Level 7 - Number and Algebra

Overview

Activity name Over- and under-estimates

Learning intention To make over- and under-estimates and calculate answers to check

reasonableness of results.

Duration 40 minutes

Links to Victorian Curriculum

These work samples are linked to Level 7 of the Mathematics curriculum.

Extract from Mathematics Level 7 achievement standard

They solve problems involving all four operations with fractions, decimals, percentages.

They make simple estimates to judge the reasonableness of results.

Relevant content descriptions

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)
- Round decimals to a specified number of decimal places (VCMNA246)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (VCMNA248)

Links to NAPLAN

Minimum standards – numeracy

Year 7: Number

Applying number

Students form estimates and make approximations. They interpret and solve practical problems using appropriate operations. For example, students can generally: ...

- solve simple rate problems involving time and distance
- select an appropriate approximation to a calculation involving money
- interpret and solve practical problems involving division, with access to a calculator.

Calculating

Students use mental and written methods with addition, subtraction, multiplication and division. They use a calculator to assist with more complex calculations.

For example, students can generally:

- solve simple problems in familiar contexts involving addition or subtraction of integers
- use knowledge of place value to multiply and divide decimals by 10 and 100
- perform calculations involving key percentages or addition and subtraction of decimal numbers with the same number of decimal places.





Student work samples – Over- and underestimates

These work samples were created by students working at Level 7. Evidence of student achievement has been annotated.

Victorian Curriculum link

Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)

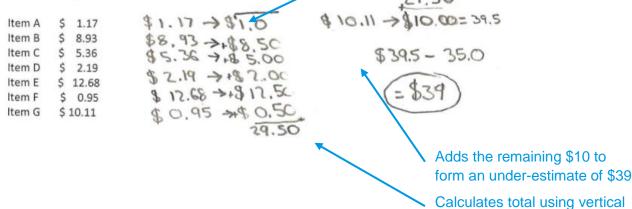
Round decimals to a specified number of decimal places (VCMNA246)

Part 1

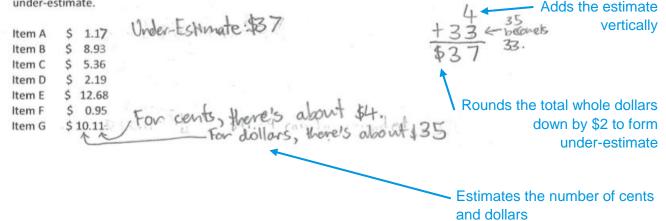
Rounds given numbers down to the nearest 50 cents

algorithm

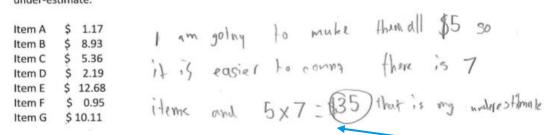
 Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.



a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.

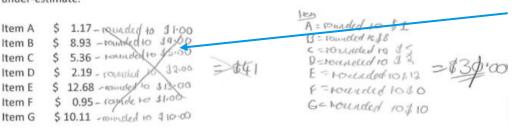


Form an under- estimate for the total price of the items shown below. Show how you obtained this
under-estimate.



Rounds all items to \$5 and multiplies by the amount of items to calculate an underestimate of \$35

Form an under- estimate for the total price of the items shown below. Show how you obtained this
under-estimate.



Rounds all items to the nearest whole dollar to form an over-estimate

Then self corrects and rounds down to the nearest dollar, calculating an |underestimate of \$30

a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.

Rounds all items down to the Item A 1.17 Item B 8.93 nearest whole dollar Item C \$ 5.36 Item D 2.19 Item E \$ 12.68 \$ 0.95 Item F Item G + \$ 10.11 Calculates an underestimate of \$38 by adding two items at a time

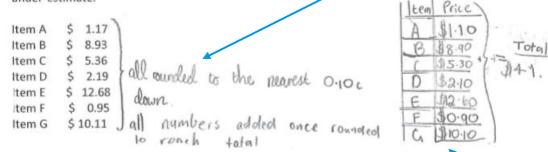
Rounds down to the nearest 10 cents

a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.

```
1,10+8,90=10
                                                     Hyunderestimate is
                   A: 1.10
Item A
        $ 1.17
                                 10+5.30 = 15.30
                   6:8.90
                                                         $31.00
Item B
        $
          8.93
                                 1530+2.10=17.40
Item C
        $ 5.36
                                 17.40+12.60=30
                   d:2.10
       $ 2.19
Item D
                   e:12.60
                                 30+0,90=30,90
       $ 12.68
Item E
                   F: 0,90
                                30 AO+10.10= 31
                                                               Calculates under-estimate
        $ 0.95
Item F
                  a: 10.10
Item G
        $ 10.11
                                                               using a running total
```

