VCE Mathematics Review: Consultation on proposed structures

Introduction

As part of the background work for the major review of VCE Mathematics, the VCAA has been considering the question:

*What could a senior secondary mathematics curriculum for a liberal democratic society in a developed country for* 2020–2030 *look like*?

This continues work undertaken as part the 2013–14 review, which was also informed by the Australian Curriculum development work and included consideration of developments in mathematics, mathematics education and society more broadly over the past two decades and reflects like work in other jurisdictions around the world.

During 2018, the VCAA undertook preliminary work to investigate various structures for senior secondary mathematics curriculum and convened an expert panel to make recommendations in preparation for broad consultation in 2019.

This consultation is seeking feedback on the structures being proposed for VCE Mathematics and includes:

* three key background papers: *New Directions*, *Working Towards Change* and
*Deeper review – directions and options*
* three curriculum structures for consultation.

The VCAA will consult with key stakeholders and interested parties to ensure that feedback is gained from organisations, groups and individuals.

Key background papers

| *New Directions* (2014) |
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| This paper was developed for the 2013–14 review of the VCE Mathematics study to provide stimulus for ongoing work in this area. |
| The paper outlines and discusses historical and philosophical developments in the discipline of mathematics, its methodologies and applications and developments in mathematics education and curriculum development from the mid-20th century to the present. It puts forward possible directions for future development of senior secondary mathematics curriculum. |
| *Working Towards Change* (2017) |
| This paper provides an historical overview of developments in senior secondary mathematics curriculum and assessment in Victoria from 1970–present and outlines the various structures from over the past fifty years. |
| The paper summarises course designs and content, examination and school-based assessment, the use of technology and the role of various studies with respect to university pre-requisite requirements. It includes links to historical Victorian curriculum documents. |
| *Deeper review – directions and options* (2018) |
| This paper outlines key considerations for curriculum design in the context of developments in society, technology, mathematics and mathematics education during the 21st century.  |
| The paper includes links to articles, papers and reports that elaborate on various aspects of the related discussion. It includes a brief overview of curriculum structures in a range of other jurisdictions, and discussion and a short meta-analysis of responses to the introductory question from several invited contributors nationally and internationally. |
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After reading the three key background papers, stakeholders are invited to consider and respond to the consultation questionnaire for each structure.

Structures

The basic construct for all VCE studies is the unit. Each unit involves at least 50 hours of scheduled classroom instruction over the duration of a semester. It is expected that students will undertake up to 50 hours of self-directed learning for each unit. The key design elements of a unit are the areas of study and the outcomes.

Three structures are being put forward for consultation, with Structure A having two models. While current and indicative nomenclature and areas of study or broad topics have been included as applicable, the detailed development of a study is subject to the work of the study review panels for VCE Mathematics and further consultation on the specific draft study design proposals developed by these panels.

A block diagram overview is provided overleaf for the three structures, followed by a more detailed outline of each.

**Structure A: Model 1 *-* current VCE Mathematics study design**

Block diagram overview of the structure

This structure and model are flexible and provide pathways covering a broad range of student abilities, dispositions, interests, and aspirations, with the exception of an everyday practical context-based Unit 3 and 4 level offering. Review based on this structure and model would likely lead to minor change and refinement.

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| **Units 1 and 2: Foundation Mathematics**Prescribed content |  |
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| **Units 1 and 2: General Mathematics**Selection of topics from areas of study Specified topics contain assumed knowledge and skills for Further Mathematics Core content | → | **Units 3 and 4: Further Mathematics**Core content plus two modules selected from four possible modules |
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| **Units 1 and 2: Mathematical Methods**Prescribed content: assumed knowledge and skills for Unit 3 and 4 | → | **Units 3 and 4: Mathematical Methods**Prescribed content |
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| **Units 1 and 2: Specialist Mathematics**Four core topics (covering additional assumed knowledge and skills for Units 3 and 4: Specialist Mathematics) plus selection of topics from areas of study  | → | **Units 3 and 4: Specialist Mathematics**Prescribed content, assumes concurrent study or previous completion of Mathematical Methods Units 3 and 4 |

