Mathematics Sample Program: Year 10



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Level 7, 2 Lonsdale Street
Melbourne VIC 3000

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Abbreviations

ABS Australian Bureau of Statistics

AMSI Australian Mathematical Sciences Institute

CIMT Centre for Innovation in Mathematical Teaching

DET Department of Education and Training

ESA Educational Services Australia

NCTM National Council Teachers of Mathematics

NLVM National Library of Virtual Manipulatives

MAV Mathematical Association of Victoria

Hyperlinks

At the time of publication the URLs (website addresses) cited were checked for accuracy and appropriateness of content. However, due to the transient nature of material placed on the web, their continuing accuracy cannot be verified. Teachers are strongly advised to prepare their own indexes of sites that are suitable and applicable to the courses they teach, and to check these addresses prior to allowing student access.

Overview

This Mathematics Sample Program: Year 10 is an example of how the Mathematics curriculum could be organised into a teaching and learning program.

This sample program provides comprehensive coverage of content descriptions from the three strands of the mathematics curriculum and is sequenced to develop knowledge and skills; however, there are many other ways that the curriculum content can be arranged to suit the learning needs of students. Note that 10A provides *optional* additional content, from which a selection may be covered as appropriate. The sample program includes a particular possible selection.

Topics, suggested time allocations and sequencing

|  |  |  |
| --- | --- | --- |
| **Week\*** | **Semester 1** | **Semester 2** |
| 1 | *10.1.1: Measurement*Strand: Measurement and GeometrySub-strand: Using units of measurement | *10.2.1: Quadratic functions*Strand: Number and AlgebraSub-strand: Linear and non-linear relationshipsSub-strand: Patterns and algebra |
| 2 |
| 3 |
| 4 | *10.1.2: Linear relationships*Strand: Number and AlgebraSub-strand: Linear and non-linear relationships | *10.2.2.: Trigonometry*Strand: Measurement and GeometrySub-strand: Pythagoras and trigonometry |
| 5 |
| 6 |
| 7 | *10.1.3: Statistics: Univariate*Strand: Statistics and ProbabilitySub-strand: Data representation and interpretationStrand: Number and AlgebraSub-strand: Patterns and algebra | *10.2.3: Geometry*Strand: Measurement and GeometrySub-strand: Geometric reasoning |
| 8 |
| 9 |
| 10 | *10.1.4: Number and financial applications*Strand: Number and AlgebraSub-strand: Real NumbersSub-strand: Money and financial mathematicsSub-strand: Patterns and algebra | *10.2.4: Probability*Strand: Statistics and ProbabilitySub-strand: Chance Strand: Number and Algebra Sub-strand: Patterns and algebra |
| 11 |
| 12 |
| 13 | *10.1.5: Coordinate geometry and simultaneous equations*Strand: Number and AlgebraSub-strand: Linear and non-linear relationshipsSub-strand: Patterns and algebra | *10.2.5: Statistics: Bivariate*Strand: Statistics and ProbabilitySub-strand: Data representation and interpretation |
| 14 |
| 15 | *10.1.6: Algebra techniques*Strand: Number and AlgebraSub-strand: Patterns and algebraSub-strand: Linear and non-linear relationships | *10.2.6: Relations and their graphs*Strand: Number and AlgebraSub-strand: Linear and non-linear relationships |
| 16 |
| 17 |
| 18 |

\* Based on 3 hours teaching time per week

Content descriptions coverage within each topic

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| --- | --- |
| **Level 10 content descriptions** | **Topic/s** |
| **Strand: Measurement and Geometry** |
| **Sub-strand: Using units of measurement** |
| Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids [(VCMMG343)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG343) | 10.1.1 |
| **Sub-strand: Pythagoras and trigonometry** |
| Solve right-angled triangle problems including those involving direction and angles of elevation and depression [(VCMMG346)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG346) | 10.2.2 |
| **Sub-strand: Geometric reasoning** |
| Formulate proofs involving congruent triangles and angle properties [(VCMMG344)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG344) | 10.2.3 |
| Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes [(VCMMG345)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG345) | 10.2.3 |
| **Strand: Number and Algebra** |
| **Sub-strand: Linear and non-linear relationships** |
| Solve problems involving linear equations, including those derived from formulas [(VCMNA335)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA335) | 10.1.2 |
| Solve linear inequalities and graph their solutions on a number line [(VCMNA336)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA336) | 10.1.2 |
| Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology [(VCMNA337)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA337) | 10.1.5 |
| Solve problems involving gradients of parallel and perpendicular lines [(VCMNA338)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA338) | 10.1.5 |
| Solve linear equations involving simple algebraic fractions [(VCMNA340)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA340) | 10.1.6 |
| Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate [(VCMNA339)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA339) | 10.2.110.2.6 |
| Solve simple quadratic equations using a range of strategies [(VCMNA341)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA341) | 10.2.1 |
| **Sub-strand: Money and financial mathematics** |
| Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies [(VCMNA328)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA328) | 10.1.4 |
| **Sub-strand: Patterns and algebra** |
| Factorise algebraic expressions by taking out a common algebraic factor [(VCMNA329)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA329) | 10.1.6 |
| Simplify algebraic products and quotients using index laws [(VCMNA330)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA330) | 10.1.4 |
| Apply the four operations to simple algebraic fractions with numerical denominators [(VCMNA331)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA331) | 10.1.6 |
| Expand binomial products and factorise monic quadratic expressions using a variety of strategies [(VCMNA332)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA332) | 10.1.6 |
| Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term [(VCMNA333)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA333)  | 10.1.2 |
| Implement algorithms using data structures in a general-purpose programming language [(VCMNA334)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA334) | 10.1.310.1.410.2.410.2.5 |
| **Strand: Statistics and Probability** |
| **Sub-strand: Data representation and interpretation** |
| Determine quartiles and interquartile range and investigate the effect of individual data values, including outliers on the interquartile range [(VCMSP349)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP349) | 10.1.3 |
| Construct and interpret box plots and use them to compare data sets [(VCMSP350)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP350) | 10.1.3 |
| Compare shapes of box plots to corresponding histograms and dot plots and discuss the distribution of data [(VCMSP351)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP351) | 10.1.3 |
| Use scatter plots to investigate and comment on relationships between two numerical variables [(VCMSP352)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP352) | 10.2.5 |
| Investigate and describe bivariate numerical data where the independent variable is time [(VCMSP353)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP353) | 10.2.5 |
| Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data [(VCMSP354)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP354) | 10.2.5 |
| **Sub-strand: Chance** |  |
| Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence [(VCMSP347)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP347) | 10.2.4 |
| Use the language of ‘if … then’, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language [(VCMSP348)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP348) | 10.2.4 |

