VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VCE Mathematical Methods 2016–2018

Written examinations 1 and 2 – End of year

Examination specifications

Overall conditions

There will be two end-of-year examinations for VCE Mathematical Methods – examination 1 and examination 2.

The examinations will be sat at a time and date to be set annually by the Victorian Curriculum and Assessment Authority (VCAA). VCAA examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.

Examination 1 will have 15 minutes reading time and 1 hour writing time. Students are not permitted to bring into the examination room any technology (calculators or software) or notes of any kind.

Examination 2 will have 15 minutes reading time and 2 hours writing time. Students are permitted to bring into the examination room an approved technology with numerical, graphical, symbolic and statistical functionality, as specified in the *VCAA Bulletin* and the *VCE Exams Navigator*. One bound reference may be brought into the examination room. This may be a textbook (which may be annotated), a securely bound lecture pad, a permanently bound student-constructed set of notes without fold-outs or an exercise book. Specifications for the bound reference are published annually in the *VCE Exams Navigator.*

A formula sheet will be provided with both examinations.

The examinations will be marked by a panel appointed by the VCAA.

Examination 1 will contribute 22 per cent to the study score. Examination 2 will contribute 44 per cent to the study score.

VICTORIA State Government

Content

The *VCE Mathematics Study Design 2016–2018* ('Mathematical Methods Units 3 and 4') is the document for the development of the examination. All outcomes in 'Mathematical Methods Units 3 and 4' will be examined.

All content from the areas of study, and the key knowledge and skills that underpin the outcomes in Units 3 and 4, are examinable.

Examination 1 will cover all areas of study in relation to Outcome 1. The examination is designed to assess students' knowledge of mathematical concepts, their skill in carrying out mathematical algorithms without the use of technology, and their ability to apply concepts and skills.

Examination 2 will cover all areas of study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

Format

Examination 1

The examination will be in the form of a question and answer book.

The examination will consist of short-answer and extended-answer questions. All questions will be compulsory.

The total marks for the examination will be 40.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

All answers are to be recorded in the spaces provided in the question and answer book.

Examination 2

The examination will be in the form of a question and answer book.

The examination will consist of two sections.

Section A will consist of 20 multiple-choice questions worth 1 mark each and will be worth a total of 20 marks.

Section B will consist of short-answer and extended-answer questions, including multi-stage questions of increasing complexity, and will be worth a total of 60 marks.

All questions will be compulsory. The total marks for the examination will be 80.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

Answers to Section A are to be recorded on the answer sheet provided for multiple-choice questions.

Answers to Section B are to be recorded in the spaces provided in the question and answer book.

Approved materials and equipment

Examination 1

- normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)

Examination 2

- normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)
- an approved technology with numerical, graphical, symbolic and statistical functionality
- one scientific calculator
- one bound reference

Relevant references

The following publications should be referred to in relation to the VCE Mathematical Methods examinations:

- *VCE Mathematics Study Design 2016–2018* (‘Mathematical Methods Units 3 and 4’)
- *VCE Mathematical Methods – Advice for teachers 2016–2018* (includes assessment advice)
- *VCE Exams Navigator*
- *VCAA Bulletin*

Advice

During the 2016–2018 accreditation period for VCE Mathematical Methods, examinations will be prepared according to the examination specifications above. Each examination will conform to these specifications and will test a representative sample of the key knowledge and skills from all outcomes in Units 3 and 4.

The following sample examinations provide an indication of the types of questions teachers and students can expect until the current accreditation period is over.

Answers to multiple-choice questions are provided at the end of examination 2.