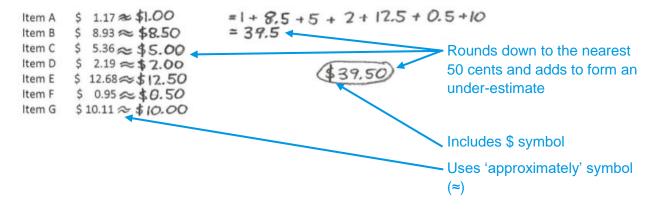
Rounds all items to the nearest 10 cents

a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.



Sorts rounded-down items vertically to form an under-estimate

Form an under- estimate for the total price of the items shown below. Show how you obtained this
under-estimate.



Rounds numbers up to the nearest dollar

b. Form an over-estimate for the total price of the items. Show how you obtained this over-estimate.

Z Wumbers rounded up = 445

Estimate

Estimated Average (ents = 60c 7 items x 50c = \$3.50

10000

Estimates the average as 50 cents

Adds the whole dollar total and the cents total to form an over-estimate

b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate.



Rounds all items up to the nearest whole dollar

Calculates vertically with regrouping to form an overestimate of \$45

Rounds all items up to the nearest 10 cents

b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate





Calculates an over-estimate using a running total

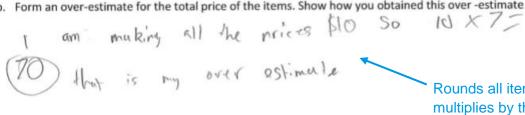
Rounds all items to the nearest whole dollar

b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate.

all the numbers up round

> Calculates over-estimate by adding in lots of two to calculate \$45 as the over-estimate

b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate



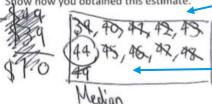
Rounds all items to \$10 then multiplies by the number of items to form an over-estimate

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Sorts the difference between the over- and under-estimates

c. Form an estimate for the total price of the items that lies between the estimates in a. and b. above. Show how you obtained this estimate.

to find the median



\$44 = Median Estimate

Uses the median to form an estimate

Calculates the difference between the over- and under-estimates using vertical algorithm and order of operations

c. Form an estimate for the total price of the items that lies between the estimates in a. and b. above. Show how you obtained this estimate.

difference \$48.50 \$41.50 \$67.00

 $\div 2 = 3.50

\$45.00

Vertically adds the difference to the under-estimate to form an in-between estimate of \$45

c. Form an estimate for the total price of the items that lies between the estimates in a. and b. above. Show how you obtained this estimate.

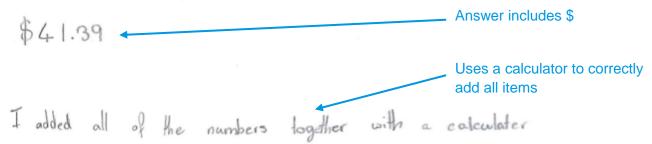
\$38+\$45=183 \$83=2=**5**41.5 together and divided it by two to find the Au

Estimates by averaging the under- and over-estimates

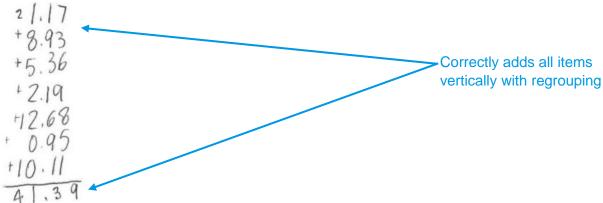
Correctly calculates the average between the estimates