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| **Level 10A content descriptions** | **Topic/s** |
| **Strand: Measurement and Geometry** |
| **Sub-strand: Using units of measurement** |
| Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids [(VCMMG365)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG365) | 10.1.1 |
| **Sub-strand: Pythagoras and trigonometry** |
| Use the unit circle to define trigonometric functions as functions of a real variable, and graph them with and without the use of digital technologies [(VCMMG368)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG368) | 10.2.2 |
| Solve simple trigonometric equations [(VCMMG369)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG369) | 10.2.2 |
| **Sub-strand: Geometric reasoning** |
| Prove and apply angle and chord properties of circles [(VCMMG366)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG366) | 10.2.3 |
| **Strand: Number and Algebra** |
| **Sub-strand: Linear and non-linear relationships** |
| Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations [(VCMNA359)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA359) | 10.2.110.2.6 |
| Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts [(VCMNA362)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA362) | 10.2.1 |
| Solve simple exponential equations [(VCMNA360)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA360) | 10.2.6 |
| **Sub-strand: Real numbers** |
| Define rational and irrational numbers and perform operations with surds and fractional indices [(VCMNA355)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA355) | 10.1.4 |
| Use the definition of a logarithm to establish and apply the laws of logarithms and investigate logarithmic scales in measurement [(VCMNA356)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA356) | 10.1.4 |
| **Sub-strand: Patterns and algebra** |  |
| Devise and use algorithms and simulations to solve mathematical problems [(VCMNA358)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA358) | 10.1.310.1.510.2.1 |
| **Strand: Statistics and Probability** |
| **Sub-strand: Data representation and interpretation** |
| Calculate and interpret the mean and standard deviation of data and use these to compare data sets. Investigate the effect of individual data values including outliers, on the standard deviation [(VCMSP372)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP372) | 10.1.3 |
| Use digital technology to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation, make predictions based on this straight line and discuss limitations [(VCMSP373)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP373) | 10.2.5 |
| **Sub-strand: Chance** |
| Investigate reports of studies in digital media and elsewhere for information on their planning and implementation [(VCMSP371)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP371) | 10.2.4 |

Achievement standards (for three levels to support planning for a continuum of learning)

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| Level 9 | Level 10 |
| **Number and Algebra**Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | **Number and Algebra**Students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, with and without the use of digital technology. Students substitute into formulas, find unknown values, manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems. |
| **Measurement and Geometry**Students solve measurement problems involving perimeter and area of composite shapes, surface area and volume of rectangular prisms and cylinders, with and without the use of digital technology. They relate three-dimensional objects to two-dimensional representations. Students explain similarity of triangles, interpret ratios and scale factors in similar figures, and apply Pythagoras's theorem and trigonometry to solve problems involving angles and lengths in right-angled triangles. | **Measurement and Geometry**Students solve and explain surface area and volume problems relating to composite solids. They use parallel and perpendicular lines, angle and triangle properties, similarity, trigonometry and congruence to solve practical problems and develop proofs involving lengths, angles and areas in plane shapes. They use digital technology to construct and manipulate geometric shapes and objects, and explore symmetry and pattern in two dimensions. |

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| Level 9 | **Level 10** |
| **Statistics and Probability**Students compare techniques for collecting data from primary and secondary sources, and identify questions and issues involving different data types. They construct histograms and back-to-back stem-and-leaf plots with and without the use of digital technology. Students identify mean and median in skewed, symmetric and bi-modal displays and use these to describe and interpret the distribution of the data. They calculate relative frequencies to estimate probabilities. Students list outcomes for two-step experiments and assign probabilities for those outcomes and related events. | **Statistics and Probability**Students compare univariate data sets by referring to summary statistics and the shape of their displays. They describe bivariate data where the independent variable is time and use scatter-plots generated by digital technology to investigate relationships between two continuous variables. Students evaluate the use of statistics in the media. They list outcomes for multi-step chance experiments involving independent and dependent events, and assign probabilities for these experiments. |

Learning in Mathematics

The proficiencies of Understanding, Fluency, Problem Solving and Reasoning are fundamental to learning mathematics and working mathematically, and are applied across all three strands Number and Algebra, Measurement and Geometry, and Statistics and Probability.

Understanding refers to students building a robust knowledge of adaptable and transferable mathematical concepts and structures. Students make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the ‘why’ and the ‘how’ of mathematics. Students build understanding when they:

* connect related ideas
* represent concepts in different ways
* identify commonalities and differences between aspects of content
* describe their thinking mathematically
* interpret mathematical information.

Fluency describes students developing skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they:

* make reasonable estimates
* calculate answers efficiently
* recognise robust ways of answering questions
* choose appropriate methods and approximations
* recall definitions and regularly use facts,
* can manipulate expressions and equations to find solutions.

Problem solving is the ability of students to make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively. Students pose and solve problems when they:

* use mathematics to represent unfamiliar or meaningful situations
* design investigations and plan their approaches
* apply their existing strategies to seek solutions
* verify that their answers are reasonable.

Reasoning refers to students developing an increasingly sophisticated capacity for logical, statistical and probabilistic thinking and actions, such as conjecturing, hypothesising, analysing, proving, evaluating, explaining, inferring, justifying, refuting, abstracting and generalising. Students are reasoning mathematically when they:

* explain their thinking
* deduce and justify strategies used and conclusions reached
* adapt the known to the unknown
* transfer learning from one context to another
* prove that something is true or false
* make inferences about data or the likelihood of events
* compare and contrast related ideas and explain their choices.

Year 10 Semester 1



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| Topic 10.1.1: Measurement |
| Strand: Measurement and Geometry | Sub-strand: Using units of measurement | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| * Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids [(VCMMG343)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG343).
* Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids [(VCMMG365)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG365).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students solve measurement problems involving perimeter and area of composite shapes, surface area and volume of rectangular prisms and cylinders, with and without the use of digital technology. They relate three-dimensional objects to two-dimensional representations. Students explain similarity of triangles, interpret ratios and scale factors in similar figures, and apply Pythagoras's theorem and trigonometry to solve problems involving angles and lengths in right-angled triangles. | **Students solve and explain surface area and volume problems relating to composite solids.** They use parallel and perpendicular lines, angle and triangle properties, similarity, trigonometry and congruence to solve practical problems and develop proofs involving lengths, angles and areas in plane shapes. They use digital technology to construct and manipulate geometric shapes and objects, and explore symmetry and pattern in two dimensions. |

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| **Activities** | **Proficiencies** |
| **Level 10*** Review of area formulas of plane shapes (including circles, trapeziums and kites) and problems involving areas of composite shapes.
* Conversion of units, e.g. between cm3 and m3
* Nets of prisms and cylinders and review of surface area.
* Surface area and volume of prisms and cylinders, including some reverse cases.
* Surface area and volume of composite solids, such as the surface area and volume of a farm shed with a gabled roof, using suitable functionalities of technology, as appropriate.
* Estimation based on familiar objects of known length, area or volume.
* Calculation of quantities of materials & costs (e.g. from house plans). Use of spreadsheets to create a costing document.
* Problem solving using approximate methods (e.g. systematic trial & error, scale drawing, counting squares).

**Level 10 A*** Explore surface area and volume of other types of solids, such as pyramids or spheres.
 | * **Understanding** through determining the volumes and surface areas of composite solids by considering the individual solids from which they are constructed.
* **Fluency** through correctly selecting the appropriate area or volume formula and applying it flexibly.
* **Problem solving** through applying knowledge of surface area and volume to solve practical problems involving calculation of materials and the like.
* **Reasoning** through estimating areas and volumes and appraising the reasonableness of answers.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Use nets of regular solids to determine their surface area, and build up volumes of regular solids from a base area using 1 cm gradations of the height.