Answers to other questions are not provided.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter

MATHEMATICAL METHODS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour)**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 2 – April 2016

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

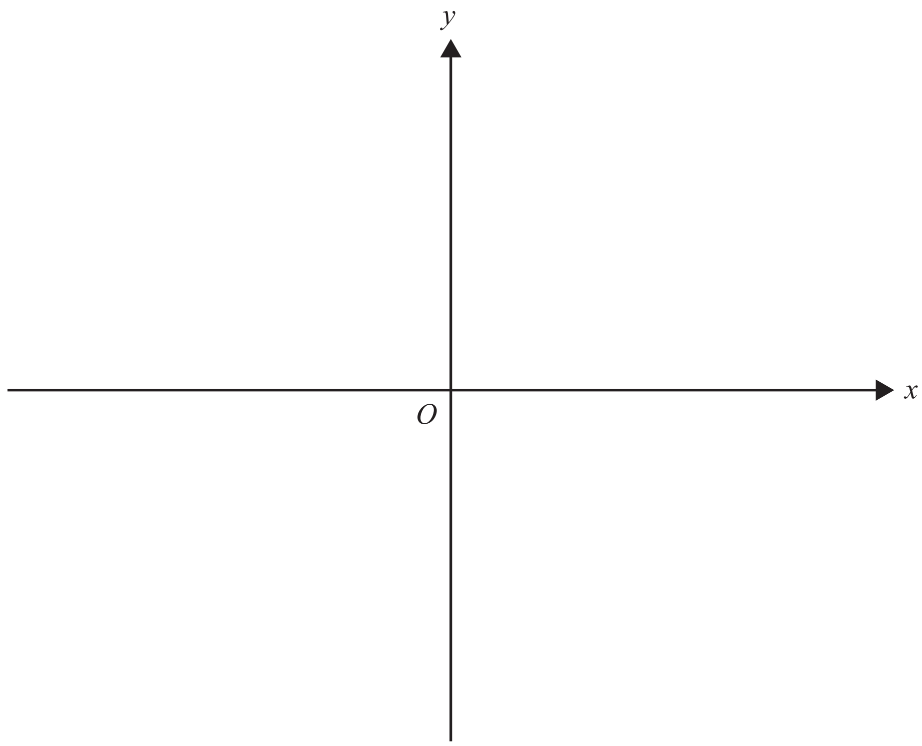
a. Differentiate $\sqrt{4-x}$ with respect to x. 1 mark

b. If $f(x)=\dfrac{x}{\sin(x)}$, find $f'\left(\dfrac{\pi}{2}\right)$. 2 marks

Question 2 (3 marks)

On the axes below, sketch the graph of $f\colon R\backslash\{-1\} \to R,\ f(x) = 2 - \dfrac{4}{x+1}$.

Label each axis intercept with its coordinates. Label each asymptote with its equation.



Question 3 (4 marks)

a. Find an antiderivative of $\frac{1}{(2x-1)^3}$ with respect to x.

2 marks

b. The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.

Given that $g(1) = \frac{1}{\pi}$, find $g(x)$.

2 marks

Question 4 (3 marks)

Let X be the random variable that represents the number of telephone calls that Daniel receives on any given day with probability distribution given by the table below.

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

a. Find the mean of X. 2 marks

b. What is the probability that Daniel receives only one telephone call on each of three consecutive days? 1 mark

Question 5 (3 marks)

The graphs of $y = \cos(x)$ and $y = a\sin(x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$.

a. Find the value of a. 2 marks

b. If $x \in [0, 2\pi]$, find the x-coordinate of the other point of intersection of the two graphs. 1 mark

Question 6 (5 marks)

a. Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. 2 marks

b. Solve $3e^{t} = 5 + 8e^{-t}$ for t. 3 marks

Question 7 (4 marks)

A student performs an experiment in which a computer is used to simulate drawing a random sample of size n from a large population. The proportion of the population with the characteristic of interest to the student is p.

a. Let the random variable $\hat{P}$ represent the sample proportion observed in the experiment.

If $p = \frac{1}{5}$, find the smallest integer value of the sample size such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{100}$. 2 marks

Each of 23 students in a class independently performs the experiment described above and each student calculates an approximate 95% confidence interval for p using the sample proportions for their sample. It is subsequently found that exactly one of the 23 confidence intervals calculated by the class does not contain the value of p.

b. Two of the confidence intervals calculated by the class are selected at random without replacement.

Find the probability that exactly one of the selected confidence intervals does not contain the value of p. 2 marks

Question 8 (4 marks)

A continuous random variable, *X*, has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The median of *X* is m.