Detailed outline of the structure

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| **Units 1 and 2** | **Units 3 and 4** |
| **Foundation Mathematics***Space, shape and designPatterns and numberDataMeasurement* | While there is no Unit 3 and 4 sequence for this study, students who had done well in Foundation mathematics Units 1 and 2, could potentially, with suitable additional preparation, subsequently undertake Further Mathematics. |
| **General Mathematics** *Algebra and structureArithmetic and numberDiscrete mathematicsGeometry, measurement and trigonometryGraphs of linear and non-linear relationsStatistics* The areas of study comprise a range of topics, for each unit to suit the range of students entering the study, content must be selected so content covers four or more topics in their entirety, selected from at least three different areas of study and courses intended as preparation for study at the Units 3 and 4 level should include a selection of topics from areas of study that provide a suitable background for these studies. | **Further Mathematics****Unit 3 (Core)***Data analysisRecursion and financial modelling***Unit 4 (Applications**)2 modules selected from the following 4 modules*Matrices and modellingNetworks and decision mathematicsGeometry and measurementGraphs and relations*Assumed knowledge and skills covered in the following topics from General Mathematics: Computation and practical arithmetic; Investigating and comparing data distributions; Investigating relationships between two numerical variables; Linear graphs and modelling; Linear relations and equations; and Number patterns and recursion. |
| **Mathematical Methods** *Functions and graphsAlgebraCalculusProbability and Statistics (distributions, sample proportions and approximate confidence intervals)*These units are a completely prescribed sequence, that are specifically designed as preparation for Mathematical Methods Units 3 and 4. They contain assumed knowledge and skills for the Unit 3 and 4 sequence.  | **Mathematical Methods** *Functions and graphsAlgebraCalculusProbability and Statistics (distributions, sample proportions and approximate confidence intervals)*These units are a completely prescribed sequence. |
| **Specialist Mathematics** *Algebra and structureArithmetic and numberDiscrete mathematicsGeometry, measurement and trigonometryGraphs of linear and non-linear relationsStatistics*The areas of study comprise a range of topics, for each unit to suit the range of students entering the study, content must be selected to content covers four or more topics in their entirety, selected from at least three different areas of study, including two of the prescribed topics: Number systems and recursion; Vectors in the plane; Geometry in the plane and proof; and Graphs of non-linear relations. | **Specialist Mathematics** *Functions and graphs (including rational and other quotient functions)Algebra (including complex numbers)Calculus (advanced - including differential equations and kinematics)Vectors (including vector calculus)MechanicsProbability and Statistics (sample means, approximate confidence intervals and hypothesis testing)*These units are a completely prescribed sequence. They assume concurrent study or prior completion of Mathematical Methods Units 3 and 4. Additional assumed knowledge and skills are contained in the four prescribed topics for Specialist Mathematics Units 1 and 2.  |

**Structure A: Model 2 - all units completely prescribed**

Block diagram overview of the structure

This structure and model incorporate provision of an everyday practical context-based Unit 3 and 4 level offering, align nomenclature, simplify and facilitate development of supporting resources, advice and assessment as content is completely prescribed.

Review based on this structure and model could facilitate a greater degree of change.

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| **Units 1 and 2: FoundationMathematics** | → | **Units 3 and 4: FoundationMathematics** |
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| **Units 1 and 2: General Mathematics** | → | **Units 3 and 4: General Mathematics** |
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| **Units 1 and 2: Mathematical Methods**Prescribed content: assumed knowledge and skills for Unit 3 and 4 | → | **Units 3 and 4: Mathematical Methods** |
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| **Units 1 and 2: Specialist Mathematic**Covers additional assumed knowledge and skills for Units 3 and 4: Specialist Mathematics and other topics  | → | **Units 3 and 4: Specialist Mathematics**Assumes concurrent study or previous completion of Mathematical Methods Units 3 and 4 |

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| Detailed outline of the structure |
| **Units 1 and 2** | **Units 3 and 4** |
| **Foundation Mathematics***Space, shape and designNumber, patterns and relationshipsDataMeasurementMathematical investigation* | **Foundation Mathematics***Space, shape and design (focus on 3-dimensional applications)Number, patterns and relationships (graphs and relations step, segment and empirical data)Chance (including practical applications of probability) and data (including bivariate data)Measurement (including earth geometry and time-zones)**Mathematical investigation* |
| Students enrolling in the calculus-based studies would **not** be permitted to enrol in these units. |
| **General Mathematics** *Statistics: investigating and comparing data distributions and relationships between two numerical variablesComputation and financial arithmeticApplications of matrices Graphs and networksNumber patterns and recursionGeometry, measurement and trigonometryLinear functions, graphs, models and equationsVariationMathematical Investigation* | **General MathematicsUnit 3** *Data analysis (including regression and time series)Recursion and financial modelling***Unit 4***Matrices and modellingNetworks and decision mathematics**Assumed knowledge and skills covered in General Mathematics Units 1 and 2.*  |
| **Mathematical Methods** *Functions and graphsAlgebraCalculusProbability and statistics (distributions, sample proportions and means)**Mathematical investigation*These units are specifically designed as preparation for Mathematical Methods Units 3 and 4. They contain assumed knowledge and skills for the Unit 3 and 4 sequence. | **Mathematical Methods** *Functions and graphsAlgebraCalculusProbability and statistics (distributions, sample proportions and approximate confidence intervals)* |
| **Specialist Mathematics** *Number systems and recursion (including complex numbers and financial applications)**Vectors, matrices and transformations**Geometry and trigonometry**Discrete mathematics (graph theory and boolean algebra)**Graphs of non-linear relations (conic, parametric)**Logic and proof, algorithms and coding (applied across topics)**Statistics (simulation and sampling distributions)Mathematical investigation* | **Specialist Mathematics** *Functions and graphs (including rational and other quotient functions)Algebra (including complex numbers)Calculus (advanced – including differential equations and kinematics)Vectors (including vector calculus)Logic and proof, algorithms and coding (applied across topics)Probability and Statistics (sample means, approximate confidence intervals, hypothesis testing and t-test)*These units assume concurrent study or prior completion of Mathematical Methods Units 3 and 4.  |

**Structure B: Core and alternative model**

Block diagram overview of the structure

This structure incorporates a more compact set of offerings at the Units 1 and 2 level with a core and alternative format at the Units 3 and 4 level, with additional breadth and depth study an option for the calculus-based studies.