Level 10AStudents who are working at this level could:* use geometry and an informal limit approach to obtain the formulas for the surface area and volume of a sphere
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| **Assessment ideas** |
| Students use spreadsheets and other methods, as appropriate, to complete a costing document based on building plans. For example, the cost of renovating part of a house or the cost of building a hayshed with sufficient volume to store a given number of bales of hay of known dimensions. |

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| **Resources** |
| AMSI[Area, volume and surface area](http://www.amsi.org.au/teacher_modules/area_volume_surface_area.html) DEECD [Measuring volume and surface area](https://fuse.education.vic.gov.au/Search/Results?AssociatedPackageId=&QueryText=volume+and+surface+area&SearchScope=All) NCTM Illuminations [Fishing for the best prism](http://illuminations.nctm.org/Lesson.aspx?id=2911) [Cubed cans](http://illuminations.nctm.org/Lesson.aspx?id=2904) FUSE: Discover resources aligned to the Victorian Curriculum[Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids](http://fuse.education.vic.gov.au/VCAA/VCMMG343) [Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids](http://fuse.education.vic.gov.au/VCAA/VCMMG365)  |

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| **Notes** |
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| Topic 10.1.2: Linear relationships |
| Strand: Number and Algebra | Sub-strand: Linear and non-linear relationships | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| * Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term [(VCMNA333)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA333)
* Solve problems involving linear equations, including those derived from formulas [(VCMNA335)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA335).
* Solve linear inequalities and graph their solutions on a number line [(VCMNA336)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA336).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | Students recognise the connection between simple and compound interest. **They solve problems involving linear equations and inequalities**, quadratic equations and pairs of simultaneous linear equations and related graphs, **with and without the use of digital technology.** **Students substitute into formulas, find unknown values, manipulate linear algebraic expressions**, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems. |

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| **Activities** | **Proficiencies** |
| * Formulas, variables and constants.
* Substituting into formulas.
* Changing the subject of a formula.
* Equations derived from substituting values in formulas.
* Representing practical situations and word problems with simple linear equations and solving them to answer questions.
* Graphs of equations derived from practical situations (such as distance-time graphs) and word problems, with and without technology.
* Solving simple linear inequalities. Representing inequalities using number lines.
* The use of technology to assist with the graphical solution of linear equations and inequalities.
 | * **Understanding** through making connections between the symbolic and graphical representation of linear relationships.
* **Fluency** through solving linear equations and inequalities using algebraic and graphical techniques.
* **Problem solving** through representing practical situations and word problems as linear equations or inequalities.
* **Reasoning** through justifying the reasonableness of solutions and using substitution to check solutions of linear equations and inequalities.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Identify integer solutions to simple linear inequalities

Level 10AStudents who are working at this level could:* Solve sets of simultaneous linear inequalities and determine the subsets of these that correspond to points with integer coordinates.
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| **Assessment ideas** |
| Students represent word problems of familiar situations as linear equations and inequalities, as appropriate, and use analytical and graphical means to solve the equations, with and without the use of technology. |

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| **Resources** |
| AMSI[Formulas](http://www.amsi.org.au/teacher_modules/Formulas.html)NCTM Illuminations [Exploring linear data](http://illuminations.nctm.org/Lesson.aspx?id=1189) nRich[Linear equations activities](http://nrich.maths.org/public/leg.php?group_id=5&code=51#results)[Simultaneous equations activities](http://nrich.maths.org/public/leg.php?group_id=5&code=53#results)FUSE: Discover resources aligned to the Victorian Curriculum[Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term](http://fuse.education.vic.gov.au/VCAA/VCMNA333) [Solve problems involving linear equations, including those derived from formulas](http://fuse.education.vic.gov.au/VCAA/VCMNA335)  |

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| **Notes** |
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| Topic 10.1.3: Statistics: Univariate  |
| Strand: Statistics and probability | Sub-strand: Data representation and interpretation | Recommended teaching time: 3 weeks (approximately 9 hours) |
| Strand: Number and algebra | Sub-strand: Patterns and algebra |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Determine quartiles and interquartile range and investigate the effect of individual data values, including outliers on the interquartile range [(VCMSP349)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP349).
* Construct and interpret box plots and use them to compare data sets [(VCMSP350)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP350).
* Compare shapes of box plots to corresponding histograms and dot plots and discuss the distribution of data [(VCMSP351)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP351)
* Implement algorithms using data structures in a general-purpose programming language [(VCMNA334)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA334)

Level 10A* Calculate and interpret the mean and standard deviation of data and use these to compare data sets. Investigate the effect of individual data values including outliers, on the standard deviation [(VCMSP372)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP372).
* Devise and use algorithms and simulations to solve mathematical problems [(VCMNA358)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA358)
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students compare techniques for collecting data from primary and secondary sources, and identify questions and issues involving different data types. They construct histograms and back-to-back stem-and-leaf plots with and without the use of digital technology. Students identify mean and median in skewed, symmetric and bi-modal displays and use these to describe and interpret the distribution of the data. They calculate relative frequencies to estimate probabilities. Students list outcomes for two-step experiments and assign probabilities for those outcomes and related events. | **Students compare univariate data sets by referring to summary statistics and the shape of their displays.** They describe bivariate data where the independent variable is time and use scatter-plots generated by digital technology to investigate relationships between two continuous variables. Students evaluate the use of statistics in the media. They list outcomes for multi-step chance experiments involving independent and dependent events, and assign probabilities for these experiments. |

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| **Activities** | **Proficiencies** |
| **Level 10*** Quartiles, five figure summary and the use of boxplots to summarise and represent data.
* Using boxplots to compare continuous data across categories
* Using statistical and spreadsheet functionalities of technology to generate boxplots, dotplots and histograms for sets of authentic data.
* Making comparisons of boxplots with the corresponding dotplots and/or histograms.
* Describing and comparing data sets from the shape of their distribution and from summary statistics.
* Algorithms and coding: Shuffle algorithm
* Develop a program to randomly shuffle the elements of a set of data in a list or matrix. The pseudo-code for the [Fisher-Yates/Knuth algorithm](https://en.wikipedia.org/wiki/Fisher%E2%80%93Yates_shuffle) may be used as a starting point. This simulates processes such as shuffling a deck of cards.
* Develop a program for a simple sorting algorithm, such as a bubble sort, as the reverse of a shuffle.
* Data investigation planned by the student, with collection or accessing of data, the use of technology to compare various graphical and numerical representations of the data and interpretation of the results.

**Level 10A*** In the data investigation planned by the student, exploring and calculating of other measures of centre and spread (including mean and standard deviation) to describe and compare data sets.
* Algorithms and coding: [Sattolo’s shuffle algorithm](https://en.wikipedia.org/wiki/Fisher%E2%80%93Yates_shuffle#Sattolo.27s_algorithm). Modify the [Fisher-Yates/Knuth algorithm](https://en.wikipedia.org/wiki/Fisher%E2%80%93Yates_shuffle) in such a way that no item remains in it’s original position. That is, the shuffle produces a random [derangement](http://mathworld.wolfram.com/Derangement.html).
 | * **Understanding** recognising that summary statistics and different displays provide different snapshots of the data, thereby selecting appropriate means of describing and displaying particular data sets.
* **Fluency** through calculating summary statistics and creating data displays, using technology when appropriate.
* **Problem solving** through identifying questions and planning and conducting a data investigation to help answer the questions.
* **Reasoning** through describing, evaluating and interpreting data summaries and displays and reaching appropriate conclusions.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Compare the mean and median for a variety of different shaped distributions and explore their sensitivity to outliers and changes in data values.

