2006 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The new structure for examination 1 started in 2006. Previously, students were required to answer 30 multiple-choice questions in Part I, and five short answer questions worth a total of 20 marks in Part II. In 2006 students were required to answer nine short and extended answer questions worth a total of 40 marks. No calculators, CAS or notes of any kind were permitted in the examination. Due to the change in the structure of the examination, where practicable the comparisons given below are of students' performance on the short answer parts of the respective examinations; that is, with respect to Part II of examination 1 in 2005 and the entire paper in 2006.

The number of students who sat for the 2006 Specialist Maths examination 1 was 5208, compared to 5626 in 2005.

The mean score for the 2006 examination 21.8 was out of 40 (54.4%), very similar to the performance on the short answer section of the 2005 paper where the mean score was 54.5%. Seven out of 17 question parts had mean scores of less than 50% of the maximum possible, which was a little worse than three out of nine question parts in 2005.

The overall mean and median scores were 21.8 (54.4%) and 22 (55%) out of 40 respectively, compared with 30.2 (60.4%) and 31 (62%) out of 50 respectively for the whole examination (multiple-choice and short answer) in 2005. For the 2006 examination 1, about 18 per cent of students scored less than 25% of the available marks. At the lower end of scores, 64 students (1.2% of the cohort) scored zero marks and 375 students (7.2%) scored less than four marks out of 40. At the higher end of scores, 133 students (2.6%) scored full marks and 573 students (11%) scored more than 36 marks out of 40.

In the comments on specific questions in the next section, common mistakes that are made year after year are highlighted. These mistakes should be brought to the attention of students so that they can take steps to avoid them in future. A particular concern is the need for students to **read the questions carefully**.

Areas of weakness included:

- poor algebraic skills. This was evident in several questions, and the inability to simplify expressions often prevented students from completing a question
- showing a given result, which was required in Question 5a. The onus is on students to include sufficient relevant working to convince the assessors that they do know how to derive the required result. Importantly, students should be reminded that they **can** use a given value in the remaining part(s) of the question, **whether or not** they were able to derive it
- recognising the need to use the chain rule when differentiating a function of a function (Question 1a.)
- using the change of variable rule when changing the variable of integration (Questions 2 and 8)
- choosing a substitution which leads to a straightforward integration (Questions 2 and 8)
- recognising the method of integration required (Questions 2, 3b. and 8)
- recognising the need for a double angle formula (Question 5a.)
- knowing the exact values for circular functions (Questions 5 and 9)
- labelling a diagram with the correct forces and only those forces (Question 4a.)
- giving answers in the required form (Questions 4b., 6c. and 9b.).

Students need to be reminded that the instruction 'sketch' (see Questions 3a. and 7b.) does **not** mean that a rough and careless attempt is acceptable or that details such as a reasonable scale, correct domain, asymptotes and asymptotic behaviour can be omitted.

As students were not permitted to bring calculators into the examination, they should be able to simplify simple arithmetic expressions. Too many students were not able to do this correctly and consequently lost marks.

SPECIFIC INFORMATION

Question 1a.

Marks	0	1	2	Average
%	17	27	56	1.5

$$\frac{dy}{dx} = \frac{y}{9y - x}$$

Most students recognised the need to use implicit differentiation and that the product rule had to be used to differentiate the term *xy*, but several could not get started. A large number were not able to make $\frac{dy}{dx}$ the subject due to poor algebraic skills.

Question 1b.

Marks	0	1	2	Average
%	33	16	52	**1.3**

$$\frac{dy}{dx} = \frac{1}{9}$$

Students were required to substitute $y = 1$ into the original equation to find that $x = 0$ and then substitute both of these values into $\frac{dy}{dx}$. Many did not substitute $y = 1$ into the original equation to find x and gave an answer in terms of x.

Question 2

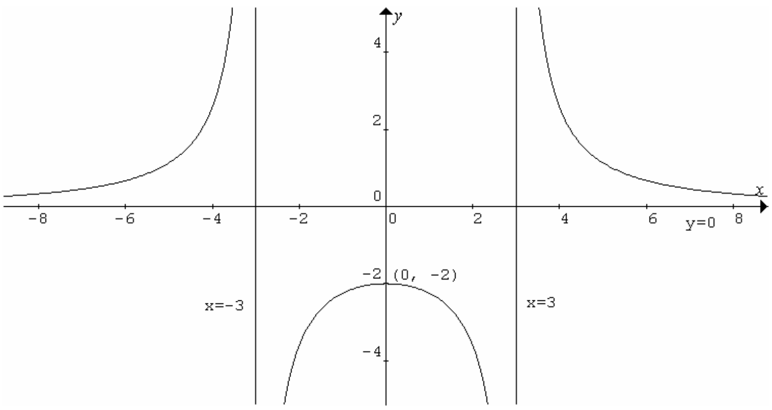
Marks	0	1	2	3	4	Average
%	20	4	7	24	45	**2.8**

$$y = \frac{1}{3}\left(x^2 - 16\right)^{\frac{3}{2}} + 4$$

To solve this differential equation it was necessary to use a substitution of variable. There were several possible substitutions, the easiest being $u = x^2 - 16$. There was a great diversity in the substitutions attempted and many were unsuccessful. Some students omitted the constant of integration and of those who did include it, a surprisingly large number were not able to evaluate it due either to arithmetic errors or an inability to simplify $9^{\frac{3}{2}}$. Some attempted to multiply both sides of the equation by 3 to find the constant of integration but multiplied all terms other than the *c*. Several students were let down by poor notation or careless errors such as taking the *x* outside the integral. A number of students used the product rule to differentiate the given expression instead of integrating.

