## Mathematics - Annotated student work samples

## Level 7 - Number and Algebra

## Overview

Activity name Over- and under-estimates
Learning intention To make over- and under-estimates and calculate answers to check reasonableness of results.

Duration
40 minutes

## Links to Victorian Curriculum

These work samples are linked to Level 7 of the Mathematics curriculum.

## Extract from Mathematics Level 7 achievement standard

They solve problems involving all four operations with fractions, decimals, percentages.
They make simple estimates to judge the reasonableness of results.

## Relevant content descriptions

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)
- Round decimals to a specified number of decimal places (VCMNA246)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (VCMNA248)


## Links to NAPLAN

## Minimum standards - numeracy

## Year 7: Number

Applying number
Students form estimates and make approximations. They interpret and solve practical problems using appropriate operations. For example, students can generally: ...

- solve simple rate problems involving time and distance
- select an appropriate approximation to a calculation involving money
- interpret and solve practical problems involving division, with access to a calculator.

Calculating
Students use mental and written methods with addition, subtraction, multiplication and division. They use a calculator to assist with more complex calculations.
For example, students can generally:

- solve simple problems in familiar contexts involving addition or subtraction of integers
- use knowledge of place value to multiply and divide decimals by 10 and 100
- perform calculations involving key percentages or addition and subtraction of decimal numbers with the same number of decimal places.


## Mathematics - Annotated student work samples

## Student work samples - Over- and underestimates

These work samples were created by students working at Level 7 . Evidence of student achievement has been annotated.

## Victorian Curriculum link

Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)
Round decimals to a specified number of decimal places (VCMNA246)

## Part 1


a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.
Hem A $\$ 1.17$ Under-Esimate $\$ 37$
Item B \$ 8.93
Item C \$ 5.36
Item D \$ 2.19
Item E \$ 12.68

 $4 \underset{35}{\leftarrow}$ Adds the estimate
+33 beronels vertically

Rounds the total whole dollars down by $\$ 2$ to form under-estimate

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a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.

a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.


Rounds all items to the nearest whole dollar to form an over-estimate rounds down to the nearest dollar, calculating an |underestimate of \$30
a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.


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Rounds down to the nearest 10 cents
a. Form an under- estimate for the total price of the items shown below. Showhow you obtained this under-estimate.

| Item A | \$ 1.17 | A $=1.10$ | $1.10+8,90=10$ | Myunderestimate is |
| :---: | :---: | :---: | :---: | :---: |
| Item B | \$ 8.93 | B:8.90 |  | 1.00 |
| Item C | \$ 5.36 | C: 5.30 | $\begin{aligned} & 15.30+2.10=17.40 \\ & 17.40+12.60=30 \end{aligned}$ |  |
| Item D | \$ 2.19 | d: 2.12 .60 | $3.40+12.00=30$ |  |
| Item E | \$ 12.68 | $\begin{aligned} & e: 12.60 \\ & F: 0.90 \end{aligned}$ | $30.90+10.10=31$ | Calcula |
| Item F Item G | \$ 0.95 | G: 10.10 |  | using a |



Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.



Sorts rounded-down items vertically to form an under-estimate
a. Form an under- estimate for the total price of the items shown below. Show how you obtained this under-estimate.

( $\approx$

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Rounds numbers up to the nearest dollar
b. Form an oyer-estimate for the total price of the items. Show how you obtained this over-estimate.

b. Form an over-estimate for the total price of the items. Show how you obtained this over-estimate.


