VCE Mathematical Methods Unit 3

Sample application task: Bezier curves

The application task is to be of 4–6 hours’ duration over a period of 1–2 weeks.

Introduction

A context such as the following could be used to develop an application task that investigates the use of a special kind of cubic polynomial function, called Bernstein polynomials, whose graphs form what are called Bezier curves. These are named after the French automobile engineer Pierre Bezier who developed their application to a new computer-aided design tool for the Renault car manufacturing corporation in the 1960s. Today, Bezier curves are a key component of graphic design applications.

Bezier curves use polynomial functions of low degree, such as cubic polynomials over a restricted domain, to specify the coordinates of the points that make up these curves. These functions, called Bernstein polynomials, provide local control of shape, based on a small set of points called *control points*, and have [graphs](http://www.e-cartouche.ch/content_reg/cartouche/graphics/en/html/unit_Curves.html) that are continuous and smooth curves for which the derivative can be found at any point on the curve. Shapes constructed using drawing packages and the outlines of letters produced by printers are typically based on a set of routines that use these curves.

A cubic Bezier curve drawn over the interval $0\leq t\leq 1$ is produced by graphing a relation that has its *x* and *y* coordinates specified respectively by the cubic polynomial functions:

$$x=a(1-t)^{3}+3ct(1-t)^{2}+3et^{2}\left(1-t\right)+gt^{3}$$

$$y=b(1-t)^{3}+3dt(1-t)^{2}+3ft^{2}\left(1-t\right)+ht^{3}$$

where the coefficients *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h* are obtained from the coordinates of the four control points
(*a*, *b*), (*c*, *d*), (*e*, *f*) and (*g*, *h*).

The gradient of the tangent to the curve for a particular value of *t* can be determined, using the chain rule for differentiation, by the relationship:

$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=provided x'(t)\ne 0$

The shape of the Bezier curve produced depends on the selection of coordinate values for the control points. Where more than one Bezier curve is used to produce a required shape, these curves will need to be joined smoothly to produce a good image. The three components for an application task could be developed as follows.

Component 1

*Introduction of the context: through specific cases or examples students should*

1. Consider the simpler case of quadratic Bezier curves.
2. Select four distinct points as control points and determine the equations for the *x* and *y* coordinates as functions of *t* to specify a particular cubic Bezier curve. Represent the corresponding Bezier curve using a table of values and the graph of the relation.
3. Consider the gradient of the tangent to the curve at various points on the curve, including the relation between the tangents to the first and last control points and the location of the second and third control points.

Component 2

*Consideration of general features of the context students should*

1. Vary the control points and consider the Bezier curves produced, including cases that lead to, for example, straight lines and loops.
2. Investigate the selection of control points to produce a reasonable representation of a particular shape: for example, how many letters of the alphabet can be reasonably approximated by a single Bezier curve?
3. Consider how well a cubic Bezier curve matches the required shape and any limitations on possible shapes that can be represented using these curves.

Component 3

*Variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context. Students should*

1. Represent more complicated shapes formed by piecing together several Bezier curves.
2. Identify the location of control points for a pair of cubic Bezier curves that are smoothly joined to represent other letters, or for several Bezier curves that are smoothly joined to represent a shape such as the outline or cross section of a car.

Areas of study

The following content from the areas of study is addressed through this task.

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| --- | --- |
| **Area of study** | **Content dot points** |
| Functions, relations and graphs | 1, 5, 6 |
| Algebra, number and structure | 1, 5 |
| Calculus | 3, 4 |
| Data analysis, probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| --- | --- | --- |
| **Outcome** | **Key knowledge dot points** | **Key skills dot points** |
| 1 | 1, 3, 4, 7, 10, 11 | 1, 2, 7, 9, 11, 12, 13 |
| 2 | 1, 2, 3, 5 | 1, 2, 3, 4, 6, 7 |
| 3 | 1, 2, 3, 4, 5, 8 | 1, 2, 3, 4, 5, 6, 7, 9, 11, 12 |