrix} 384.000 \\ 191.999 \end{bmatrix}$ and $T^{51}S_1 = \begin{bmatrix} 384.000 \\ 191.999 \end{bmatrix}$ both give the same result

Questions 3–4

Marks	0	1	2	3	4	5	Average
%	26	26	17	15	8	7	1.8

3a.
$\begin{bmatrix} 360 \\ 250 \end{bmatrix}$

$$O_{2009} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix} \begin{bmatrix} 456 \\ 350 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

3b.
248

$$O_{2009} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 500 \\ 360 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 360 \\ 250 \end{bmatrix}$$

$$O_{2010} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 360 \\ 250 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 248 \\ 162 \end{bmatrix}$$

The added column matrix at each year is applied after each transition matrix is applied. Consequently, simply squaring the transition matrix is not appropriate in this question.

Question 4
42

$$S_{2009} = \begin{bmatrix} 0.88 & 0.52 & 0.65 \\ 0.10 & 0.44 & 0.10 \\ 0.02 & 0.04 & 0.25 \end{bmatrix}^2 \begin{bmatrix} 880 \\ 230 \\ 120 \end{bmatrix} = \begin{bmatrix} 996.9 \\ 191.4 \\ 41.7 \end{bmatrix}$$

Few students wrote down the relevant transition matrix which would have earned at least one mark if correct. Of those who did write it down, 2% and 4% were sometimes incorrectly written as 0.2 and 0.4.