Stage 1 Consultation Phase 2: Mathematical reasoning and working mathematically

Background

Mathematics and its applications in fields from STEM to academia, business, government, industry and research, involves computation and proof, experiment and investigation, formulating questions, conjecturing, hypothesising and problem-posing, estimating, calculating and computing, simulating, analysing, inferring and proving, modelling and problem-solving.

These aspects of mathematics are often broadly characterised as ‘mathematical reasoning’ and ‘working mathematically’ (see appendix for a more detailed description) and the abstraction that is fundamental to mathematics gives rise to its applicability as its concepts, structures, processes (including experimental mathematics and proof) and skills are used to model and explore different contexts and situations and formulate and solve related problems.

Mathematics underpins, develops and uses the digital technology of our contemporary globalised world to deal with large amounts of different types of data and the algorithms that are employed to accurately and efficiently carry out related numerical, graphical, symbolic, geometric, statistical and other computations that would otherwise not be possible.

In the five years from the 2013 – 2014 review of VCE senior secondary mathematics, computational thinking, algorithms and coding, have even more become part of mainstream discourse, along with data analysis, data science, and stochastic reasoning. Their representation and expression in the various VCE Mathematics studies has an emphasis and focus related to the nature, purpose and scope of those studies. The current study design, based on the 2013-14 review, incorporated aspects of each of these across the Units 1–4 studies, in particular with respect to algorithms for recurrence relations, numerical methods, data analysis and statistical inference.

VCE Mathematics major review

The current major review provides the opportunity to further address these aspects in the light of ongoing developments in the discipline, education and society more generally, and to enhance the VCE Mathematics study design.Following on from the work of an Expert Panel in 2018, and publication of three background papers, the VCAA undertook consultation on structures and models for senior secondary mathematics.

Analysis of consultation responses showed support for further development of aspects of mathematical reasoning and processes for working mathematically as discussed in some detail in the background papers and outlined in the questionnaire for the first phase of Stage 1 consultation. In this second phase of consultation, the VCAA is seeking your feedback and comment on how the various aspects of mathematical reasoning and working mathematically should be represented, and with what focus or emphasis, in the different types of VCE Mathematics study, and by what means in relation to areas of study and content, outcomes and key knowledge and key skills, and assessment tasks.

Issues and considerations

Attention to reasoning and working mathematically can be further developed in the senior secondary mathematics curriculum by a combination of approach to covering existing content and also revision of this content including re-allocation of some content across a four unit sequence, inclusion of new content, and removal of some existing content.

The following indicates, for example, some areas of mathematics where these could apply:

* iterative processes: numerical and graphical solutions to equations, systems of equations, derivatives and differential equations, integration
* recursion and induction: sequences, recurrence relations, number, graphs and networks
* simulations: probability, kinematics, graphs and networks
* logic and proof (including induction): number, algebra, calculus, probability and geometry.

Detailed work on such development would take place through study specific review panels informed by feedback from consultation. This would involve linking content, key knowledge and key skills and processes through the areas of study, outcomes and related assessment.

Content and process are closely linked in mathematics, within broad areas of the discipline there are topics that naturally lend themselves to focus on various aspects of mathematical reasoning and working mathematically. For example, work on algorithms and coding can be readily incorporated in such areas as numerical methods, recurrence relations, networks and decision mathematics, optimization. Experimental approaches can be readily used in number theory, probability and differential equations, and proofs can be suitably incorporated in algorithm and program design and checking as well as number, algebra, logic, geometry, graph theory, calculus, probability and statistics.

In the *current* study design, it is specified in the assessment for Units 1 and 2, that demonstration of achievement of Outcome 2 should be based on the student’s performance on a selection of the following assessment tasks: modelling tasks, problem-solving tasks and mathematical investigations. Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

The inclusion of a mathematical investigation over a one-two week period as a *prescribed* component of both Units 1 and 2 of a revised study would provide a natural vehicle for addressing aspects of mathematical reasoning and working mathematically in a context based on one are more areas of study from a unit. It would focus on specific content from the areas of study, be technology active and involve experimental, computational or proof-based approaches, and could incorporate some element of guided student choice in context, within an overall theme. These investigations would also provide preparatory experience for school-assessed coursework applications tasks and modelling or problem-solving tasks in Units 3 and 4. The following table provides an example of some possible content connections for an investigation with respect to the current studies:

| Study | Unit 1 | Unit 2 |
| --- | --- | --- |
| **Foundation Mathematics** | Use of data in the media | Design and measurement |
| **General Mathematics** | Association in data | Financial modelling |
| **Mathematical Methods** | Experiments and simulations in probability  | Modelling with polynomial functions including optimisation |
| **Specialist Mathematics** | Graphs of non-linear relations | Proof in geometry |

Appendix

The following are key aspects of working mathematically.

***Reasoning*** as it relates to logic and proof considers the relationship between natural and symbolic language, concepts, assumptions, definitions and structures, and how various agreed principles of logic can be applied to deduce necessary consequences, or theorems. This is the particular hypothetical-deductive nature of mathematical reasoning (including mathematical induction) and provides the basis for certainty in the discipline. Conjectures are the statements to which experiment, investigation and proof can be applied.

***Experimental mathematics*** involves exploring mathematical ideas through the use of experiments, simulations and the like, often with technology, to carry out computations that generate results, data, examples, counterexamples and similar to test propositions, explore patterns, and develop conjectures or hypotheses. In particular, experimental mathematics can be used to generate and/or explore *new* aspects of mathematics and its applications.

***Computational thinking*** involves the strategic use of technology for the purposes of mathematical analysis that may be directed towards investigation of a theoretical or practical problem, or context, including proof. It involves [four processes](https://computationalthinkingcourse.withgoogle.com/unit):

* *decomposition*: breaking data, processes, or problems into smaller, manageable parts
* *pattern recognition*: observing patterns, trends, and regularities in data
* *abstraction*: identifying the general principles that generate these patterns
* *algorithm design*: developing the step-by-step instructions for solving a problem and like problems.

The development of a language specific implementation for an algorithm is referred to as *coding*. Related fields of mathematics are mathematical logic and computability theory.

***Stochastic reasoning*** involves contexts with uncertainty and randomness, and relates to probability, statistics, and data analysis and data science. Inference with respect to hypotheses is an example of stochastic reasoning and has many applications in STEM, business and other fields.

***Modelling and problem-solving*** mathematical modelling is the process of using mathematical constructs, structures and techniques to represent and describe a real-world context or system, in a simple and concise way that enables one to investigate features and characteristics of its behaviour, analyse particular aspects or solve problems of interest, and to make predictions related to the context or system. Problem solving is a process that occurs in a context where a question, task or issue needs to be solved or resolved, and there is a motivation, but not yet the means, to do so. Questions or tasks for which there are already recognised methods or approaches for solution or resolution, do not require problem-solving in this sense.

In mathematics, problems are generated from questions, conjectures and hypotheses within and across areas of study. New problems may arise in their own right, or as a variation, re-formulation, extension or generalisation of a known problem or class of problems. Mathematical modelling and problem-solving are [complementary processes](https://www.immchallenge.org.au/supporting-resources/what-is-mathematical-modelling) - developing a model may be a strategy that is employed to solve a problem, and problem-solving may be required in developing and applying aspects of a model.