Level 10AStudents who are working at this level could:* Investigate the use of different summary statistics for various contexts, in particular the notion of robust statistics.
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| **Assessment ideas** |
| Students plan and conduct a data investigation, having identified questions or issues from everyday life and familiar situations. They collect data or access secondary data, analyse, evaluate and interpret the data, using technology as appropriate. |

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| **Resources** |
| Australian Bureau of Statistics[Activities for Level 10/10A](http://www.abs.gov.au/websitedbs/CaSHome.nsf/Home/Year%2B10%2BResources#maths) AMSI [[Data investigation and interpretation Year 10](http://www.amsi.org.au/teacher_modules/Data_investigation_year_10.html)](http://www.amsi.org.au/teacher_modules/Formulas.html#Substitution_into_a_formula_resulting_in_an_equation) nRich[Representing data](http://nrich.maths.org/public/leg.php?group_id=44)FUSE: Discover resouces aligned to the Victorian CurriculumLevel 10[Determine quartiles and interquartile range and investigate the effect of individual data values, including outliers on the interquartile range](http://fuse.education.vic.gov.au/VCAA/VCMSP349) [Construct and interpret box plots and use them to compare data sets](http://fuse.education.vic.gov.au/VCAA/VCMSP350) [Compare shapes of box plots to corresponding histograms and dot plots and discuss the distribution of data](http://fuse.education.vic.gov.au/VCAA/VCMSP351) [Implement algorithms using data structures in a general-purpose programming language](http://fuse.education.vic.gov.au/VCAA/VCMNA334)Level 10A[Calculate and interpret the mean and standard deviation of data and use these to compare data sets. Investigate the effect of individual data values including outliers, on the standard deviation](http://fuse.education.vic.gov.au/VCAA/VCMSP372)  |

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| Topic 10.1.4: Number and financial applications |
| Strand: Number and Algebra | Sub-strand: Money and financial mathematicsPatterns and algebraReal numbers | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10Money and financial mathematics* Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies [(VCMNA328)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA328).

Patterns and algebra* Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term [(VCMNA333)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA333).
* Implement algorithms using data structures in a general-purpose programming language [(VCMNA334)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA334)

Level 10 AReal numbers* Define rational and irrational numbers and perform operations with surds and fractional indices [(VCMNA355)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA355).
* Use the definition of a logarithm to establish and apply the laws of logarithms and investigate logarithmic scales in measurement [(VCMNA356)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA356).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | **Students recognise the connection between simple and compound interest.** They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, with and without the use of digital technology. Students substitute into formulas, find unknown values, manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems. |

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| **Activities** | **Proficiencies** |
| **Level 10*** Applying index laws to simplifying algebraic expressions using both positive and negative integral indices.
* Examples of growth and decay involving repeated application of a process, such as doubling or halving.
* Review of simple interest.
* Compound interest and its connection to repeated applications of simple interest.
* Algorithms and coding:
* use a spreadsheet to explore growth and decay models by recursion
* create a program that uses the recurrence relation\* for compound interest to display the amount in a bank account at the end of each compounding period, for a total *n* compounding periods. \*If is invested at *r* % p.a. compounding *k* times per year, then at the end of the first compounding period . In general, the recurrence relation is

.**Level 10A** * Exploring irrational numbers, including surds and their location on a number line.
* The four operations with surds, including simplifying surds and rationalising simple denominators.
* Interpretation of simple non-integer rational indices and use of index laws with rational indices.
* Introduce the logarithm as another name for an index, therefore
* Explore the connection between index laws and the corresponding logarithm law.
* Solving simple exponential equations in the context of exploring authentic growth and decay models, using analytical and graphical means, including with the aid of technology.
 | * **Understanding** through making connections between compound interest and repeated applications of simple interest.
* **Fluency** through applying index laws and calculating simple and compound interest in various contexts.
* **Problem solving** through investigating and representing growth and depreciation as mathematical models and interpreting results.
* **Reasoning** through making generalisations regarding growth and decay models from investigation of specific cases.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Use repeated multiplication by decimals of the form  to generate a sequence of compound interest values

Level 10AStudents who are working at this level could:* Explore what happens as the number of compoundings for a given annual interest rate increases indefinitely within a specified time interval
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| **Assessment ideas** |
| * Students carry out a guided investigation of growth and decay in the context of finance, including investigation of compound interest and depreciation (e.g. investigating the value of a car over time, if each year it loses 20% of the previous year’s value), using technology as appropriate.
* Students undertaking the topic at Level 10A can explore more complexities of the contexts, including investigating and solving exponential equations derived from growth and decay models.
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| **Resources** |
| AMSI [Indices and logarithms](http://www.amsi.org.au/teacher_modules/Indices_and_logarithms.html)  [Surds](http://www.amsi.org.au/teacher_modules/Surds.html)NLVM [Loan calculator](http://nlvm.usu.edu/en/nav/frames_asid_321_g_4_t_5.html)[Savings calculator](http://nlvm.usu.edu/en/nav/frames_asid_320_g_4_t_5.html)+Plus magazine (University of Cambridge)[Have we caught your interest?](https://plus.maths.org/content/have-we-caught-your-interest)FUSE: Discover resources aligned to the Victorian CurriculumLevel 10[Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies](http://fuse.education.vic.gov.au/VCAA/VCMNA328) [Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term](http://fuse.education.vic.gov.au/VCAA/VCMNA333) [Implement algorithms using data structures in a general-purpose programming language](http://fuse.education.vic.gov.au/VCAA/VCMNA334) Level 10 A[Use the definition of a logarithm to establish and apply the laws of logarithms and investigate logarithmic scales in measurement](http://fuse.education.vic.gov.au/VCAA/VCMNA356)  |

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| Topic 10.1.5: Coordinate geometry and simultaneous equations |
| Strand: Number and Algebra | Sub-strand: Linear and non-linear relationships | Recommended teaching time: 2 weeks (approximately 6 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| * Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology [(VCMNA337)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA337).
* Solve problems involving gradients of parallel and perpendicular lines [(VCMNA338)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA338).

Level 10A* Devise and use algorithms and simulations to solve mathematical problems [(VCMNA358)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA358)
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | Students recognise the connection between simple and compound interest. **They solve problems involving linear equations** and inequalities,quadratic equations **and pairs of simultaneous linear equations and related graphs, with and without the use of digital technology.** Students substitute into formulas, find unknown values, manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems. |

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| **Activities** | **Proficiencies** |
| * Exploring graphs of families of linear equations.
* Solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel
* Graphical solution of pairs of linear equations, including using graphical functionalities of technology.
* The geometric interpretation of a pair of linear equations having a unique solution, no solution or infinitely many solutions.
* Solving pairs of linear equations by substitution and by elimination. Verifying results by substitution and graphically, with the aid of graphical functionalities of technology.
* Exploring the gradients perpendicular lines, with graphical and dynamic geometry functionalities of technology.
* Solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular.

Level 10 A* Investigate [Pick’s theorem as a system of equations](https://illuminations.nctm.org/lesson.aspx?id=2089).
* Algorithms and coding: for any polygon with vertices at lattice points (i.e. points with integer coordinates), create a program to calculate the number of boundary lattice points. This could be done by taking the coordinates of the vertices from a matrix and calculating the sum of the highest common factor of the rise and run of each side of the polygon.
 | * **Understanding** through making connections between the symbolic and graphical representation of pairs of linear equations.
* **Fluency** through solving simultaneous linear equations using algebraic and graphical techniques.
* **Problem solving** through representing practical situations and word problems as families of linear equations and solving problems using facts about gradients of parallel and perpendicular lines.
* **Reasoning** through justifying the reasonableness of solutions and using graphical representations and substitution to check solutions of simultaneous linear equations.
 |
| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Use digital technology to plot families of lines that are parallel or perpendicular to a given line.

