VCE Mathematical Methods Unit 1

Unit 1 Area of Study 1: Calculus

Example of learning activity: Filling vases with water

Introduction

There are many different shaped vases. If a vase is filled by pouring in water at a constant rate, the depth of water in the vase will be a function of how long the water has been pouring in from empty, and the shape of the vase.

Assuming that a vase has rotational symmetry about its central axis, its shape can be represented by a planar cross-section that contains this axis.

This task has two parts and involves qualitative analysis to tackle two problems and explain reasoning:

1. Given the cross-section of a vase, what do the graphs of the depth of water in the vase as a function of time, and the corresponding graph of the rate of change of the depth of water in the vase as a function of time look like?
2. Given a graph of the depth of water in the vase as a function of time, and the graph of the rate of change of the depth of water in the vase as a function of time, what is the shape of the vase?

Note that both functions will be strictly increasing functions as water is being poured in at a constant rate and the vases have a finite volume.

Part 1

Consider a collection of different shaped vases by means of their cross-section and construct a corresponding depth-time graph and rate of change in depth-time graph.

There are a number of [related interactive demonstrations](https://www.geogebra.org/m/S4Yc2fda) available online.

Students could be asked to draw an estimate of the rate of change in depth-time graph before each of the animations are shown.

Part 2

Consider a collection of different depth-time graphs and rate of change in depth-time graphs and construct corresponding cross-sections for vases.

There are various images of rate of change in [depth-time graphs](https://tasks.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2082) available online.

Students could be asked to draw an approximation of the shape of the container before its revealed.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Unit 1** | |
| **Area of study** | **Content dot point** |
| Functions, relations and graphs | – |
| Algebra, number and structure | – |
| Calculus | 2 |
| Data analysis, probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Unit 1** | | |
| **Outcome** | **Key knowledge dot point(s)** | **Key skills dot point(s)** |
| 1 | 1, 8 | 15 |
| 2 | 1, 5 | 5, 6 |
| 3 | 3 | 6 |