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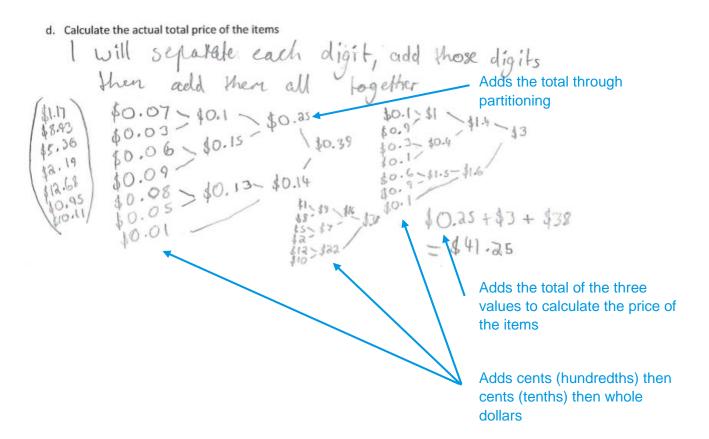
d. Calculate the actual total price of the items



d. Calculate the actual total price of the items



d. Calculate the actual total price of the items $\frac{18.93}{10.10}$ $\frac{15.46}{2.19}$ Uses a vertical algorithm with regrouping as a cumulative total to calculate the actual price of the items $\frac{131.28}{11.39}$



The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:

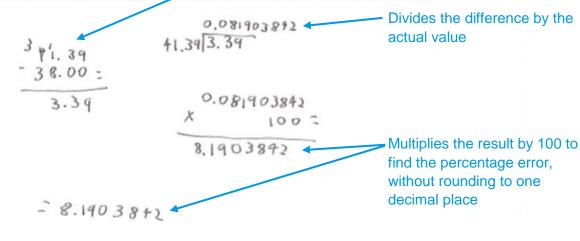
Step 1: calculate the difference: estimate - actual value

Step 2: divide this difference by the actual value

Step 3: multiply the answer to Step 2 by 100

Calculates the difference between the estimate and the actual value using vertical subtraction with regrouping

c. Calculate the percentage error using the estimate from c. and round this correct to one decimal place.



The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:

Step 1: calculate the difference: estimate - actual value

Step 2: divide this difference by the actual value

Step 3: multiply the answer to Step 2 by 100

Correctly uses a calculator to complete steps 1–3 to identify percentage error

the nearest tenth

e. Calculate the percentage error using the estimate from c. and round this correct to one decimal place.

Step 1: Actual Value - Estimate = Pifference
$$$41.39 - $41.25 = $0.14$$
 Clearly labels the steps to complete the calculation Step 2: Difference \div Actual Value = Answer $$0.14 \div $41.39 = 0.00338246$ Step 3: Answer $(Step 2) \times 100 (\%) = Final$ Answer $0.00338246 \times 100 = 0.338246$ Includes percentage symbol in calculation, without rounding to

The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:

- Step 1: calculate the difference: estimate actual value
- Step 2: divide this difference by the actual value
- Step 3: multiply the answer to Step 2 by 100

Correctly includes steps in the line of working

e. Calculate the percentage error using the estimate from c. and round this correct to one decimal place

$$[\$39.50 - \$41.42] \div \$41.42] \times 100$$

= $[-4.6]$

Shows the order of steps within one line of working

Uses a calculator to determine percentage error as a negative number

Correctly rounds to the nearest tenth, without percentage symbol

The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:

- Step 1: calculate the difference: estimate actual value
- Step 2: divide this difference by the actual value
- Step 3: multiply the answer to Step 2 by 100

Calculates the difference as a positive value, accurately using the \$ symbol

e. Calculate the percentage error using the estimate from c. and round this correct to one decimal place.

Percentage away from the correct

\$41.5 -\$41.39=\$0.11 \$0.11-\$41.39=0.00265764677 0.00265765764677 X100=0.26564677 With the Answer above, 1 rounded it to 0

Uses a calculator to divide the difference by the actual value

Rounds calculation to the nearest tenth and includes percentage symbol

Identifies calculation as a positive percentage error, explaining how 'far away' their estimation is from the actual value

Part 2

Consider the docket for a weekend grocery shopping trip (shown below).

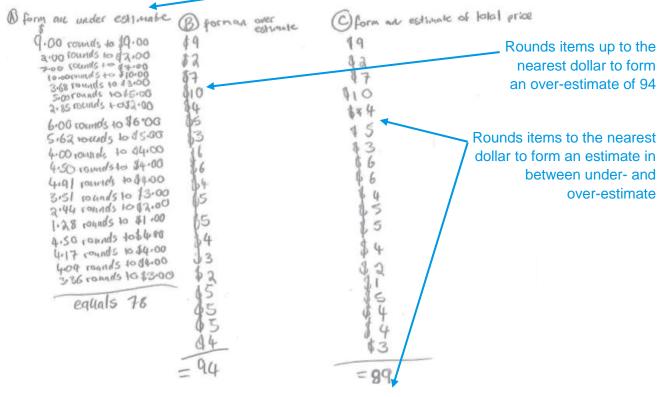
Carry out a. to e. from Part 1 for this list [see below]. Briefly discuss the accuracy of the estimate for Part 2.