Level 10AStudents who are working at this level could:* Explore using the gradient of pairs of lines to determine the angle of intersection between two lines.
* Investigate Pick’s theorem as a system of equations.
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| **Assessment ideas** |
| Students complete tasks where they represent word problems as pairs of linear equations and use analytical and graphical means to solve the equations simultaneously, with and without the use of technology. |

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| **Resources** |
| AMSI[Coordinate geometry (parallel, perpendicular lines)](http://www.amsi.org.au/teacher_modules/Introduction_to_coordinate_geometry.html) NCTM Illuminations[Pick's theorem as a system of equations](https://illuminations.nctm.org/lesson.aspx?id=2089) nRich[Simultaneous equations activities](http://nrich.maths.org/public/leg.php?codesearch=simultaneous+equations&search=search)[Pick’s theorem](https://nrich.maths.org/1867)FUSE: Discover resources aligned to the Victorian Curriculum[Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology](http://fuse.education.vic.gov.au/VCAA/VCMNA337)  |
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| Topic 10.1.6: Algebra techniques |
| Strand: Number and Algebra | Sub-strand: Patterns and algebraLinear and non-linear relationships | Recommended teaching time: 4 weeks (approximately 12 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| * Factorise algebraic expressions by taking out a common algebraic factor [(VCMNA329)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA329).
* Apply the four operations to simple algebraic fractions with numerical denominators [(VCMNA331)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA331).
* Expand binomial products and factorise monic quadratic expressions using a variety of strategies [(VCMNA332)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA332).
* Solve linear equations involving simple algebraic fractions [(VCMNA340)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA340).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | Students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, with and without the use of digital technology. **Students** substitute into formulas, find unknown values, **manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology.** They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems. |

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| **Activities** | **Proficiencies** |
| * Expressing the sum and difference of algebraic fractions with a common denominator.
* Using the index laws to simplify products and quotients of algebraic fractions.
* Solving linear equations involving simple algebraic fractions, and checking solutions by substitution.
* Representing word problems involving algebraic fractions as equations and solving the equations.
* Review of expansion of binomial products, including use of area models (such as algebra tiles) to visualise expansion and connecting factorisation as the reverse of expansion.
* Using common factors and the index laws to factorise algebraic expressions.
* Using the identities for perfect squares and the difference of squares to factorise quadratic expressions and applying the null factor law to solve related equations.
* Identifying linear factors for monic quadratic trinomials, such as and other simple trinomials, such as , and solving related equations.
* Use of area models to explore completing the square to factorise monic quadratic expressions and solve related equations.
* Using common factors, including binomial expressions, to factorise by grouping in pairs, such as , and solving related equations.
 | * **Understanding** through connecting the distributive law to factorisation and expansion, including interpretation of related area models.
* **Fluency** through applying the distributive and null factor laws to binomial expressions and expanding and factorising quadratic expressions and solving related equations.
* **Problem solving** through modelling expansion and factorisation of quadratic expressions with algebra tiles and other area models.
* **Reasoning** through explaining the relationship between expansion and factorisation and identifying algebraic factors in algebraic expressions.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* use algebra blocks (algebra tiles) to factorise monic and some non-monic quadratics by the geometric area model

Level 10AStudents who are working at this level could:* use the rational root theorem to investigate when a quadratic expression has linear factors with integer coefficients
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| **Assessment ideas** |
| Students respond to a set of problems, including word problems, requiring them to simplify algebraic fractions and solve related linear equations. They respond to a set of problems involving expanding and factorising quadratic expressions using various techniques, and use the null factor law to solve related equations.  |

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| **Resources** |
| AMSI [Special expansion and algebraic fractions](http://www.amsi.org.au/teacher_modules/special_expansions_algbrc_fracs.html)[Factorisation](http://www.amsi.org.au/teacher_modules/Factorisation.html) [Quadratic equations](http://www.amsi.org.au/teacher_modules/Quadratic_Equations.html)NLVM (Note: Some web browsers may not support the NLVM Java plug-in. An off-line NLVM app is available for download) [Algebra tiles](http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html) nRich[Algebraic factorisation activities](http://nrich.maths.org/public/leg.php?group_id=5&code=-57#results)FUSE: Discover resources aligned to the Victorian Curriculum[Factorise algebraic expressions by taking out a common algebraic factor](http://fuse.education.vic.gov.au/VCAA/VCMNA329) [Apply the four operations to simple algebraic fractions with numerical denominators](http://fuse.education.vic.gov.au/VCAA/VCMNA331) [Expand binomial products and factorise monic quadratic expressions using a variety of strategies](http://fuse.education.vic.gov.au/VCAA/VCMNA332) [Solve linear equations involving simple algebraic fractions](http://fuse.education.vic.gov.au/VCAA/VCMNA340)  |

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Year 10 Semester 2



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| Topic 10.2.1: Quadratic functions |
| Strand: Number and Algebra | Sub-strands: Linear and non-linear relationshipsPatterns and algebra | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate [(VCMNA339)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA339).
* Solve simple quadratic equations using a range of strategies [(VCMNA341)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA341).

Level 10A* Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations [(VCMNA359)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA359).
* Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts [(VCMNA362)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA362).
* Devise and use algorithms and simulations to solve mathematical problems [(VCMNA358)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA358)
 |
| **Achievement standard (excerpt in bold)** |
|  Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology. | Students recognise the connection between simple and compound interest. **They solve problems** **involving** linear equations and inequalities, **quadratic equations** and pairs of simultaneous linear equations **and related graphs,** **with and without the use of digital technology**. Students substitute into formulas, find unknown values, manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. **They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems.** |

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| **Activities** | **Proficiencies** |
| * Words ↔ symbols, associated with quadratic relationships, from number patterns, ‘stories’, etc.
* Using symbolic, numerical and graphical functionalities of technology to investigate different forms of quadratics, the solution of quadratic equations and the graphical illustration of this.
* Using technology to exploring transformations of

 and related families of curves, such as , .Sketching graphs of expressed in the form , and graphs of the form and  as a special case.* The connection between linear factors, solutions to the equation  and the x-axis intercepts of the graph of .
* Sketching graphs of expressing as  and completing the square to find coordinates of the vertex and x-intercepts, respectively.
* Application of quadratic functions to problem solving and modelling, including consideration of domain.

Level 10A* Algorithms and coding: infinitely nested square roots
* Create a program to evaluate rational approximations to expressions of the form , where is a positive integer. The program should use a loop structure to recursively evaluate and display a sequence of approximations: , up to 10 nested square roots.
* Explore the value, , to which a sequence converges and the values of  for which  is an integer.
* Explore the connection to the quadratic equation .
* For the case where , explore the connection to the golden ratio, *φ*, the related continued fraction, the quadratic equation  and the equality .
 | * **Understanding** through recognising that every quadratic graph is a transformation of  and connecting the *x*-axis intercepts of a graph to the related linear factors and equation.
* **Fluency** through plotting points accurately and sketching graphs from equations and labelling key features.
* **Problem solving** and applying knowledge of quadratic functions to formulate, model and investigate unfamiliar non-routine situations.
* **Reasoning** through explaining how different equivalent forms of an equation highlight different features of the corresponding graph.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Use technology to solve quadratic equations graphically and/or numerically, by systematics guess-check-refine approaches

Level 10AStudents who are working at this level could:* Explore the use of continued fractions to solve quadratic equations and determine a sequence of rational approximations to a surd.
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| **Assessment ideas** |
| * Students carry out a non-routine quadratic modelling task based on a practical or theoretical context. The task should encourage the use of graphical, numerical and symbolic functionalities of technology to explore the context and carry out further analysis.

