VCE Specialist Mathematics   
Units 3 and 4

Mathematical investigation – curves in the complex plane

Formulation

You can use locus properties to describe curves in the complex plane. For example, | z | = 1 is the unit circle. It is the set of points which are 1 unit from 0+0i.

Different lines and curves in the complex plane can be describe using modulus and Argument.

This is an investigation of these subsets of the complex plane.

Exploration

Part 1

1. Draw graphs of relations |*z* – a| = *b* in the complex plane and describe their nature. Specify the corresponding cartesian forms.
2. Draw graphs of relations |*z* – *h*| = |*z* – *k*| in the complex plane and describe their nature. Specify the corresponding cartesian forms.
3. Draw graphs of relations |*z* – *h*| + |*z* – *k*| = *c* in the complex plane and describe their nature. Specify the corresponding cartesian forms.
4. Draw graphs of relations |*z* – *h*| - |*z* – *k*| = *c* in the complex plane and describe their nature. Specify the corresponding cartesian forms.

Part 2

1. Find the cartesian equation of the locus of points associated with the complex number *z* such that |*z* – *a*| = *k* |*z* – *b*|, where *a* = 1, *b* = 1 + *I* and *k* is a positive real constant. Illustrate with suitable diagrams.
2. Vary the value of *k* and draw the corresponding family of graphs.

Part 3

Let *A* be the point in the complex plane corresponding to *a* = 1 + 0*i* and let ***a*** be the complex vector associated with *a*. For a complex number *z* let *Z* be the associated point in the complex plane. Let *θ* be the magnitude of angle *OZA*.

*O*

*Z*

Im(*z*)

Re(*z*)

*A*

1. Copy the diagramand show the vector ***z* – *a*** starting at *O*. Mark in the position of   
   *z* – *a.* Complete the parallelogram joining *O*, *A*, *Z* and the point corresponding to   
   *z* – *a.*
2. Find *θ* in terms of Arg(*z* – *a*) and Arg(*z*)
3. Let *z* = *x* + *iy*.If *θ* = use the scalar product of two suitable vectors to find a cartesian equation in terms of *x* and *y* of the locus of points which satisfy the relation Arg(*z* – *a*) – Arg(*z*) = .

A circle theorem states that:

‘*angles subtended by a chord at the circle in the same segment are equal in magnitude*.’

The converse of this theorem also holds:

‘*If the line joining two points A and B subtends equal magnitude angles at two other points on the same side of it then the four points lie on a circle.’*

1. Use the converse to help establish the cartesian equation, the range and domain of the locus of points such that Arg(*z* – *a*) – Arg(*z*) = , where *a* = 1 + 0*i*
2. Also find the cartesian equation, the range and domain of the locus of points such that Arg(*z*) –   
   Arg(*z* – *a*) = , where *a* = 1 + 0*i.*

Conclusions

Draw together your findings and summarise how you obtain circles, straight lines, ellipses and hyperbolas. You could construct a table which classifies the different relations.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Area of study** | **Topic** | **Content dot point** |
| Algebra, number and structure. | Complex numbers | 1, 2, 3, 4, 5, 6, 7 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 8, 14, 15, 17 | 14, 15 |
| **2** | 1, 2, 3, 4, 6 | 1, 2, 3, 4 |
| **3** | 1, 2, 3, 4 | 1, 2, 3, 4, 5 |