## Mathematics - Annotated student work samples

Rounds all items up to the nearest 10 cents
b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate


Rounds all items to the nearest whole dollar
b. Form an overestimate tor the total price of the items. Show how you obtained this over-estimate.
$2+9=11=20$
$6+3=9=20=45 \quad \$ 45$
$13+1=14-25$
$+11=11$


Calculates over-estimate
by adding in lots of
two to calculate \$45
as the over-estimate
b. Form an over-estimate for the total price of the items. Show how you obtained this over -estimate

1 am mukiry all the prices $\$ 10$ So $1 \mathrm{~d} \times 7=$ (70) that is my over astimute Rounds all items to $\$ 10$ then multiplies by the number of items to form an over-estimate

## Mathematics - Annotated student work samples

Sorts the difference between the over- and under-estimates c. Form an estimate for the total price of the items that lies been the estimates in a. and b. above. to find the median


c. Form an estimate for the total price of the items that lies between the estimates in a . and b . above.

Show how you obtained this estimate.



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d. Calculate the actual total price of the items

## 1. Answer includes \$


d. ${ }^{\text {calculate the actual total price of the items }}$
21.17
$+8.93$
$\begin{aligned} &+ 5.36 \\ &+ 2.19 \\ &+12.68 \\ & 0.95 \\ &+10.11\end{aligned}$
d. Calculate the actual total price of the items \(\begin{aligned} \& 1.177 <br>

\& +\)| 1.93 |
| :--- |
| 5.36 |
| 5.46 |\end{aligned}

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d. Calculate the actual total price of the items

I will separate each digit, add those digits then add them all together Adds the total through
 partitioning


Adds the total of the three values to calculate the price of the items

Adds cents (hundredths) then cents (tenths) then whole dollars

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The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:


The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:
Step 1: calculate the difference: estimate - actual value
Step 2: divide this difference by the actual value
Step 3: multiply the answer to Step 2 by 100
. Calculate the percentage error using the estimate from c. and round this correct to one decimal place.

Step 1: Actual Value - Estimate $=$ Difference
$\$ 41.39-\$ 41.25=\$ 0.14 \quad$ Clearly labels the steps to complete the calculation
 $\$ 0.14 \div \$ 41.39=0.00338246$
Step 3: Answer (Step 2) $\times 100(\%)=$ Final Answer

$$
0.00338246 \times 100=0.338246
$$



Includes percentage symbol in calculation, without rounding to the nearest tenth

## Mathematics - Annotated student work samples

The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:
Step 1: calculate the difference: estimate - actual value
Step 2: divide this difference by the actual value
Step 3: multiply the answer to Step 2 by 100 Correctly includes steps in the line of working
e. Calculate the percentage error using the estimate from c. and round this correct to one decimal place
$[\$ 39.50-\$ 41.42) \div \$ 41.42] \times 100$
$=(-4.6) \gg$
The percentage error of an estimate if found by comparing the difference between the estimate and the actual value to the actual value and representing this as a percentage. The algorithm for this is given below:
Step 1: calculate the difference: estimate - actual value
Calculates the difference as a Step 2: divide this difference by the actual value
Step 3: multiply the answer to Step 2 by 100 positive value, accurately using the \$ symbol
e. Calculate the percentage error using the estimate form c. and round this correct to one decimal place.

$\$ 0.110$ ot h $1.39=0.00265764677$
$0.00265765764672 \times 100=0.26564677 \%$ Wi th the nearest tenth with the Answer doves 1 rounded if $100.3 \%$ and includes My total Percentage away from the correct Identifies calculation as a positive percentage error, explaining how 'far away' their estimation is from the actual value

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## Part 2

## Consider the docket for a weekend grocery shopping trip

 (shown below).
## Carry out a. to e. from Part 1 for this list [see below]. Briefly

 discuss the accuracy of the estimate for Part 2.- Form an under-estimate
- Form an over-estimate
- Form an estimate that lies between $\mathbf{a}$. and b.
- Calculate the actual price
- Calculate the percentage error