Some possible modelling tasks are included in the ‘resources’ section below, including [Egg Launch Contest](http://illuminations.nctm.org/Lesson.aspx?id=2650), [Princess Dido and the Ox Skin](http://illuminations.nctm.org/dido/).* Students undertaking the topic at Level 10A can explore additional complexities of the contexts, such as exploring a more general case.
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| **Resources** |
| AMSI [The quadratic function](http://www.amsi.org.au/teacher_modules/Quadratic_Function.html) NCTM Illuminations [Princess Dido and the Ox Skin](http://illuminations.nctm.org/dido/)[Egg Launch Contest](http://illuminations.nctm.org/Lesson.aspx?id=2650)nRich[Continued fractions and the golden ratio](https://nrich.maths.org/2737)FUSE: Discover resources aligned to the Victorian CurriculumLevel 10[Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate](http://fuse.education.vic.gov.au/VCAA/VCMNA339) [Solve simple quadratic equations using a range of strategies](http://fuse.education.vic.gov.au/VCAA/VCMNA341) Level 10A[Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations](http://fuse.education.vic.gov.au/VCAA/VCMNA359) [Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts](http://fuse.education.vic.gov.au/VCAA/VCMNA362)  |

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| Topic 10.2.2: Trigonometry |
| Strand: Measurement and Geometry | Sub-strand: Pythagoras and trigonometry | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Solve right-angled triangle problems including those involving direction and angles of elevation and depression [(VCMMG346)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG346).

Level 10A* Use the unit circle to define trigonometric functions as functions of a real variable, and graph them with and without the use of digital technologies [(VCMMG368)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG368).
* Solve simple trigonometric equations [(VCMMG369)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG369).
 |
| **Achievement standard (excerpt in bold)** |
|  Level 9 | Level 10 |
| Students solve measurement problems involving perimeter and area of composite shapes, surface area and volume of rectangular prisms and cylinders, with and without the use of digital technology. They relate three-dimensional objects to two-dimensional representations. Students explain similarity of triangles, interpret ratios and scale factors in similar figures, and apply Pythagoras's theorem and trigonometry to solve problems involving angles and lengths in right-angled triangles. | Students solve and explain surface area and volume problems relating to composite solids. **They use parallel and perpendicular lines, angle and triangle properties, similarity, trigonometry and congruence to solve practical problems** and develop proofs involving lengths, angles and areas in plane shapes. They use digital technology to construct and manipulate geometric shapes and objects, and explore symmetry and pattern in two dimensions. |

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| **Activities** | **Proficiencies** |
| Level 10* Review of similarity of right-angled triangles and the three basic trigonometric ratios, including unit circle definitions.
* Review of solving simple right-angled triangle problems by comparison to a similar triangle in the unit circle, and making links to the trigonometric ratios.
* Special angles and exact values for 0°, 30°, 45°, 60° and 90°.
* Solving word and other problems involving finding side lengths of right-triangles when the angles are known.
* Solving word and other problems involving finding angles of right-triangles when the angles are known.
* Applications of trigonometry, such as surveying and design.

Level 10A* Exploring trigonometric ratios for angles in the four quadrants of the unit circle.
* Exploring symmetry properties and periodicity of trigonometric functions.
* Further applications of trigonometric ratios, including problems involving compass bearings and 3D.
* Solving simple trigonometric equations using symmetry and periodicity.
 | * **Understanding** through applying knowledge of trigonometric ratios flexibly to solve unfamiliar problems.
* **Fluency** through choosing appropriate trigonometric ratios and procedures to solve problems involving right-angled triangles.
* **Problem solving** through investigating problem situations that can be modelled with right-angled triangles and formulating solutions.
* **Reasoning** through communicating and justifying their solution method in problem situations modelled with right-angles triangles.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Use enlargements of a right angled triangle with unit hypotenuse and trigonometry to solve practical problems in two dimensions.

Level 10AStudents who are working at this level could:* Investigate exact values of sin, cos and tan for other angles (see, for example: [Exact Trigonometric Function Values](http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/simpleTrig.html) )
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| **Assessment ideas** |
| * Students carry out an application task that requires them to apply knowledge of ratios to solve a range of problems in various practical contexts.
* Students undertaking the topic at Level 10A can explore more sophisticated contexts, such as exploring the periodicity of circular functions to model motion.
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| **Resources** |
| AMSI[Introductory trigonometry](http://www.amsi.org.au/teacher_modules/Introductory_trigonometry.html) [Further trigonometry](http://www.amsi.org.au/teacher_modules/further_trigonometry.html)[Trigonometric functions](http://www.amsi.org.au/teacher_modules/The_trigonometry_functions.html) NCTM Illuminations [Seeing music](http://illuminations.nctm.org/Lesson.aspx?id=2359) nRich[Trigonometry activities](http://nrich.maths.org/public/leg.php?group_id=35)FUSE: Discover resources aligned to the Victorian CurriculumLevel 10[Solve right-angled triangle problems including those involving direction and angles of elevation and depression](http://fuse.education.vic.gov.au/VCAA/VCMMG346) Level 10A[Use the unit circle to define trigonometric functions as functions of a real variable, and graph them with and without the use of digital technologies](http://fuse.education.vic.gov.au/VCAA/VCMMG368) [Solve simple trigonometric equations](http://fuse.education.vic.gov.au/VCAA/VCMMG369)  |

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| Topic 10.2.3: Geometry |
| Strand: Measurement and Geometry | Sub-strand: Geometric reasoning | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Formulate proofs involving congruent triangles and angle properties [(VCMMG344)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG344).
* Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes [(VCMMG345)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG345).

Level 10A* Prove and apply angle and chord properties of circles [(VCMMG366)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMMG366).
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| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 | Level 10A |
| Students solve measurement problems involving perimeter and area of composite shapes, surface area and volume of rectangular prisms and cylinders, with and without the use of digital technology. They relate three-dimensional objects to two-dimensional representations. Students explain similarity of triangles, interpret ratios and scale factors in similar figures, and apply Pythagoras's theorem and trigonometry to solve problems involving angles and lengths in right-angled triangles. | Students solve and explain surface area and volume problems relating to composite solids. **They use parallel and perpendicular lines, angle and triangle properties, similarity**, trigonometry **and congruence to solve practical problems and develop proofs involving lengths, angles and areas in plane shapes. They use digital technology to construct and manipulate geometric shapes and objects, and explore symmetry and pattern in two dimensions.** |  |

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| **Activities** | **Proficiencies** |
| Level 10* Review of congruent triangles and congruence tests for triangles.
* Review of similarity and similarity tests.
* Applying an understanding of relationships to deduce properties of geometric figures (for example: the base angles of an isosceles triangle are equal; a rhombus is a special kite and a special parallelogram).
* Use of dynamic geometry functionality to informally explore conjectures, e.g. explore the conjecture:
* for any arbitrary triangle ABC whose sides have midpoints D, E and F, the inscribed triangle DEF divides Δ ABC into four congruent triangles
* for any arbitrary quadrilateral ABCD whose sides have midpoints E, F, G and H, the inscribed quadrilateral EFGH is a parallelogram.
* Distinguishing between a practical demonstration and a proof (e.g. demonstrating triangles are congruent by placing them on top of each other, as compared to using congruence tests).
* Performing a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next.
* Communicating a proof using a sequence of logically connected statements, such as the proof that the diagonals of a kite are perpendicular (see worked examples of [proofs involving congruent triangles](http://www.regentsprep.org/Regents/math/geometry/GP4/PracCongTri.htm)).