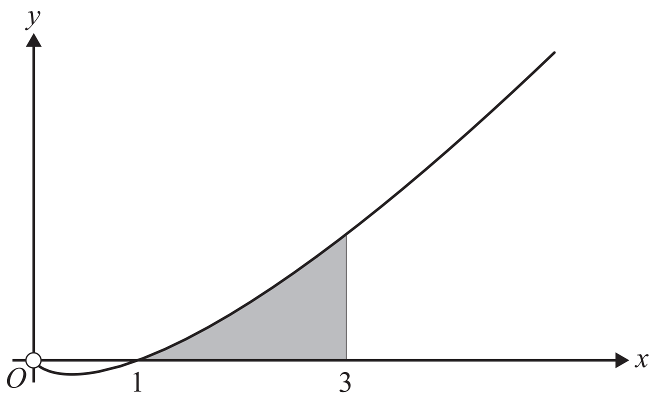
a. Determine the value of m. 2 marks

b. The value of m is a number greater than 1.

Find $\Pr(X < 1 | X \leq m)$. 2 marks

Question 9 (4 marks)

Part of the graph of $f: R^+ \rightarrow R, f(x) = x \log_e(x)$ is shown below.

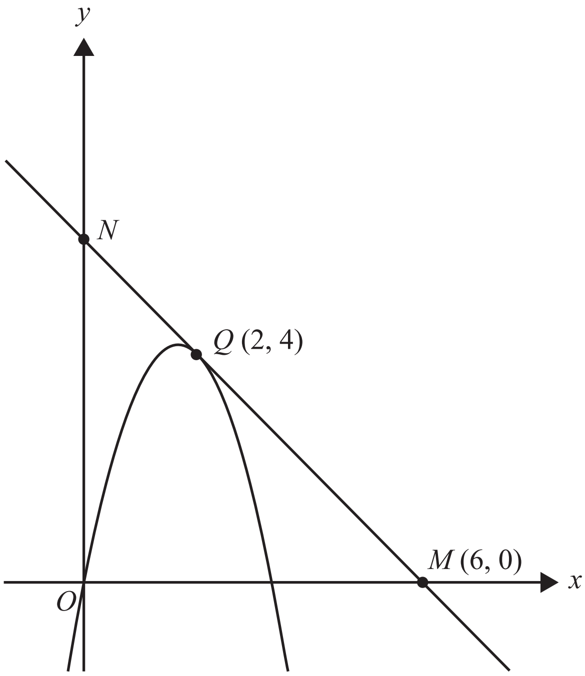


a. Find the derivative of $x^2 \log_e(x)$. 1 mark

b. Use your answer to **part a.** to find the area of the shaded region in the form $a \log_e(b) + c$, where a, b and c are non-zero real constants. 3 marks

Question 10 (7 marks)

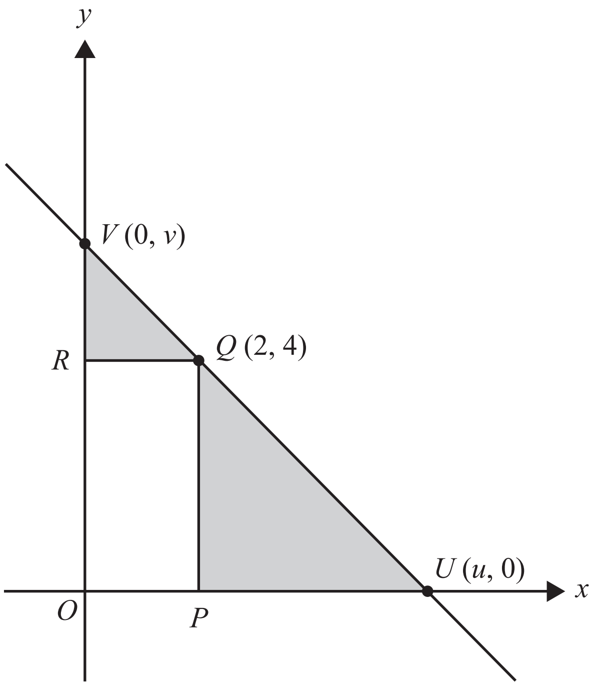
The line containing the points M and N intersects the x-axis at the point M with coordinates (6, 0). The line is also a tangent to the graph of $y = ax^2 + bx$ at the point Q with coordinates (2, 4), as shown below.



