Specialist Mathematics Examination 2

2024 Formula Sheet

You may keep this Formula Sheet.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Mensuration

area of a circle segment	$\dfrac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\dfrac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\dfrac{1}{2}bc\sin(A)$
volume of a cone	$\dfrac{1}{3}\pi r^2 h$	sine rule	$\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$
volume of a pyramid	$\dfrac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}\left(\theta_1 + \theta_2\right)$	
$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \dots X_n$	$\begin{aligned} &E(aX_1 + b) = a\,E(X_1) + b \\ &E(a_1 X_1 + a_2 X_2 + \dots + a_n X_n) \\ &= a_1 E(X_1) + a_2 E(X_2) + \dots + a_n E(X_n) \end{aligned}$	
	$\begin{aligned} &\mathrm{Var}(aX_1 + b) = a^2 \mathrm{Var}(X_1) \\ &\mathrm{Var}(a_1 X_1 + a_2 X_2 + \dots + a_n X_n) \\ &= a_1^2 \mathrm{Var}(X_1) + a_2^2 \mathrm{Var}(X_2) + \dots + a_n^2 \mathrm{Var}(X_n) \end{aligned}$	
for independent identically distributed variables $X_1, X_2 \dots X_n$	$E(X_1 + X_2 + \dots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \dots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}}, \ \bar{x} + z\dfrac{s}{\sqrt{n}} \right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

Calculus

$\dfrac{d}{dx}\left(x^n\right) = n\,x^{n-1}$	$\displaystyle\int x^n dx = \dfrac{1}{n+1}x^{n+1} + c, \;\; n \neq -1$
$\dfrac{d}{dx}\left(e^{ax}\right) = a\,e^{ax}$	$\displaystyle\int e^{ax} dx = \dfrac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$	$\displaystyle\int \dfrac{1}{x} dx = \log_e|x| + c$
$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}\left(\cot(ax)\right) = -a\,\mathrm{cosec}^2(ax)$	$\displaystyle\int \mathrm{cosec}^2(ax)\,dx = -\dfrac{1}{a}\cot(ax) + c$
$\dfrac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\displaystyle\int \sec(ax)\tan(ax)\,dx = \dfrac{1}{a}\sec(ax) + c$
$\dfrac{d}{dx}\left(\mathrm{cosec}(ax)\right) = -a\,\mathrm{cosec}(ax)\cot(ax)$	$\displaystyle\int \mathrm{cosec}(ax)\cot(ax)\,dx = -\dfrac{1}{a}\mathrm{cosec}(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right) = \dfrac{a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right) = \dfrac{-a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right) = \dfrac{a}{1+(ax)^2}$	$\displaystyle\int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$
	$\displaystyle\int (ax+b)^n\,dx = \dfrac{1}{a(n+1)}(ax+b)^{n+1} + c, \;\; n \neq -1$
	$\displaystyle\int \dfrac{1}{ax+b}\,dx = \dfrac{1}{a}\log_e|ax+b| + c$

Calculus – continued

product rule	$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
quotient rule	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
chain rule	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
integration by parts	$\displaystyle\int u\dfrac{dv}{dx}\,dx = uv - \int v\dfrac{du}{dx}\,dx$
Euler's method	If $\dfrac{dy}{dx} = f(x,\,y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f\left(x_n,\,y_n\right).$
arc length parametric	$\displaystyle\int_{t_1}^{t_2} \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\displaystyle\int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\displaystyle\int_{y_1}^{y_2} 2\pi x \sqrt{1 + \left(\dfrac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\displaystyle\int_{t_1}^{t_2} 2\pi y \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\displaystyle\int_{t_1}^{t_2} 2\pi x \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \dfrac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$	$\|\mathbf{r}(t)\| = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\mathbf{r}}(t) = \dfrac{d\mathbf{r}}{dt} = \dfrac{dx}{dt}\mathbf{i} + \dfrac{dy}{dt}\mathbf{j} + \dfrac{dz}{dt}\mathbf{k}$
for $\mathbf{r}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{r}_2 = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$	vector scalar product $\mathbf{r}_1 \cdot \mathbf{r}_2 = \|\mathbf{r}_1\|\|\mathbf{r}_2\|\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$ vector cross product $\mathbf{r}_1 \times \mathbf{r}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\mathbf{i} + (x_2z_1 - x_1z_2)\mathbf{j} + (x_1y_2 - x_2y_1)\mathbf{k}$
vector equation of a line	$\mathbf{r}(t) = \mathbf{r}_1 + t\mathbf{r}_2 = (x_1 + x_2t)\mathbf{i} + (y_1 + y_2t)\mathbf{j} + (z_1 + z_2t)\mathbf{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\mathbf{r}(s, t) = \mathbf{r}_0 + s\mathbf{r}_1 + t\mathbf{r}_2$ $= (x_0 + x_1s + x_2t)\mathbf{i} + (y_0 + y_1s + y_2t)\mathbf{j} + (z_0 + z_1s + z_2t)\mathbf{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t, \; y(s, t) = y_0 + y_1s + y_2t, \; z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1 - \cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1 + \cos(2ax)\right)$

End of Formula Sheet