Question 3a.

Marks	0	1	2	3	Average
%	22	12	28	39	**1.9**



Some strange graphs were seen for this routine reciprocal function question; however, most students made a good attempt at the shape of the graph. The main error was a failure to mention and/or label the horizontal asymptote. Students should be encouraged to use a pencil to draw graphs; some graphs were very messy and hard to read, with multiple lines drawn in some cases.

Question 3b.

Marks	0	1	2	3	4	Average
%	30	9	18	21	22	**2.1**

$6\log_e(5)$

$3\log_e(25)$ and other equivalent answers were also accepted.

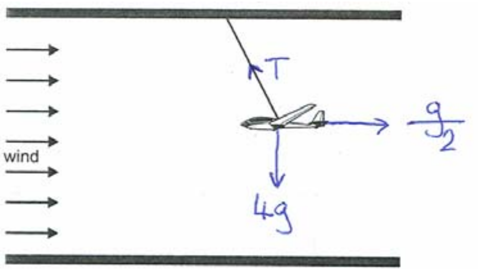
This was a straightforward question involving the use of partial fractions to find the specified area. A large proportion of students did not account for the fact that the area specified was below the *x*-axis, and therefore $-\int_{-2}^{2} y\,dx$ was required. This was disappointing given that this concept is taught in Mathematical Methods/Mathematical Methods (CAS) also.

Leaving out the negative sign led to answers such as $6\log_e\left(\frac{1}{5}\right)$, which many students did not recognise is a negative number and hence cannot represent an area. Another common error was to neglect the modulus (absolute value) sign in $\int \frac{1}{x}\,dx = \log|x| + c$. This led to numbers such as $\log_e(-5)$ or $\log_e\left(-\frac{1}{5}\right)$, which are not real numbers. Some students achieved the correct answer by neglecting the modulus sign and then having the negatives inside the logs cancel when dividing. This was not awarded full marks.

Too many students gave an answer involving arctan or even the logarithm of a quadratic expression. A good alternative used by some students was to take the negative out of the $x-3$ before integrating so that the modulus signs were not necessary.

Question 4a.

Marks	0	1	Average
%	30	70	**0.7**



This easy question was not answered as well as expected. The majority of students were able to place the forces in the correct positions, although many invented imaginative forces such as normal and friction forces. The tension force was sometimes not shown or its direction was not indicated.

Question 4b.

Marks	0	1	2	Average
%	42	15	42	**1.0**

$\frac{g\sqrt{65}}{2}$

Many students correctly used Pythagoras' Theorem but quite a few of these made careless errors. The success rate among those who resolved forces was not high, with many getting lost along the way and some trying to find an angle rather than the tension force. Some students showed correct working but did not give their answer in the required form.

Far too many students added the two vectors using the scalar value only to get the incorrect answer of

$$T = W + F = \frac{g}{2} + 4g = \frac{9g}{2} = \frac{g\sqrt{81}}{2}$$

Question 5a.

Marks	0	1	2	3	4	Average
%	59	6	6	7	22	**1.3**

$\sqrt{2} - 1$ (given)

This question was poorly answered. Students could apply either the double angle formula using $\tan\left(\frac{\pi}{4}\right) = \tan\left(2 \times \frac{\pi}{8}\right)$ or the compound angle formula using $\tan\left(\frac{\pi}{8}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{8}\right)$ and, in either case, then use $\tan\left(\frac{\pi}{4}\right) = 1$.

Many algebraic errors were seen and the given answer often appeared from nowhere. Quite a few students whose working was correct failed to complete their solution, giving no proper explanation as to why the negative answer should be rejected, or not mentioning the negative answer at all. Substituting $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$ into either the double angle formula or the compound angle formula was not sufficient to achieve full marks. Some students converted to sin and cos but they generally made little progress. This question was often left blank.

Question 5b.

Marks	0	1	2	Average
%	61	11	28	**0.7**

$$\frac{\pi}{2\left(\sqrt{2} - 1\right)}$$

It was disappointing that this question was not answered better as it was similar to a question on the sample examination. A large number of students did not seem to recall that inverse tan function has range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Several of those who did answer this question correctly used a sketch of the inverse tan function to assist them. Some students thought that to find the minimum value they had to differentiate rather than consider the range of the function involved. Others, after arriving at a correct answer, tried to go further and incorrectly rationalised the denominator of the fraction. Many students left this question blank.

Question 6a.