- Form an under-estimate
- Form an over-estimate
- Form an estimate that lies between a. and b.
- Calculate the actual price
- Calculate the percentage error

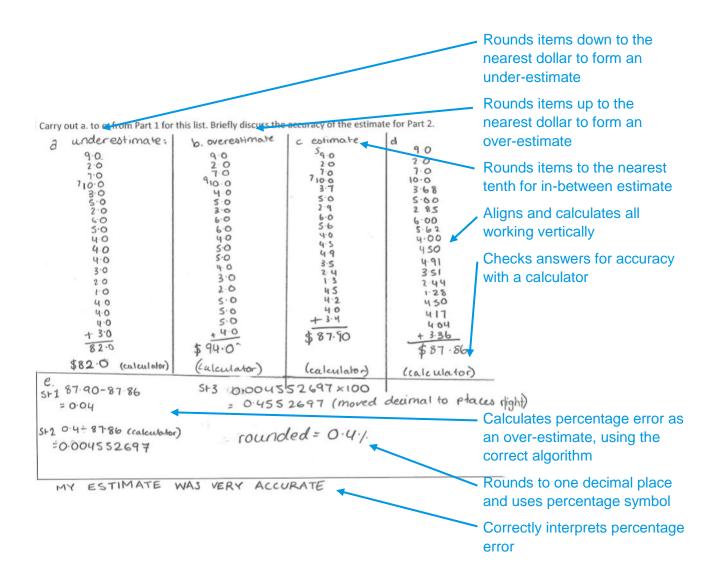


Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2.

 Rounds items down to the nearest dollar and adds to form an under-estimate of 78



Uses a calculator to correctly calculate the actual total of the items, without \$ symbol calculatos Follows algorithm to accurately calculate pecentage error. calculate percentage error Calculates the difference as a calculate difference positive value difference 1.14 Step a divide differce by Divides correct values Multiplies by 100 to correctly calculate percentage error, without rounding or Step 3) times answer for step 6 percentage symbol 1.2808989 The accuracy of the estimate for part (a) was not as good as the accuracy of the estimate for part (D. It had a difference of 1.14, while the other part (part () had a difference of 0:39. Correctly interprets percentage error results, comparing results from parts 1 and 2 and identifying the lower decimal as a more accurate estimate



Rounds items down to the nearest 10 cents to form an under-estimate

Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2.

A: 900+7.00+7.00+10.00+3.60+5.00+2.80+6.00+5.60+4.00+4.50+4.90+ 3.50 +2.40+1.20+4.50+4.10+4.00+3.30 = 82.5 Rounds items up to the nearest 10 cents to form an My underestimate is 82.5 over-estimate B:9.00+2.00+7.00+10.00+3.70+5.00+2.90+6.00+5.70+4.00+4.50+ 5.00+3.60+2.50+1.30+4.50+4.20+4.10+3.40=88.4 My overestimate is 88.4 C:9.00 + 2.00 + 7.00 + 10.00 + 3.70 + 5.00 + 2.90 + 6.00 + 5.60 + 4.00 + 4.50 + 4.90 + 8.00 + 3.50 + 2.40 + 1.30 + 4.50 + 4.20 + 4.00 + 3.40 = 87.9Rounds items to the nearest 10 cents and adds the items My estimate is 87.9 D:9,00+2.00+7.00+10.00+3.68+5.00+2.85+600+5.62+4.00+4.50+4.91+3.5H 2.44+1.28+4.50+4.17+4:04+336=87.86 E: 87,9-87,86-0.04 Correctly adds actual cost using 0,04:87,86=0,00045526974 ×100 a calculator 0.045526974 rounded to lolp = 0.0