| Description | \$ |
| :---: | :---: |
| *SSCHWEPPES MIN WTR 1.1LITRE $6 \$ \$ 1.50$ EACH | 9.00 |
| COLES DAIRY DIP 200GRAM <br> * MOCCONA INSTANT MED 100GRAM | 2.00 |
| MEDJOOL DATES 454GRAM | 7.00 10.00 |
| D/L ELCO OLIVES KALA PERKG | 3.68 |
| Maggie beer dairy Pa 100GRAM | 5.00 |
| AUSTRALIAN FETTA PERKG | 2.85 |
| MERSEY VALLEY VINTAG 180GRaM | 6.00 |
| TAS HERITAGE DL BRIE 250GRAM | 5.62 |
| COLES DRIED APRICOTS 400GRiM | 4.00 |
| PERINO TOMATOES 200GRAM | 4.50 |
| TRUSS TOMATOES PERKG | 4.91 |
| EED PLUMS PERKG |  |
| 0.717 kg NET \& \$4 | 3.51 |
| BANANAS PERKG | 2.44 |
| $0.976 \mathrm{~kg} \mathrm{NET} 0 \$ 2.50 / \mathrm{kg}$ | 2.44 |
| GARLIC | 1.28 |
| CELERY 0.051 kg NET © $\$ 25.00 / \mathrm{kg}$ |  |
| CELERY 1EACH RED GRAPES PERKG |  |
| 1.070 kg NET \& $\$ 3.90 / \mathrm{kg}$ | 4.17 |
| 0 GRAPES PERKG | 4.04 |
| 1.035 kg NET $0 \$ 3.90 / \mathrm{kg}$ |  |
| RUBY RED GRAPEFRUIT PERKG 0.746 kg NET \& $\$ 4.50 / \mathrm{kg}$ | 3.36 |

Rounds items down to the nearest dollar and adds to
Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2. rm an under-estimate of 78


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(e) calculate pecentage error

Follows algorithm to accurately calculate percentage error
Step (1)
calculate difference
difference 1.14
Step(2) divide differce by
actual value

$=1.2808989$
The accuracy of the estimate for pact (2) was not as good as the accuracy of the estimate for part (1). It had a difference of 1.14 , while the other part (part (1)) had ? difference of 0.39 .

Correctly interprets percentage error results, comparing results from parts 1 and 2 and identifying the lower decimal as a more accurate estimate

## Mathematics - Annotated student work samples



## Mathematics - Annotated student work samples

Carry out a. to e. from Part 1 for this list. Briefly discuss the accuracy of the estimate for Part 2.
$A: 900+2.00+7.00+10.00+3.60+5.00+2.80+6.00+5.60+4.00+4.50+4.90+$ $3.50+2.40+1.20+4.50+4.10+4.00+3.30=82.5$ Rounds items up to the My underestimate is 82.5 nearest 10 cents to form an over-estimate
$B: 9.00+2.00+7.00+10.00+3.70+5.00+2.90+6.00+5.70+4.00+4.50+$ $5.00+3.60+2.50+1.30+4.50+4.20+4.10+3.40=88.4$
Hy overestimate is 88.4
$0: 9.00+2.00+7.00+10.00+3.70+5.00+2.90+6.00+5.60+4.00+4.50+4.90+$ Rounds items to the nearest
$3.50+2.40+1.30+4.50+4.20+4.00+3.40=87.9 / \begin{aligned} & \text { Rounds items to the nearest } \\ & 10 \text { cents and adds the items }\end{aligned}$ Myestimate is 87.9 using a calculator
$D: 9.00+2.00+7.00+10.00+3.68+5.00+2.85+6.00+5.62+4.00+4.50+4.91 .+3.51+$ $2.44+1.28+4.50+4.14+4.04+3.36=87.86$
E: 87,9-87,86= 0,04
$0,04 \div 87,86=0,00045526974 \times 100$
0.045526974 rounded to $101 p=$

0.0

Correctly calculates percentage error to one decimal place, without the percentage symbol, using the given algorithm