Review based on this structure could facilitate a greater degree of change.

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| **Units 1 and 2: Non-calculus** Core and selection of topics from areas of study  | → | **Units 3 and 4: Non-calculus** Core plusAlternative A (everyday focus) *or* Alternative B (business focus) |
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| **Units 1 and 2: Calculus**Prescribed content | → | **Units 3 and 4: Calculus** Core plus Alternative A (Modelling applications)*or* Alternative B (Mathematical structure) |
| **+ optional**  |  | **+ optional** |
| **Units 1 and 2: Additional breadth and depth**Core and selection of topics from areas of study  | → | **Units 3 and 4: Additional breadth and depth**Core and selection of topics from areas of study  |

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| Detailed outline of the structure |
| **Units 1 and 2: Non-calculus**  | **Units 3 and 4: Non-calculus**  |
| **Common core***Computation and financial arithmetic**Linear relations, graphs, models and equations**Statistics: investigating and comparing data distributions and relationships between two numerical variables**Mathematical investigation**Selection of topics covering a range of Foundation and General interests and background* | **Common core** *Data analysis (including regression and time series)Computation and financial modelling*Plus Select either ***Alternative A*** or ***Alternative B*****Alternative A (Everyday focus)***Space, shape and design (focus on 3-dimensional applications)Number, patterns and relationships (graphs and relations step, segment and empirical data)**Applications folio* **Alternative B (Business focus)***Matrices and modellingNetworks and decision mathematics* |
| **Units 1 and 2: Calculus**  | **Units 3 and 4: Calculus**  |
| **Common content***Functions and graphsAlgebraCalculusProbability and statistics (elementary probability, distributions of sample proportions and means)**Mathematical investigation* | **Common core** *Probability and statistics (Binomial and normal distributions, sample proportions and means, approximate confidence intervals)**Functions, algebra and calculus, differentiation of combined functions, integration of basic and transformed functions*Plus Select either ***Alternative A*** or ***Alternative B*****Alternative A (Modelling applications)***Modelling with combined functions**Modelling with other probability distributions**Matrices, transformations of the plane and Markov sequences***Alternative B (Mathematical structure)***Rational and quotient functions**Advanced calculus**Differential equations and kinematics* |
| **Units 1 and 2: Additional breadth and depth** | **Units 3 and 4: Additional breadth and depth** |
| *An additional pair of units covering topics such as number systems, sets and logic, geometry, coordinate systems and non-linear relations, algorithms and coding, coordinate systems, introduction to graph theory, and proof could also be offered for students wishing to develop their background in mathematical structure and proof.**This would also include a mathematical investigation into the emergence and development of a key mathematical idea.* | *An additional pair of units offered for students wishing to develop their background in mathematical structure and proof, covering topics such as graph theory, boolean algebra, complex numbers, matrices and vectors, geometry, algorithms and coding.**This would also include a mathematical investigation into the emergence and development of a key mathematical proof.* |

**Structure C: problem-centred computer-based mathematics incorporating data science**

Block diagram overview of the structure

This structure comprises five areas of mathematics: Data science, Geometry, Information theory, Modelling techniques and Architecture of mathematics, developed across each of four 50-hour blocks using specifically designed and sequenced problem sets. For each problem, a module of work would be developed and mapped to mathematical concepts, techniques, tools and outcomes.

This would be a new and distinctive structure and approach.

Review based on this structure would likely represent significant change with respect to content and approach on a whole of program level compared to Structure A (Model 1 or Model 2) or Structure B.

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| **Areas of mathematics**Data scienceGeometryInformation theoryModelling techniquesArchitecture of mathematics  | **Blocks**Mathematical modelling and tools of mathematicsMathematics for problems in the physical worldMathematics for problems in the natural world Mathematics for business and life  |
| **Problems related to areas of mathematics apply across all blocks** |
| **Unit 1** | **Unit 2** | **Unit 3** | Unit 4 |
| **+ optional** |
| **Extensions** | **Extensions** | **Extensions** | Extensions |

Detailed outline of the structure

The distinctive proposal by the Wolfram Research team (problem centred computer-based mathematics) and the proposed Data science study contain aspects which the Expert Panel found valuable.

There was support for these aspects, indeed, many of the invited paper respondents independently included elements of them in their considerations, within more familiar structures and models. These were: a greater focus on problem-solving, modelling and mathematical investigation, greater use of the affordances of technology and experimental approaches, further development of computational thinking, algorithms and coding across the mathematics curriculum, increased focus on statistics/probability/data science, discrete mathematics and logic and proof. It would be possible for the structure chosen for the first half of the next decade to be developed so that it supports transition into a more flexible problem-centred structure in the second half of the decade.

The detail of the Wolfram Research team possible structure can be accessed on each of the [VCE Mathematics study pages](https://www.vcaa.vic.edu.au/Pages/vce/studies/index.aspx).