VCE Specialist Mathematics
Units 3 and 4

Sample application task – cubic and quartic polynomials

Introduction

A context such as the following can be used to develop an application task that investigates the number and nature of factors and real and complex roots of some cubic and quartic polynomial functions of a single variable with integer coefficients.

Part 1

1. Randomly select a non-zero integer *a* between -10 and 10 and define a cubic polynomial function of the form where  has non-zero integer coefficients and is irreducible over *R*. Draw the graph of and identify all complex roots.
2. Vary the value of the constant term in the rule of and determine the sets of values of this term for which the function has 1, 2 or 3 distinct real roots. Illustrate these cases graphically.
3. Show that the cubic polynomial function with rule  has no rational roots but three irrational real roots. Draw the graph of the function and state the approximate values of its roots to a suitable accuracy.
4. Find a monic cubic polynomial function with non-zero integer coefficients that has three distinct irrational real roots. Use Newton’s method to find an approximate value for each of these roots to a suitable accuracy. Investigate the sensitivity of Newton’s method with respect to the initial value for the approximation for each of the roots of this function.

Part 2

A biquadratic function has rule of the form  where *a* is non-zero.

1. Investigate the graphs of various families of biquadratic functions, including  and identify their key features.
2. Investigate the representation of biquadratic functions in the form and consider the number of real roots. Use a range of examples to illustrate various combinations of real and complex roots and related graphs.

Part 3

Consider the relation between a quartic polynomial function of a single variable and its possible representation as a product of two irreducible quadratic functions with integer coefficients.

1. Give several examples of quartic polynomial function of a single variable that can be expressed as a product of two irreducible quadratic functions with integer coefficients and state their complex roots. Show that can be expressed in this form and explain how the coefficients of the quadratic factors are determined.
2. Show that  can be expressed as a product of irreducible quadratic functions with coefficients including rational numbers and the surd. Provide some other examples of the same kind, for other surd values.
3. Consider other examples and special cases to see if criteria for expression of quartic polynomial functions of a single variable with integer coefficients as a product of two irreducible quadratic functions with integer coefficients can be obtained.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Areas of study** | **Topics** | **Content dot point** |
| Functions, relations and graphs | Functions, relations and graphs | 1, 2, 3 |
| Calculus | Differential and integral calculus | 2, 3 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 3, 6, 7, 8 | 2, 6, 7 |
| **2** | 1, 2, 3, 4, 5 | 1, 2, 3, 5, 6 |
| **3** | 1, 2, 3, 5, 6, 7 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 |