VCE Specialist Mathematics
Units 3 and 4

Sample modelling or problem-solving task – parachute

Introduction

This task considers modelling and problem-solving related to motion under the effect of gravity and air resistance.

Part 1

A parachutist drops from a plane which is flying horizontally at 2000 metres above a point *O* (the origin). Assume the parachutist drops in free fall for 600 metres vertically, and has the same initial velocity as the plane. The parachute opens after the free fall of 600 metres. And the parachutist then drops vertically with constant speed. Assume there is no horizontal component of velocity after the parachute opens.

a. Find where the parachutist lands with respect to *O*, and find the velocity with which the parachutist reaches the ground.

b. Comment on this model for the parachutist.

Part 2

In **Part 1** the parachutist’s free fall does not consider wind resistance or the parachutist’s mass. In fact, both of these will influence the speed of the descent.

In the following table the motion of a parachutist is shown for the free fall. These figures are recorded from an actual jump. Free fall means before the parachute opens.

|  |
| --- |
| **Vertical distance travelled during free fall** |
| **Time in seconds after jumping from the plane** | **Distance travelled vertically in metres** |
| 1 | 4.88 |
| 2 | 18.89 |
| 3 | 42.06 |
| 4 | 73.76 |
| 5 | 111.56 |
| 6 | 153.62 |
| 7 | 198.73 |
| 8 | 246.28 |
| 9 | 295.96 |
| 10 | 346.86 |
| 11 | 398.98 |
| 12 | 452.01 |
| 13 | 505.05 |
| 14 | 558.09 |
| 15 | 611.13 |

a. Prepare a table comparing the results of the model introduced in Part 1 with the experimental results given in the table and comment briefly. Consider an 11-second period.

**Now assume that air resistance is proportional to the square of the velocity.**

In Part 1 only the vertical component of motion was considered. Vertical position is relative to where the parachutist leaves the plane.

1. The differential equation describing the motion for the free fall in the vertical direction is
 = *g* – *kv*2 where *v* m/sis thevertical speed of the parachutist at time *t* seconds after leaving the plane and *v* = 0 when *t* = 0

c. Find the vertical speed and vertical distance travelled, *y* m,at time *t* seconds after leaving the plane, of the parachutist during free fall (each of these expressions will involve *k*).

Let the terminal velocity of the parachutist be *vT* .

d. Show that *k* = and that the expressions for *v* and the position at time *t* byreplacing

*k*  with are respectively

 *v* = *v*−and *y* = −

e. It has been recorded that the terminal velocity for a parachutist is 54 m/s. Use this result to obtain a table of values for the vertical position, relative to where the parachutist leaves the plane, of the parachutist for the first 15 seconds of his motion under free fall. Compare this with the experimental values and illustrate with suitable graphs. This should include the following:

* a table of values comparing values from the experiment and from this model
* a position (vertical distance travelled) versus time graph with the experimental data also shown
* a vertical speed versus time graph.

Part 3

In this part the path of the parachutist is considered using the information obtained in Part 2. Position is to be measured relative to the origin as shown in the diagram. The parachutist leaves the plane travelling at 200 km/h at an altitude of 2000 m travelling in an easterly direction. The vertical component of the motion is the same as in part 2-d. while the parachutist is in free fall. In part 2-e. the horizontal motion is considered.

Plane

 ***j***

2000 metres

 ***i***

### O

The differential equation describing the motion for the free fall in the horizontal direction is
 = – *k2w*2 where *w* m/sis thehorizontal speed of the parachutist at time *t* seconds after leaving the plane and when *t* = 0 the parachutist has the horizontal speed of the plane. The horizontal speed of the parachutist is 1m/s after 15 seconds.

a. Find *w* in terms of *t* and find the horizontal position, *x m*, of the parachutist relative to *O*.

b. Plot the graph of *w* versus *t* and the graph of *x* versus *t* for the first 15 seconds of motion.

c. Give the velocity vector and position vectors at time *t* seconds after leaving the plane, use technology to produce the position vectors for *t* = 0, 1, 3, 6, 9, 12, 15 giving values correct to one decimal place and plot the graph of the path of the parachutist for the first 15 seconds of motion. Show the position vectors on the graph.

At the end of 15 seconds the parachute opens and the parachutist drops vertically to ground, the final vertical speed is 1m/s.

d. The acceleration for this section of motion is –*pj* m/s2 where *p* is a constant. Find the value of *p* correct to two decimal places and obtain expressions for position, relative to the point on the ground directly below where the parachute opens, and velocity *t* seconds after the parachute opens.

Areas of study

The following content from the areas of study is addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Area of study** | **Topic** | **Content dot point** |
| Calculus | Kinematics | 12 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 6, 7, 8, 9 | 2, 6, 7, 9, 12 |
| **2** | 1, 2, 3, 4, 5 | 1, 2, 3, 5, 6 |
| **3** | 1, 2, 3, 5, 6, 7 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 |