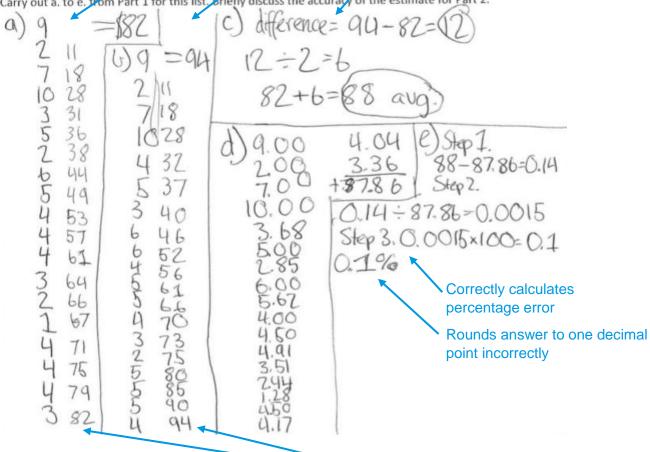
> Correctly calculates percentage error to one decimal place, without the percentage symbol, using the given algorithm

Rounds items down to the nearest whole dollar to form an under-estimate

Rounds items up to the nearest whole dollar to form an over-estimate

Calculates the difference, and uses the middle value between the under- and over-estimate to form an estimate in the middle

Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2.



Adds up all items vertically with a running total recorded

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Correctly uses 'approximate' symbol (≈)

Rounds items to the nearest \$ to form an over- and under- estimate

Uses a calculator to find each total

Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2.

Item:	Actual \$	Under.	over	a.=9+2+7+10+3.5+5+2.5+ 6+5.5+4+4.5+4.5+3.5+2+ +
A	9.00 ≈	9.00	9.00	4.5+4+4+3 = 85.5 (\$85.50)
B	2.00 ≈	2.00	2.00	
C	7.00 ≈	7.00	7.00	b. = 9+2+7+10+4+5+3+6+6+4+
DE	10.00 ≈	10.00	10.00	4.5+5+4+2.5+1.5+4.5+
	3.68≈	3,50	4.00	= 87.5 (\$87.50)
F	5.00≈	5.00	5.00	
G	2.85≈	2.50	2 00	c. (85.5+87.5) ÷ 2=86.5
H	6.00≈		3.00	\$86.50
(2) pro	1		6.00	d. = 9+2+7+10+3.68+5+
I	5.62 ≈	5.50	6.00	2.85 + 6+5.62 + 4+4.5 +4.914
J	4.00≈	4.00	4.00	3.51+2.44+1.28+4.5+
K	4.50≈	4.50	4.50	4.17+4.04+3.36
L	4.91 ≈	100000000000000000000000000000000000000	5.00	= 76.13 (\$76.13)
M	3.51≈		4.00	Uses symbols to assist
N	2.44		2.50	Step 1: E - AV=D calculation
0	1.28 =		1.50	\$86.5-\$76.13=\$10.73
P	4.50 %		4.50	Stop2: D = AV = A
0	4.17	4.00	4.50	\$10.73 - \$76.13 = 0.140943124
R	1 4.04	4.00	4.50	Stan 3: A(S2) x 100/%) = FA
QKS	3.36 ≈	3.00	3.50	0.140943124 × 100 = 14.094312496

Uses the given algorithm to identify percentage error as a positive number with percentage symbol

Where to next for the teacher?

When the task on which these annotated student work samples is based has been used as a classroom activity, there is opportunity to gather data on student achievement and to help inform further teaching.

An analysis of student responses, on an individual, group or whole class basis, can be used to develop and direct student learning with respect to the following content.

For students needing to review underpinning knowledge and skills at Level 6

- Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers and make estimates for these computations (VCMNA209)
- Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (VCMNA214)

For students consolidating knowledge and skills at <u>Level 7</u>

Investigate and calculate 'best buys', with and without digital technologies (VCMNA250)

For students moving on to new knowledge and skills at <u>Level 8</u>

- Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies and make estimates for these computations (VCMNA273)
- Solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital technologies (VCMNA276)
- Use algorithms and related testing procedures to identify and correct errors (VCMNA282)

Resources

- Numeracy Learning Progressions, Victorian Curriculum and Assessment Authority (VCAA) –
 The Numeracy Learning Progressions amplify, extend and build on the numeracy skills in the
 Victorian Curriculum F–10: Mathematics and support the application of numeracy learning
 within other learning areas.
- <u>FUSE</u>, Victorian Department of Education and Training (DET) The FUSE website provides
 access to digital resources that support the implementation of the Victorian Curriculum F–10,
 including an extensive range of activities and other resources for <u>Primary Mathematics</u> and
 Secondary Mathematics.
- <u>Mathematics Curriculum Companion</u>, Victorian Department of Education and Training (DET)
- Aligned Australian Curriculum Resources (Mathematics), Australian Curriculum, Assessment and Reporting Authority (ACARA)