a. If a and b are non-zero real numbers, find the values of a and b. 3 marks

b. The line containing the points U and V intersects the coordinate axes at the points U and V with coordinates $(u, 0)$ and $(0, v)$, respectively, where u and v are positive real numbers and $\frac{5}{2} \leq u \leq 6$, as shown below.

The rectangle $OPQR$ has a vertex at Q on the line. The coordinates of Q are $(2, 4)$, as shown below.



i. Find an expression for v in terms of u. 1 mark

ii. Find the **minimum** total shaded area and the value of u for which the area is a minimum. 2 marks

iii. Find the **maximum** total shaded area and the value of u for which the area is a maximum. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

MATHEMATICAL METHODS
Written examination 2

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (2 hours)**

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 20 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 2 – April 2016

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The point $P\ (4, -3)$ lies on the graph of a function f. The graph of f is translated four units vertically up and then reflected in the y-axis.

The coordinates of the final image of P are

A. $(-4, 1)$
B. $(-4, 3)$
C. $(0, -3)$
D. $(4, -6)$
E. $(-4, -1)$

Question 2

The linear function $f: D \to R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is

A. $[-2, 6)$
B. $(-2, 2]$
C. R
D. $(-2, 6]$
E. $[-6, 2]$

Question 3

The function with rule $f(x) = -3\tan(2\pi x)$ has period

A. $\frac{2}{\pi}$
B. 2
C. $\frac{1}{2}$
D. $\frac{1}{4}$
E. 2π

Question 4

If $x + a$ is a factor of $7x^3 + 9x^2 - 5ax$, where $a \in R\backslash\{0\}$, then the value of a is

A. -4

B. -2

C. -1

D. 1

E. 2

Question 5

The function $g: [-a, a] \to R, g(x) = \sin\left(2\left(x - \frac{\pi}{6}\right)\right)$ has an inverse function.
The maximum possible value of a is

A. $\frac{\pi}{12}$

B. 1

C. $\frac{\pi}{6}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{2}$

Question 6

If $\int_1^4 f(x)\,dx = 6$, then $\int_1^4 (5 - 2f(x))\,dx$ is equal to

A. 3

B. 4

C. 5

D. 6

E. 16

Question 7

For events A and B, $\Pr(A \cap B) = p$, $\Pr(A' \cap B) = p - \frac{1}{8}$ and $\Pr(A \cap B') = \frac{3p}{5}$.
If A and B are independent, then the value of p is

A. 0

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

E. $\frac{3}{5}$

Question 8

Which one of the following functions satisfies the functional equation $f\left(f(x)\right) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 9

A bag contains five red marbles and four blue marbles. Two marbles are drawn from the bag, without replacement, and the results are recorded.

The probability that the marbles are different colours is

A. $\frac{20}{81}$

B. $\frac{5}{18}$

C. $\frac{4}{9}$

D. $\frac{40}{81}$

E. $\frac{5}{9}$

Question 10

The transformation $T : R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$

B. $x + 4y = 0$

C. $-x - y = 4$

D. $x + 4y = -6$

E. $x - 2y = 1$

Question 11

If the tangent to the graph of $y = e^{ax}$, $a \neq 0$, at $x = c$ passes through the origin, then c is equal to

A. 0

B. $\dfrac{1}{a}$

C. 1

D. a

E. $-\dfrac{1}{a}$

Question 12

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for

A. $a = 3$

B. $a = -3$

C. both $a = 3$ and $a = -3$

D. $a \in R\backslash\{3\}$

E. $a \in R\backslash[-3, 3]$

Question 13

It is known that 26% of the 19-year-olds in a region do not have a driver's licence.

