VCE Mathematical Methods Unit 2

Unit 2 Area of Study 2: Algebra, number and structure

Example of learning activity: Linear equations for cubic roots

Introduction

This learning activity uses linear functions that are tangents to a graph to determine an approximate value for an irrational root of a cubic polynomial function. That is, the horizontal axis intercept of the tangent to the graph of the function at a point close to the root approximates this root.

Part 1

Consider the cubic polynomial function .

1. Plot the graph of the function and show that it only has one real root.
2. Find the unit interval [a, b] with integer endpoints that contains this root. By varying the plotting domain, find the approximate value of the root correct to one decimal place.
3. Use the rational root theorem to show that this root is irrational.

Part 2

1. Find the horizontal axis intercept, *b*1, of the tangent to the graph of *f* at (*b*, *f*(*b*)). Plot the graph of the tangent on the same set of axes as the graph of the function, with a suitable graphing domain to show the root and this approximation.
2. Find the horizontal axis intercept, *b*2, of the tangent to the graph of *f* at (*b*1, *f*(*b*1)). Plot the graph of the tangent on the same set of axes as the graph of the function, with a suitable graphing domain to show the root and this second approximation.
3. Find the horizontal axis intercept, *b*3, of the tangent to the graph of *f* at (*b*2, *f*(*b*2)). Plot the graph of the tangent on the same set of axes as the graph of the function, with a suitable graphing domain to show the root and this third approximation.

Part 3

1. Use technology to implement the algorithm for Newton’s method, using both *a* and *b* as initial values.
2. Explore how quickly Newton’s method works for other initial values.
3. Apply Newton’s method to a cubic polynomial function with three real roots and explore the variation in sensitivity of the method based on the choice of initial value.

Areas of study

The following content from the areas of study is addressed through this task.

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| --- | --- |
| **Unit 2** | |
| **Area of study** | **Content dot point** |
| Functions, relations and graphs | – |
| Algebra, number and structure | 3 |
| Calculus | – |
| Data analysis, probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| --- | --- | --- |
| **Unit 2** | | |
| **Outcome** | **Key knowledge dot point(s)** | **Key skills dot point(s)** |
| 1 | 8 | 5 |
| 2 | 2, 4, 5 | 3, 4 |
| 3 | 1, 2, 5 | 1, 2, 3, 4, 5, 7, 11 |