VCE Mathematical Methods Unit 2

Unit 2 Area of Study 3: Calculus

Example of learning activity: Numerical approximations for derivatives

Introduction

This learning activity looks at numerical approximations to derivatives by left and right secants and central difference.

If the derivative of a function *f* is defined, then it can be evaluated from first principles by either of the two limits:

 $f^{'}\left(x\right)=lim\_{h\rightarrow 0}\frac{f\left(x\right)-f(x-h)}{h}$ or $f^{'}(x)=lim\_{h\rightarrow 0}\frac{f\left(x+h\right)-f(x)}{h}$

For small positive values of *h*, these correspond to the left secant (backward difference) and right secant (forward difference) approximations for the derivative, that is:

 $f'(x)≈\frac{f\left(x\right)-f(x-h)}{h}$ or $f'(x)≈\frac{f\left(x+h\right)-f(x)}{h}$

The central difference is the average of these, and is used by technology to calculate numerical values for derivatives:

 .$f'(x)≈\frac{f\left(x+h\right)-f(x-h)}{2h}$

In the following work let $ h= 0.0001$.

Part 1

Consider the quadratic function.

1. Construct a table of values for the left secant, right secant and central difference approximations for this function for *x* from –2 to 5 in steps of 0.5.
2. Plot the corresponding points for the central difference approximation, and draw a straight line through them, stating its rule.
3. Repeat steps a. and b. for several other quadratic functions.

Part 2

Repeat Part 1 for several simple cubic polynomial functions, the square root function and the basic hyperbola.

Part 3

Carry out similar analysis for $f\left(x\right)=\sin(\left(x\right))$ over the interval$\left[0, 2π\right] $ in steps of 0.1. What does the graph of the approximate derivative function look like? Repeat this analysis for $f\left(x\right)=\sin(\left(x\right))$ over the interval $\left[0, 2π\right] $in steps of 0.1. What does the graph of the approximate derivative function look like?

Areas of study

The following content from the areas of study is addressed through this task.

|  |
| --- |
| **Unit 2** |
| **Area of study** | **Content dot points** |
| Functions, relations and graphs | – |
| Algebra, number and structure |  |
| Calculus | 1, 2, 3 |
| Data analysis, probability and statistics  | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |
| --- |
| **Unit 2** |
| **Outcome** | **Key knowledge dot point(s)** | **Key skills dot point(s)** |
| 1 | 8 |  8 |
| 2 | 2, 4 | 2, 4, 5 |
| 3 | 2, 5 | 1, 2, 3, 5 |