If a random sample of ten 19-year-olds from the region is taken, the probability, correct to four decimal places, that more than half of them will not have a driver's licence is

A. 0.0239

B. 0.0904

C. 0.2600

D. 0.9096

E. 0.9761

Question 14

An opinion pollster reported that for a random sample of 574 voters in a town, 76% indicated a preference for retaining the current council.

An approximate 90% confidence interval for the proportion of the total voting population with a preference for retaining the current council can be found by evaluating

A. $\left(0.76 - \sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + \sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

B. $\left(0.76 - 1.65\sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + 1.65\sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

C. $\left(0.76 - 2.58\sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + 2.58\sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

D. $\left(436 - 1.96\sqrt{0.76 \times 0.24 \times 574},\ 436 + 1.96\sqrt{0.76 \times 0.24 \times 574}\right)$

E. $\left(0.76 - 2\sqrt{0.76 \times 0.24 \times 574},\ 0.76 + 2\sqrt{0.76 \times 0.24 \times 574}\right)$

Question 15

The cubic function $f: R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b and c are positive constants, has no stationary points when

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

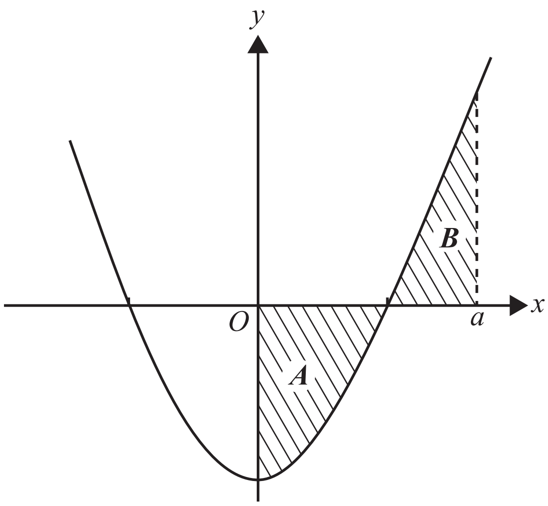
C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

E. $c < \frac{b^2}{3a}$

Question 16

A part of the graph of $g: R \to R, g(x) = x^2 - 4$ is shown below.



The area of the region labelled A is the same as the area of the region labelled B.

The exact value of a is

A. 0

B. 6

C. $\sqrt{6}$

D. 12

E. $2\sqrt{3}$

Question 17

The equation $x^3 - 9x^2 + 15x + w = 0$ has only one solution for x when

A. $-7 < w < 25$

B. $w \leq -7$

C. $w \geq 25$

D. $w < -7$ or $w > 25$

E. $w > 1$

Question 18

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x-axis at (2, 0) and (–3, 0). The maximum value of the derivative function is 10.

The value of x for which the graph of $y = f(x)$ has a local maximum is

A. -2

B. 2

C. -3

D. 3

E. $-\frac{1}{2}$

Question 19

Butterflies of a particular species die T days after hatching, where T is a normally distributed random variable with a mean of 120 days and a standard deviation of σ days.

If, from a population of 2000 newly hatched butterflies, 150 are expected to die in the first 90 days, then the value of σ is closest to

A. 7 days.

B. 13 days.

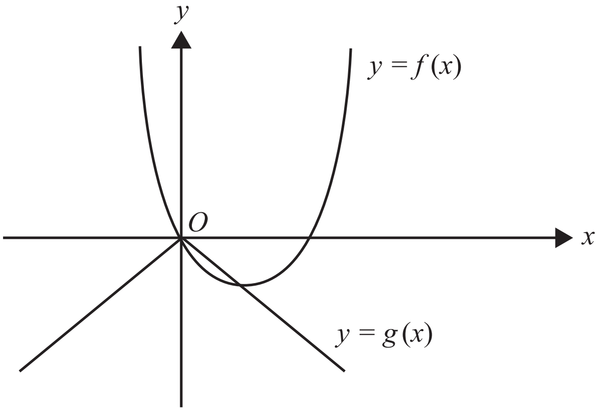
C. 17 days.

