2025
NHT

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Write your **student number** in the boxes above. **Letter**

General Mathematics Examination 2

Question and Answer Book

VCE (NHT) Examination – Friday 23 May 2025

- Reading time is **15 minutes:** 10.30 am to 10.45 am
- Writing time is **1 hour 30 minutes:** 10.45 am to 12.15 pm

Approved materials

- One bound reference that may be annotated
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (5 marks)

Table 1 shows the number of children per family, *children,* in 200 families. For example, there are 23 families with 2 children.

Table 1

children	frequency
1	33
2	23
3	22
4	29
5	27
6	18
7	16
8	16
9	7
10	4
11	4
12	0
13	0
14	0
15	1

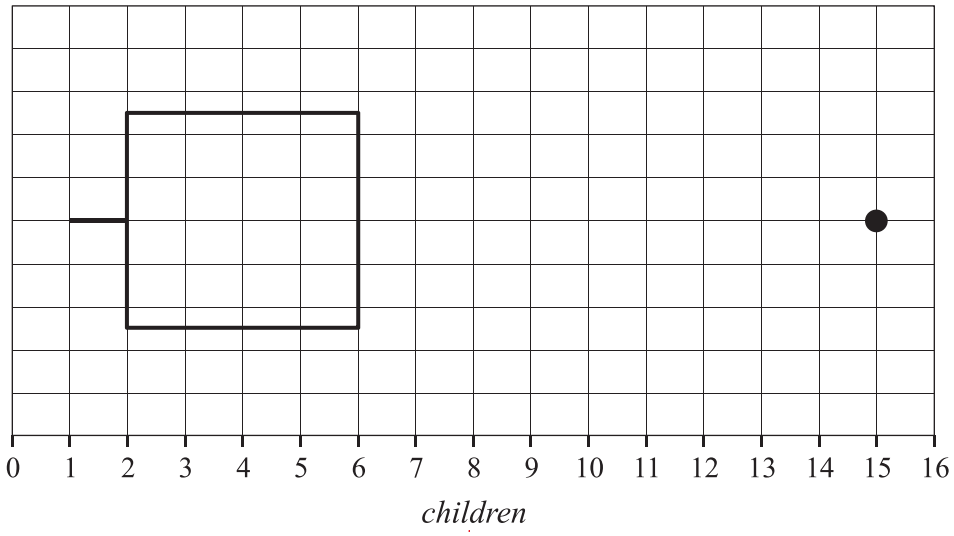
Data: adapted from <https://vincentarelbundock.github.io/Rdatasets/csv/HistData/GaltonFamilies>

a. Determine the range of *children*. 1 mark

b. Determine the percentage of families in the sample that have more than six children. 1 mark

c. The boxplot below displays the distribution of *children*.

The boxplot is incomplete.



i. The boxplot has an outlier at 15.

Write a calculation that shows that the *children* value of 15 is an outlier. 1 mark

ii. Use the information in Table 1 to determine the median *children* value.

Draw the median on the boxplot above. 1 mark

(*Answer on the boxplot above.*)

iii. Draw the upper whisker on the boxplot above. 1 mark

(*Answer on the boxplot above.*)

Question 2 (2 marks)

Data was collected to investigate the height, in metres, of the sons in 200 families.

An analysis of this data produced the following:

- The mean is 1.76 m
- The standard deviation is 0.07 m
- The standardised score for the first quartile (Q1) is $z = -0.71$
- The interquartile range (IQR) is 0.09

Complete the five-number summary in the table below by writing in the values for Q1 and Q3.

Round your answers to two decimal places.

Measure	Minimum	Q1	Median	Q3	Maximum
Value	1.52		1.76		2.01

Data: adapted from <https://vincentarelbundock.github.io/Rdatasets/csv/HistData/GaltonFfamilies>

Question 3 (6 marks)

The height measurements of the families that contained at least one daughter were recorded.

Table 2 below shows that there were 48 daughters who have no brothers and 394 daughters who have at least one brother.

Each daughter's height was recorded as taller, equal to or shorter than that of her mother.

Table 2

	Number of brothers	
Daughter's height	**None**	**One or more**
Taller	30	156
Equal to	6	65
Shorter	12	173
Total	48	394

Do not write in this area.

a. Use the data from Table 2 to complete the percentage frequency table, Table 3.

Round the percentages to one decimal place. 2 marks

Table 3

	Number of brothers	
Daughter's height	**None (%)**	**One or more (%)**
Taller		
Equal to	12.5	16.5
Shorter		
Total	100	100

b. Does the information in Table 3 support the contention that there is a difference in the height of daughters relative to their mothers based on the number of brothers (none, one or more)?