Marks	0	1	Average
%	30	70	**0.7**

$$V = \pi \int_0^a \left(e^{-x}\right)^2 dx$$

This question was generally very well done, with most errors being due to carelessness. It was a straightforward solid of revolution question. Amongst the careless errors, many left the 'dx' off the integral, some forget to square e^{-x}, a few omitted π, and a few had wrong terminals.

Question 6b.

Marks	0	1	Average
%	40	60	**0.6**

$$V = \frac{\pi}{2}\left(1 - e^{-2a}\right)$$

Most of the students who succeeded with part a. were also able to answer this part. A few made careless errors such as integrating e^{-2x} to get $-2e^{-2x}$, or taking the factor of $\frac{1}{2}$ out of the bracket incorrectly.

Question 6c.

Marks	0	1	2	Average
%	21	29	49	**1.4**

$\log_e\left(\frac{3}{2}\right)$

$\frac{1}{2}\log_e\left(\frac{9}{4}\right)$ and other equivalent answers were also accepted.

The question was generally well done with the main issues being transcription errors, algebraic errors, and an inability of some students to evaluate $\frac{5}{9}-1$. Some students substituted the given value of $\frac{5\pi}{18}$ into *a*, instead of *V*.

Question 7a.

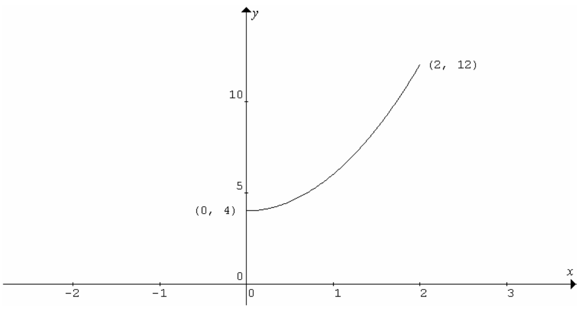
Marks	0	1	2	Average
%	20	6	74	**1.6**

$y = 2x^2 + 4$

This question was well done by most students. While some could not get started, the vast majority recognised that *x* and *y* had been given in parametric form and worked accordingly. Some attempts were confounded by algebraic errors, often involving the square root. Quite a few students appeared to have only attempted similar questions that result in circles, as there were many attempts to find $x^2 + y^2$. A small minority inexplicably thought that this question involved complex numbers.

Question 7b.

Marks	0	1	2	Average
%	48	21	31	**0.9**



Surprisingly, many students who correctly answered part a. were unable to sketch the parabolic shape. Some just plotted two or three points and connected them, some drew a straight line and others drew some other shape. A very large proportion of students ignored the restriction on the domain. Of those who did restrict the domain, many used the given *t*-values rather than converting to cartesian using $2 \le x \le 6$, while several used $x \ge 0$. Far too many students were not careful with their vertical scaling of the graph, placing the 4 and 12 at disproportionate intervals, and some did not mark the endpoints. Students need to be reminded that the instruction ‘sketch’ does not mean that a rough and careless attempt is acceptable.

Question 8

Marks	0	1	2	3	4	Average
%	47	8	5	9	31	**1.7**

$2\arcsin\left(\frac{x}{2}\right)-6\sqrt{4-x^2}+c$

Students were expected to split the integral into the sum of two integrals and then recognise that one part was in the form for inverse sine and the other required a substitution. Several substitutions were possible, the easiest being $u=4-x^2$. Many students used brackets poorly and made errors in expanding. Some students found $\frac{d}{dx}\left(\frac{2+6x}{\sqrt{4-x^2}}\right)$ rather than the antiderivative, while others used the partial fractions $\frac{A}{\sqrt{2+x}}+\frac{B}{\sqrt{2-x}}$.

Overall this question was not well done and there was a clear difference between the students who knew how to approach this type of question and those who did not. Many failed to see that the integral should **first** be split into the sum of two integrals, and many of these proceeded with unproductive substitutions and nonsensical manipulations.

Question 9a.

Marks	0	1	Average
%	21	79	0.8

$2\text{cis}\left(\frac{\pi}{3}\right)$

This question was well done by most students, although it was clear that some had not learned the exact values for trigonometry as the most common mistake was to use $\frac{\pi}{6}$ as the argument.

Question 9b.

Marks	0	1	2	3	Average
%	39	37	8	16	1.0

$z=\frac{\sqrt{6}-2+\sqrt{2}\,i}{2},\frac{-\sqrt{6}-2-\sqrt{2}\,i}{2}$

Many of the attempts at this question were very disappointing, with far too many students having no idea how to get started. Students were expected to either complete the square or use the quadratic formula. In either case, they then had to use the information from part a. and apply de Moivre's Theorem. Few students were able to get past the first stage and it was disappointing to see how many students either failed to use the quadratic formula correctly, did not complete the square correctly, or could not simplify their result. A large proportion of students stopped after completing the square or using the quadratic formula, making it clear that many did not know that cartesian form means in the form $x+iy$ where x and y are real numbers. Several students assumed that $1+i\sqrt{3}$ was a solution to the quadratic equation.