Level 10A* Explore proofs of circle theorems and apply circle theorems to solve problems.
 | * **Understanding** through defining shapes from minimum (sufficient) properties recognising relationships between geometric properties
* **Fluency** through selecting and using geometric facts and angle properties to determine unknown angles.
* **Problem solving** through communicating a logical sequence of steps in informal deduction.
* **Reasoning** through discovering new properties by deduction, and providing and justifying logically connected statements.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Investigate the relationship between the location of the short diagonal of a kite along its long diagonal, and the perimeter and area of the kite

Level 10AStudents who are working at this level could:* Explore diifferent proofs of Pythagoras theorem and discuss their distinctive assumptions, constructions, features and approaches
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| **Assessment ideas** |
| Students use dynamic geometry functionality of technology to explore guided geometry activities such as those found in the journal article [Geometric Explorations with Dynamic Geometry Applications based on van Hiele Levels](http://www.cimt.org.uk/journal/olkun.pdf), (Olkun et al) (PDF) with the aim of helping students progress from the informal deductive level to simple proofs.Initially, students explore a context and make and test conjectures, discover new properties by deduction and give informal arguments. They then reflect on their findings and attempt simple formal proofs of some of their findings. |

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| **Resources** |
| AMSI [Congruence](http://www.amsi.org.au/teacher_modules/Congruence.html)[Quadrilaterals – TIMES module 21](http://www.amsi.org.au/teacher_modules/Rhombuses_Kites_and_Trapezia.html#Symmetries_of_triangles,_parallelograms_and_rectangles)NCTM Illuminations Perplexing parallelogramsnRich[2D geometry, shape and space activities](http://nrich.maths.org/public/leg.php?group_id=39)ScootleLevel 10[Formulate proofs involving congruent triangles and angle properties](http://fuse.education.vic.gov.au/VCAA/VCMMG344) [Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes](http://fuse.education.vic.gov.au/VCAA/VCMMG345) Level 10A[Prove and apply angle and chord properties of circles](http://fuse.education.vic.gov.au/VCAA/VCMMG366)   |

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| Topic 10.2.4: Probability |
| Strand: Statistics and ProbabilityStrand: Number and algebra | Sub-strand: Chance Sub-strand: Patterns and algebra | Recommended teaching time: 3 weeks (approximately 9 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence [(VCMSP347)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP347).
* Use the language of ‘if ....then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language [(VCMSP348)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP348).
* Implement algorithms using data structures in a general-purpose programming language [(VCMNA334)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA334)

Level 10A* Investigate reports of studies in digital media and elsewhere for information on their planning and implementation [(VCMSP371)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP371).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students compare techniques for collecting data from primary and secondary sources, and identify questions and issues involving different data types. They construct histograms and back-to-back stem-and-leaf plots with and without the use of digital technology. Students identify mean and median in skewed, symmetric and bi-modal displays and use these to describe and interpret the distribution of the data. They calculate relative frequencies to estimate probabilities. Students list outcomes for two-step experiments and assign probabilities for those outcomes and related events. | Students compare univariate data sets by referring to summary statistics and the shape of their displays. They describe bivariate data where the independent variable is time and use scatter-plots generated by digital technology to investigate relationships between two continuous variables. Students evaluate the use of statistics in the media. **They list outcomes for multi-step chance experiments involving independent and dependent events, and assign probabilities for these experiments.** |

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| **Activities** | **Proficiencies** |
| * Analysis of chance situations involving two and three steps, such as tossing multiple coins or drawing coloured lollies at random from a bag.
* Carrying out and analysing two and three steps experiments and simulations.
* Selection with and without replacement.
* Using arrays and tree diagrams to determine probabilities in multi-step events.
* Real-life conditional statements and the language of conditional events.
* Using two-way tables and Venn diagrams to understand conditional statements.
* The concept of independence and problems involving independent events.
* Applications of the law of total probabilities.
* Algorithms and coding: nGreedy Pig simulation
* Develop a program to simulate playing the game [Greedy Pig](https://nzmaths.co.nz/resource/greedy-pig-0), and explore the effectiveness of using various strategies.
* Students compare expected scores using different strategies, from simulation data and from theoretical probabilities. For example, students find expected scores for using a strategy of staying in the game:
* for *n* rolls of the die, where *n* = 1, 2, … 10;
* until a player has a total of at least *m*, where *m* = 10, 11, … 25.
 | * **Understanding** through making connections between conditional probability and independence or dependence of two events.
* **Fluency** through using arrays, tree diagrams, and the like, to determine probabilities of multi-step events.
* **Problem solving** through planning and designing probability simulations.
* **Reasoning** through interpreting conditional statements and through making inferences from the results of probability simulations.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Carry out experiments involving simple sampling (eg two and three step expriments with coloured balls from a jar) with and without replacement

Level 10AStudents who are working at this level could:* Explore problems that are counter-intuitive to judmental probability, such as the Monty-Hall problem and derangements, including the use of simulations
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| **Assessment ideas** |
| Students plan and carry out a probability simulation of a two or three stage event, using the key steps of simulation. They analyse and interpret the results and communicate conclusions. |

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| **Resources** |
| AMSI [Chance: Year 10](http://www.amsi.org.au/teacher_modules/Chance_year10.html)[Probability](http://www.amsi.org.au/ESA_Senior_Years/PDF/Probability4a.pdf)NCTM Illuminations [Random drawing tool](http://illuminations.nctm.org/Activity.aspx?id=3532)NRICH Maths [Probability archive](http://nrich.maths.org/public/leg.php?group_id=40) FUSE: Discover resources aligned to the Victorian Curriculum Level 10[Implement algorithms using data structures in a general-purpose programming language](http://fuse.education.vic.gov.au/VCAA/VCMNA334) Level 10A[Investigate reports of studies in digital media and elsewhere for information on their planning and implementation](http://fuse.education.vic.gov.au/VCAA/VCMSP371)  |

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| Topic 10.2.5: Statistics: bivariate |
| Strand: Statistics and Probability | Sub-strand: Data representation and interpretation | Recommended teaching time: 2 weeks (approximately 6 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Use scatter plots to investigate and comment on relationships between two numerical variables [(VCMSP352)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP352).
* Investigate and describe bivariate numerical data where the independent variable is time [(VCMSP353)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP353).
* Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data [(VCMSP354)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP354).
* Implement algorithms using data structures in a general-purpose programming language [(VCMNA334)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA334)

Level 10A* Use digital technology to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation, make predictions based on this straight line and discuss limitations [(VCMSP373)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMSP373)
 |
| **Achievement standard (excerpt in bold)** |
|  Level 9 | Level 10 |
| Students compare techniques for collecting data from primary and secondary sources, and identify questions and issues involving different data types. They construct histograms and back-to-back stem-and-leaf plots with and without the use of digital technology. Students identify mean and median in skewed, symmetric and bi-modal displays and use these to describe and interpret the distribution of the data. They calculate relative frequencies to estimate probabilities. Students list outcomes for two-step experiments and assign probabilities for those outcomes and related events. | Students compare univariate data sets by referring to summary statistics and the shape of their displays. **They describe bivariate data where the independent variable is time and use scatter-plots generated by digital technology to investigate relationships between two continuous variables. Students evaluate the use of statistics in the media.** They list outcomes for multi-step chance experiments involving independent and dependent events, and assign probabilities for these experiments. |