D. 21 days.

E. 37 days.

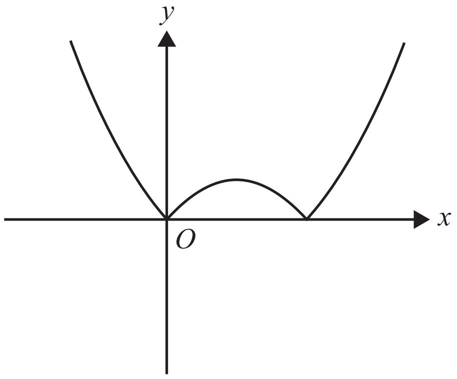
Question 20

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

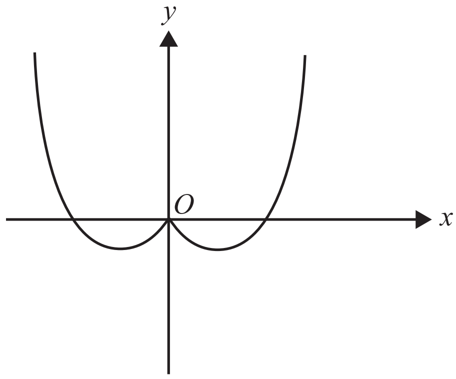


The graph of $y = f(g(x))$ is best represented by

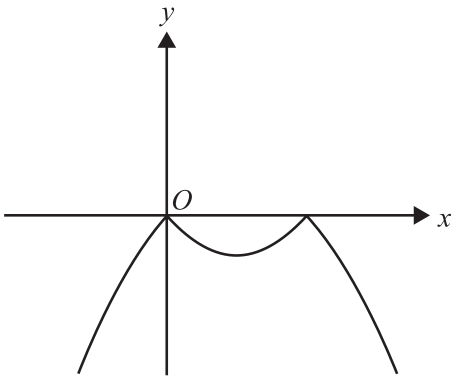
A.



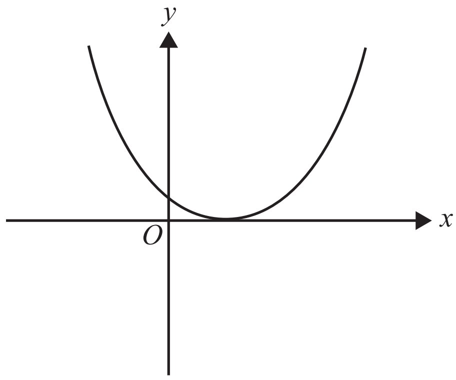
B.



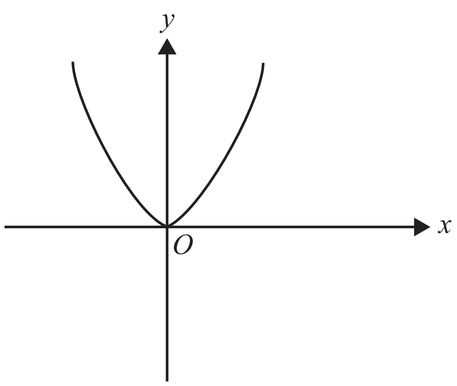
C.



D.



E.



END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

The population of wombats in a particular location varies according to the rule $n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2013.

a. Find the period and amplitude of the function n. 2 marks

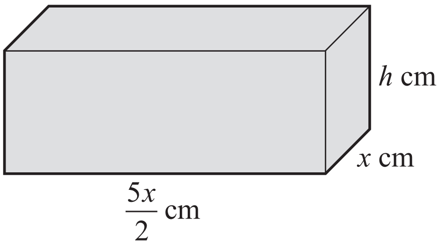
b. Find the maximum and minimum populations of wombats in this location. 2 marks

c. Find $n(10)$. 1 mark

d. Over the 12 months from 1 March 2013, find the fraction of time when the population of wombats in this location was less than $n(10)$. 2 marks

Question 2 (10 marks)

A solid block in the shape of a rectangular prism has a base of width x centimetres. The length of the base is two-and-a-half times the width of the base.