## Mathematics - Annotated student work samples



| Item: | Actual $\$$ | Inder. | Over.. |
| :---: | :---: | :---: | :---: |
| A | $9.00 \approx$ | 9.00 | 9.00 |
| B | $2.00 \approx$ | 2.00 | 2.00 |
| $C$ | $7.00 \approx$ | 7.00 | 7.00 |
| D | $10.00 \approx$ | 10.00 | 10.00 |
| E | $3.68 \approx$ | 3.50 | 4.00 |
| $F$ | $5.00 \approx$ | 5.00 | 5.00 |
| $G$ | $2.85 \approx$ | 2.50 | 3.00 |
| H | $6.00 \approx$ | 6.00 | 6.00 |
| I | $5.62 \approx$ | 5.50 | 6.00 |
| J | $4.00 \approx$ | 4.00 | 4.00 |
| $K$ | $4.50 \approx$ | 4.50 | 4.50 |
| $L$ | $4.91 \approx$ | 4.50 | 5.00 |
| $M$ | $3.51 \approx$ | 3.50 | 4.00 |
| N | $2.44 \approx$ | 2.00 | 2.50 |
| 0 | $1.28 \approx$ | \% 1.00 | 1.50 |
| $P$ | $4.50 \approx$ | $=4.50$ | 4.50 |
| Q | $4.17=$ | $\approx 4.00$ | 4.50 |
| $R$ | $4.04 \approx$ | \% 4.00 | 4.50 |
| 5 | $3.36 \approx$ | 3.00 | 3.50 |

$$
\begin{aligned}
a . & =9+2+7+10+3.5+5+2.5+ \\
& 6+5.5+4+4.5+4.5+3.5+2+1+ \\
& 4.5+4+4+3 \\
& =85.5 \quad \$ 85.50
\end{aligned}
$$

b. $=9+2+7+10+4+5+3+6+6+4+1$
$4.5+5+4+2.5+1.5+4.57$
$=4.5+4.5+3.5$
$=87.5$
\$87.50
c. $(85.5+87.5) \div 2=86.5$
$\$ 86.50$
d. $=9+2+7+10+3.68+5+$
$2.85+6+5.62+4+4.5+4.91 *$
$3.51+2.44+1.28+4.5+$
$4.17+4.04+3.36$
$=76.13$
e.
step 1: $E-A V=D \quad$ calculation
$\$ 76.13=\$ 10.73$
Step 2: $D \div A V=A$
$\$ 10.73 \div \$ 76.13=0.140943124$
Step 3: $A(52) \times 100(\%)=F A$
$0.140943124 \times 100=14.0943124 \%$

Uses the given algorithm to identify percentage error as a positive number with percentage symbol

## Mathematics - Annotated student work samples

## Where to next for the teacher?

When the task on which these annotated student work samples is based has been used as a classroom activity, there is opportunity to gather data on student achievement and to help inform further teaching.

An analysis of student responses, on an individual, group or whole class basis, can be used to develop and direct student learning with respect to the following content.

## For students needing to review underpinning knowledge and skills at Level 6

- Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers and make estimates for these computations (VCMNA209)
- Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (VCMNA214)


## For students consolidating knowledge and skills at Level 7

- Investigate and calculate 'best buys', with and without digital technologies (VCMNA250)


## For students moving on to new knowledge and skills at Level 8

- Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies and make estimates for these computations (VCMNA273)
- Solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital technologies (VCMNA276)
- Use algorithms and related testing procedures to identify and correct errors (VCMNA282)


## Resources

- Numeracy Learning Progressions, Victorian Curriculum and Assessment Authority (VCAA) The Numeracy Learning Progressions amplify, extend and build on the numeracy skills in the Victorian Curriculum F-10: Mathematics and support the application of numeracy learning within other learning areas.
- FUSE, Victorian Department of Education and Training (DET) - The FUSE website provides access to digital resources that support the implementation of the Victorian Curriculum F-10, including an extensive range of activities and other resources for Primary Mathematics and Secondary Mathematics.
- Mathematics Curriculum Companion, Victorian Department of Education and Training (DET)
- Aligned Australian Curriculum Resources (Mathematics), Australian Curriculum, Assessment and Reporting Authority (ACARA)

