This is the third video in the five videos that we've got, that we're presenting of how to write a SAC in Mathematical Methods, looking at Unit 4, a modelling or problem-solving task. And this one is the functions, algebra and calculus task.

So this is the second part of the task where we identify questions of interest. We state related analysis and we identify relevant content. So we've already looked at what the graphs of xy equals one looks like. And we're already looked to the simple case of y = ax + b over ax + c. So we've begun to introduce the parameters. Now we're going to look at what Part 2 of this task looks like.

Again, the questions of interest we're looking at this time are varying parameters values and areas of discontinuity. We're looking at the hyperbola of the form x minus h, y minus k is m, for some constant m. The centre of the hyperbola is at the point hk, where h and k, at this stage, we're going to assume are positive reals, but you can always change that to make it negative reals or a mixture.

In Part 2, I've introduced composite functions, stationary points and points of inflection, tangents and perpendicular lines. We could introduce area under the graph but I haven't for this one, this particular task, but you as a teacher could introduce that. Distance from a certain point to the graph and maxima and minima, in this context. In Part 2, do you remember in Part 1, we had the coefficient of x being the same. Well, clearly the next question is, if we change the coefficient of x, what happens?

So again, let's start with numbers. The graph of y = 2x + 3, over 4x + 5. Note, I've got a coefficient of x is two on the top and four on the bottom. State the domain and range and identify key features. And we already know in the key features, we're going to be looking at the centre, the asymptotes, where the branches are, and where the asymptotes are. And then similarly analyse, and I haven't actually flipped the top and bottom here, but all I've done is I've changed the magnitude of the coefficient of x. And on the top, I've made the magnitude higher than on the bottom. And it's interesting to see what happens.

Here, I've got these two graphs together. We have a vertical asymptote, of the one on the left of x is -5 on four, which is what we would expect by looking at the denominator. The right-hand graph. We have a vertical asymptote of x is equal -5 on two, which again, we would expect looking at the denominator. But what's interesting is the symmetric branches are on all alternate sides, and it would be very interesting to investigate why that happens.

Similarly, we're now going to go into parameters. Let's consider y = ax - b and I've put that minus in there on purpose, so it gives us an interesting generalised form. Y = ax - b over p minus q. So I've got four different numbers here, a, b, p and q. And again, using our division facility on our calculator or by hand, we have very interesting relationships between bp and aq. So again, can you see it's more complicated than it was in Part 1? But ask the students to explore the three cases of bp is greater than aq, or bp is less than aq, or in fact, where bp equals aq. There's lots of investigation that can go there.

And the possible extension, I'll put this in because it might make the task a bit long, but it is very interesting. I've gone back to the graph of 2x + 3, over 2x + 1. And I wanted to consider where the composite function, f of f of x exists, and to analyse the graph of a composite function. Now, this is a fascinating investigation in its own right. Because you have the original discontinuities of any hyperbola. And then what you've got there is new discontinuities, so that the range of the inner is contained in the domain of the outer. And then if you draw it, you'll find you'll have two discontinuities there. One will be a point and one will be an asymptote. And then if you want to, you can keep going and look at what f of f of f of x looks like, et cetera.

So it's a fascinating analysis of the hyperbola, in its own right, using composite functions. So the indicative content here is added on to what we've looked at before. It's composition of functions, f of g of x, or f o g can be used. So I'm looking at composite functions with existing and new discontinuities. So that's the end of Part 2 of this task.

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