The block has a total surface area of 6480 cm^2.

a. Show that if the height of the block is h centimetres, $h = \dfrac{6480 - 5x^2}{7x}$. 2 marks

b. The volume, V cubic centimetres, of the block is given by $V(x) = \dfrac{5x(6480 - 5x^2)}{14}$.

Given that $V(x) > 0$ and $x > 0$, find the possible values of x. 3 marks

c. Find $\dfrac{dV}{dx}$, expressing your answer in the form $\dfrac{dV}{dx} = ax^2 + b$, where a and b are real numbers. 3 marks

d. Find the exact values of x and h if the block is to have maximum volume. 2 marks

Question 3 (20 marks)

FullyFit is an international company that owns and operates many fitness centres (gyms) in several countries. It has more than 100 000 members worldwide. At every one of FullyFit's gyms, each member agrees to have their fitness assessed every month by undertaking a set of exercises called **S**. If someone completes **S** in less than three minutes, they are considered fit.

a. It has been found that the probability that any member of FullyFit will complete **S** in less than three minutes is $\frac{5}{8}$. This is independent of any other member. A random sample of 20 FullyFit members is taken. For a sample of 20 members, let X be the random variable that represents the number of members who complete **S** in less than three minutes.

i. Find $\Pr(X \geq 10)$ correct to four decimal places. 2 marks

ii. Find $\Pr(X \geq 15 \mid X \geq 10)$ correct to three decimal places. 3 marks

For samples of 20 members, $\hat{P}$ is the random variable of the distribution of sample proportions of people who complete **S** in less than three minutes.

iii. Find the expected value and variance of $\hat{P}$. 3 marks

iv. Find the probability that a sample proportion lies within two standard deviations of $\frac{5}{8}$. Give your answer correct to three decimal places. Do not use a normal approximation. 3 marks

v. Find $\Pr(\hat{P} \geq \frac{3}{4} \mid \hat{P} \geq \frac{5}{8})$. Give your answer correct to three decimal places. Do not use a normal approximation. 2 marks

b. Paula is a member of FullyFit's gym in San Francisco. She completes **S** every month as required, but otherwise does not attend regularly and so her fitness level varies over many months. Paula finds that if she is fit one month, the probability that she is fit the next month is $\frac{3}{4}$, and if she is not fit one month, the probability that she is not fit the next month is $\frac{1}{2}$.

If Paula is not fit in one particular month, what is the probability that she is fit in exactly two of the next three months? 2 marks

c. When FullyFit surveyed all its gyms throughout the world, it was found that the time taken by members to complete another exercise routine, **T**, is a continuous random variable W with a probability density function g, as defined below.

$$g(w) = \begin{cases} \dfrac{(w-3)^3 + 64}{256} & 1 \leq w \leq 3 \\ \dfrac{w+29}{128} & 3 < w \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

i. Find $E(W)$ correct to four decimal places. 2 marks

ii. In a random sample of 200 FullyFit members, how many members would be expected to take more than four minutes to complete **T**? Give your answer to the nearest integer. 2 marks

d. From a random sample of 100 members, it was found that the sample proportion of people who spent more than two hours per week in the gym was 0.6