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| **Activities** | **Proficiencies** |
| Level 10* Collection of bivariate data and time series data from published sources and from experiments. Identification of dependent and independent variables.
* Use of scatterplots to represent bivariate data and judging the strength of the relationship between variables.
* Line fitting ‘by eye’, making predictions and drawing conclusions.
* The difference between statistical association of variables and a causal relationship.
* Construction and interpretation of data displays representing bivariate data over time (time series), and making informal inferences and predictions from trend lines.
* Use of appropriate functionalities of technology to construct displays of bivariate data, analyse strength of relationship between variables, determine equations of lines of best fit and make predictions through interpolation.
* Interpretation and evaluation of information collected from published data.
* Algorithms and coding: Hailstone sequences and the Collatz conjecture.
* Explore the [Collatz conjecture](https://en.wikipedia.org/wiki/Collatz_conjecture) by developing a program that generates the [hailstone sequence](https://plus.maths.org/content/mathematical-mysteries-hailstone-sequences) from an initial starting number.
* Explore the distribution of length of run of hailstone sequences for initial starting numbers in a certain range (e.g. for starting numbers < 50). Use appropriate data structures to display the results, and obtain appropriate graphs and plots from the resultant data.
* Explore the relationship between the starting number and the largest number reached in its hailstone sequence.

Level 10A* Use of technology to investigate different methods for finding a line of best fit.
 | * **Understanding** through analysing displays of bivariate data and making connections between the strength of a relationship and the reliability of predictions.
* **Fluency** through constructing scatterplots, fitting trend lines making predictions through interpolation.
* **Problem solving** through planning and designing an investigation of authentic bivariate data.
* **Reasoning** through interpreting, making inferences and communicating conclusions about the relationship between variables.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Form simple scatterplots from data that has a linear trend and draw in a line of good fit by eye

Level 10AStudents who are working at this level could:* Compare median and mean based approaches for obtaining a line of best fit to scatterplot data which shows reasonable linear association
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| **Assessment ideas** |
| Students carry out an investigation using authentic data within which various relationships can be explored. Students consider relationships that might be explored. They choose two particular variables to explore, construct data displays, analyse and interpret the data and draw conclusions. For example, from a random sample selected from ABS CensusAtSchool, a student might investigate length of foot and arm-span, or arm-span and belly button height. |

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| **Resources** |
| Australian Bureau of Statistics: CensusAtSchool[Activities for Level 10/10A](http://www.abs.gov.au/websitedbs/CaSHome.nsf/Home/Year%2B10%2BResources#maths) AMSI [Data Investigation and Interpretation Year 10](http://www.amsi.org.au/teacher_modules/Data_investigation_year_10.html) nRich[Statistics activities](http://nrich.maths.org/public/leg.php?group_id=44)FUSE: Discover resources aligned to the Victorian CurriculumLevel 10[Use scatter plots to investigate and comment on relationships between two numerical variables](http://fuse.education.vic.gov.au/VCAA/VCMSP352) [Investigate and describe bivariate numerical data where the independent variable is time](http://fuse.education.vic.gov.au/VCAA/VCMSP353) [Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data](http://fuse.education.vic.gov.au/VCAA/VCMSP354) [Implement algorithms using data structures in a general-purpose programming language](http://fuse.education.vic.gov.au/VCAA/VCMNA334) Level 10A[Use digital technology to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation, make predictions based on this straight line and discuss limitations](http://fuse.education.vic.gov.au/VCAA/VCMSP373)  |

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| Topic 10.2.6: Relations and their graphs |
| Strand: Number and Algebra | Sub-strand: Linear and non-linear relationships | Recommended teaching time: 4 weeks (approximately 12 hours) |

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| **Mapping to F–10 curriculum in Victoria** |
| **Content descriptions** |
| Level 10* Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate [(VCMNA339)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA339)

Level 10A* Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations [(VCMNA359)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA359).
* Solve simple exponential equations [(VCMNA360)](http://victoriancurriculum.vcaa.vic.edu.au/Curriculum/ContentDescription/VCMNA360).
 |
| **Achievement standard (excerpt in bold)** |
| Level 9 | Level 10 |
| Students apply the index laws using integer indices to variables and numbers, express numbers in scientific notation, solve problems involving very small and very large numbers, and check the order of magnitude of calculations. They solve problems involving simple interest. Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment using a range of strategies including the use of digital technology. Students sketch and draw linear and non-linear relations, solve simple related equations and explain the relationship between the graphical and symbolic forms, with and without the use of digital technology | Students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, with and without the use of digital technology. Students substitute into formulas, find unknown values, manipulate linear algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital technology. **They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems.** |

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| **Activities** | **Proficiencies** |
| Level 10* The concept of a function as a relationship that uniquely associates elements of one set with elements of another set: domain, codomain and range of a function.
* Ways of representing functions, including words, algebraic equations, tables and graphs.
* Review of graphs of quadratic functions as linear transformations of
* Tables and plots of exponential functions obtained from modelling contexts.
* Using technology to explore functions of the form and using of transformations of a basic exponential function  to sketch graphs of the form .
* Using Pythagoras’ theorem to establish the equations of circles centred at the origin.
* Using technology to explore graphs of the form  as examples of relations that are not functions. Using transformations to sketch circles of the form .

Level 10A* Graphical and algebraic solution of simple exponential equations derived from modelling contexts.
* Exploring the reciprocal of linear graphs using technology.
* The graph  and simple transformation of the forms  and .
 | * **Understanding** through making connections between algebraic and graphical representations of relations.
* **Fluency** through determining key features of graphs of relations, such as coordinates of axes intercepts and equations of asymptotes.
* **Problem solving** through planning and carrying out an investigation of relations arising from practical or theoretical contexts.
* **Reasoning** through generalising ideas about transformations and applying these ideas to different functions and relations.
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| **Considering different levels of student ability** |
| Level 9Students who are working at this level could:* Investigate the relationship distance travelled = average speed × time for different distances and average speeds, and use technology to lot corresponding families of graphs.

Level 10AStudents who are working at this level could:Investigate the behaviour of the asymptotes of hyperbolas and exponential functions under various transformations |

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| **Assessment ideas** |
| * Students carry out an investigation of relations arising from practical or theoretical contexts, such as the Tower of Hanoi and other recursive situations leading to exponential functions. They plot and sketch graphs with and without the aid of technology.
* Students respond to a set of questions requiring then to find key features of graphs of parabolas, circles, exponential functions and, for Level 10A students, rectangular hyperbolas. They sketch the corresponding graphs, labelling the key features of the graphs.
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| **Resources** |
| AMSI [Functions 1](http://www.amsi.org.au/ESA_Senior_Years/PDF/Functions12b.pdf)[Functions 2](http://www.amsi.org.au/ESA_Senior_Years/PDF/Functions22c.pdf)DET eResources[Functions and modelling](https://fuse.education.vic.gov.au/Search/Results?AssociatedPackageId=&QueryText=functions+and+modelling&SearchScope=All)nRich[Functions and graphs resources](http://nrich.maths.org/public/leg.php?group_id=7)FUSE: Discover resources aligned to the Victorian CurriculumLevel 10[Explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital technology as appropriate](http://fuse.education.vic.gov.au/VCAA/VCMNA339) Level 10A[Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations](http://fuse.education.vic.gov.au/VCAA/VCMNA359) [Solve simple exponential equations](http://fuse.education.vic.gov.au/VCAA/VCMNA360)  |

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