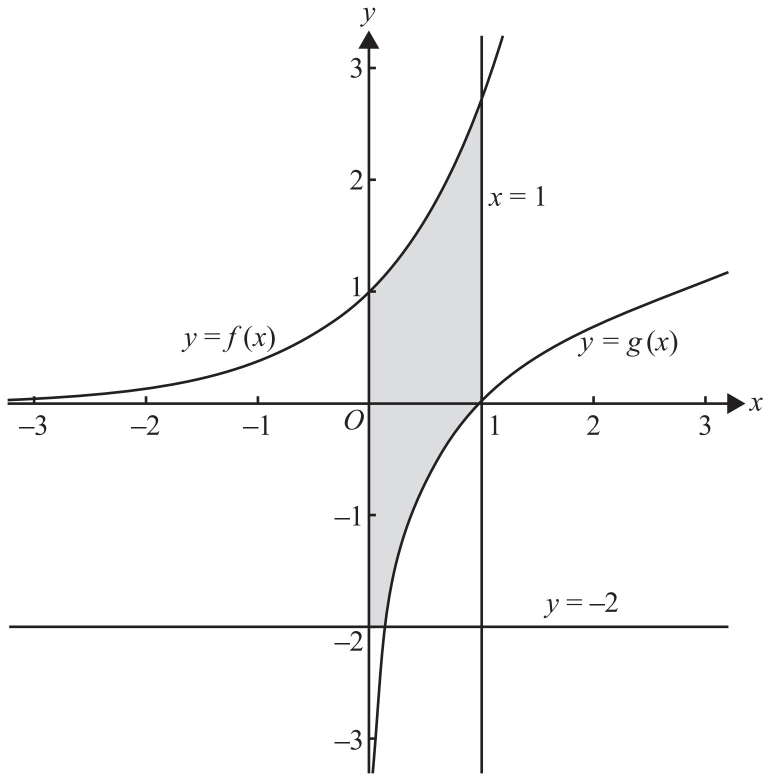
Find an approximate 95% confidence interval for the population proportion corresponding to this sample proportion. Give values correct to three decimal places.

1 mark

Question 4 (8 marks)

The shaded region in the diagram below is the plan of a mine site for the Black Possum mining company. All distances are in kilometres. Two of the boundaries of the mine site are in the shape of the graphs of the following functions.

$$f: R \to R, f(x) = e^x \qquad g: R^+ \to R, g(x) = \log_e(x)$$



a. **i.** Evaluate $\int_{-2}^{0} f(x)dx$. 1 mark

ii. Hence, or otherwise, find the area of the region bounded by the graph of g, the x-axis and y-axis, and the line $y = -2$. 1 mark

iii. Find the total area of the shaded region. 1 mark

b. The mining engineer, Victoria, decides that a better site for the mine is the region bounded by the graph of g and that of a new function $k: (-\infty, a) \to R$, $k(x) = -\log_e(a - x)$, where a is a positive real number.

i. Find, in terms of a, the x-coordinates of the points of intersection of the graphs of g and k. 2 marks

ii. Hence, find the set of values of a for which the graphs of g and k have two distinct points of intersection. 1 mark

c. For the new mine site, the graphs of g and k intersect at two distinct points, A and B. It is proposed to start mining operations along the line segment AB, which joins the two points of intersection.

Victoria decides that the graph of k will be such that the x-coordinate of the midpoint of AB is $\sqrt{2}$.

Find the value of a in this case. 2 marks

Question 5 (15 marks)

Let $f: R \to R,\ f(x) = (x-3)(x-1)(x^2+3)$ and $g: R \to R,\ g(x) = x^4 - 8x$.

a. Express $x^4 - 8x$ in the form $x(x-a)\left((x+b)^2+c\right)$. 2 marks

b. Describe the translation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. 2 marks

c. Find the values of d such that the graph of $y = f(x+d)$ has

i. one positive x-axis intercept 1 mark

ii. two positive x-axis intercepts. 1 mark

d. Find the value of n for which the equation $g(x) = n$ has one solution. 1 mark

e. At the point $(u, g(u))$, the gradient of $y = g(x)$ is m and at the point $(v, g(v))$, the gradient is $-m$, where m is a positive real number.

i. Find the value of $u^3 + v^3$. 2 marks

ii. Find u and v if $u + v = 1$. 1 mark

f. i. Find the equation of the tangent to the graph of $y = g(x)$ at the point $(p, g(p))$. 2 marks

ii. Find the equations of the tangents to the graph of $y = g(x)$ that pass through the point with coordinates $\left(\frac{3}{2}, -12\right)$. 3 marks

END OF QUESTION AND ANSWER BOOK

Answers to multiple-choice questions

Question	Answer
1	A
2	D
3	C
4	E
5	A
6	A
7	C
8	E
9	E
10	C
11	B
12	B
13	A
14	B
15	D
16	E
17	D
18	B
19	D
20	E