Refer to the appropriate percentages in your answer. 2 marks

Question 3 continues on the next page.

c. The height of a mother, *mother's height*, and the height of her daughter, *daughter's height*, were recorded for a sample of 12 mother–daughter pairs.

Table 4 lists the heights, in metres, for this sample.

Table 4

Mother's height	1.66	1.47	1.65	1.61	1.60	1.70	1.63	1.49	1.61	1.63	1.57	1.52
Daughter's height	1.56	1.60	1.55	1.61	1.68	1.70	1.65	1.60	1.61	1.65	1.57	1.57

Data: adapted from <https://vincentarelbundock.github.io/Rdatasets/csv/HistData/GaltonFamilies>

A least squares line can be used to model the association between *mother's height* and *daughter's height*.

Using the data from Table 4, determine the equation of this least squares line.

Use the template below to write your answer.

Round the values of the intercept and slope to three significant figures. 2 marks

daughter's height = ☐ + ☐ × *mother's height*

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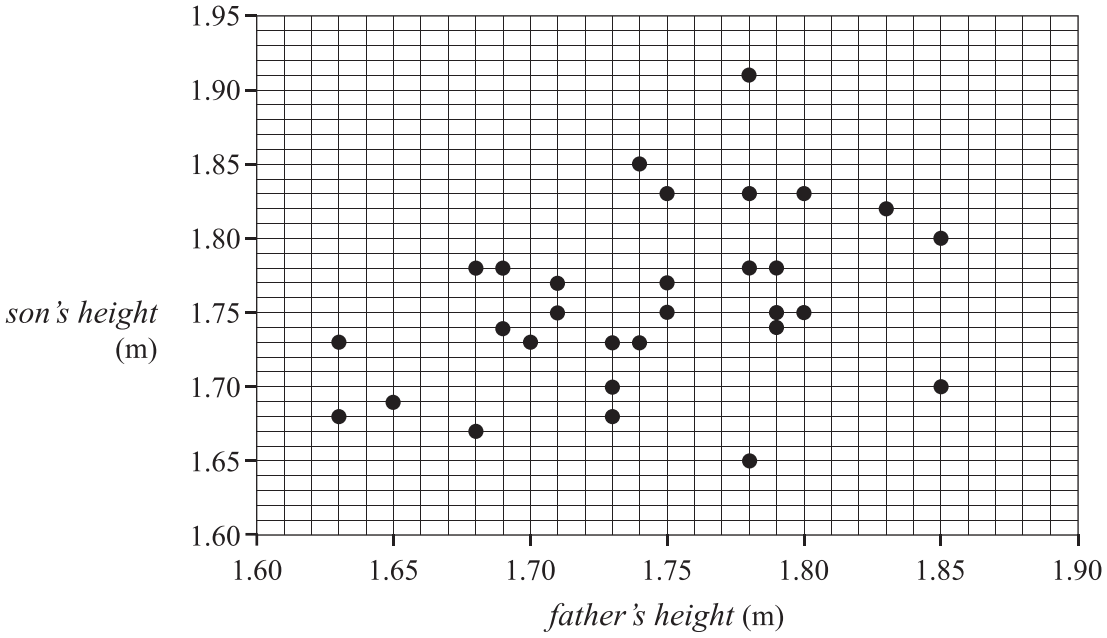
Examination continues on the next page.

Question 4 (8 marks)

The height of a father, *father's height*, and the height of his son, *son's height*, were recorded, for a sample of 30 father–son pairs.

A scatterplot of this data is shown below. All heights were measured in metres.



Data: adapted from <https://vincentarelbundock.github.io/Rdatasets/csv/HistData/GaltonFamilies>

When a least squares line is fitted to the scatterplot, the equation is found to be:

$$son's\ height = 1.10 + 0.375 \times father's\ height$$

a. Name the response variable in this equation. 1 mark

b. Draw the graph of this least squares line on the scatterplot above. 1 mark

(*Answer on the scatterplot above.*)

c. The correlation coefficient is 0.3744

What percentage of the variation in *son's height* is **not** explained by the variation in *father's height?*

Round your answer to the nearest percentage. 1 mark

Do not write in this area.

d. Describe the association between *son's height* and *father's height* in terms of strength and direction. 1 mark

strength	
direction	

e. Complete the following sentence by filling in the box provided.

The equation of the least squares line predicts that, on average, each 10 centimetre increase in *father's height* is associated with a [ ] centimetre increase in *son's height*. 1 mark

f. In this sample, one father–son pair had a *father's height* value of 1.85 m and a *son's height* value of 1.70 m.

Determine the residual value for this point when the least squares line is fitted to the scatterplot.

Round your answer to two decimal places. 2 marks

g. A sample of 30 mother–son pairs was selected to examine the association between the height of a mother, *mother's height*, and the height of her son, *son's height*.

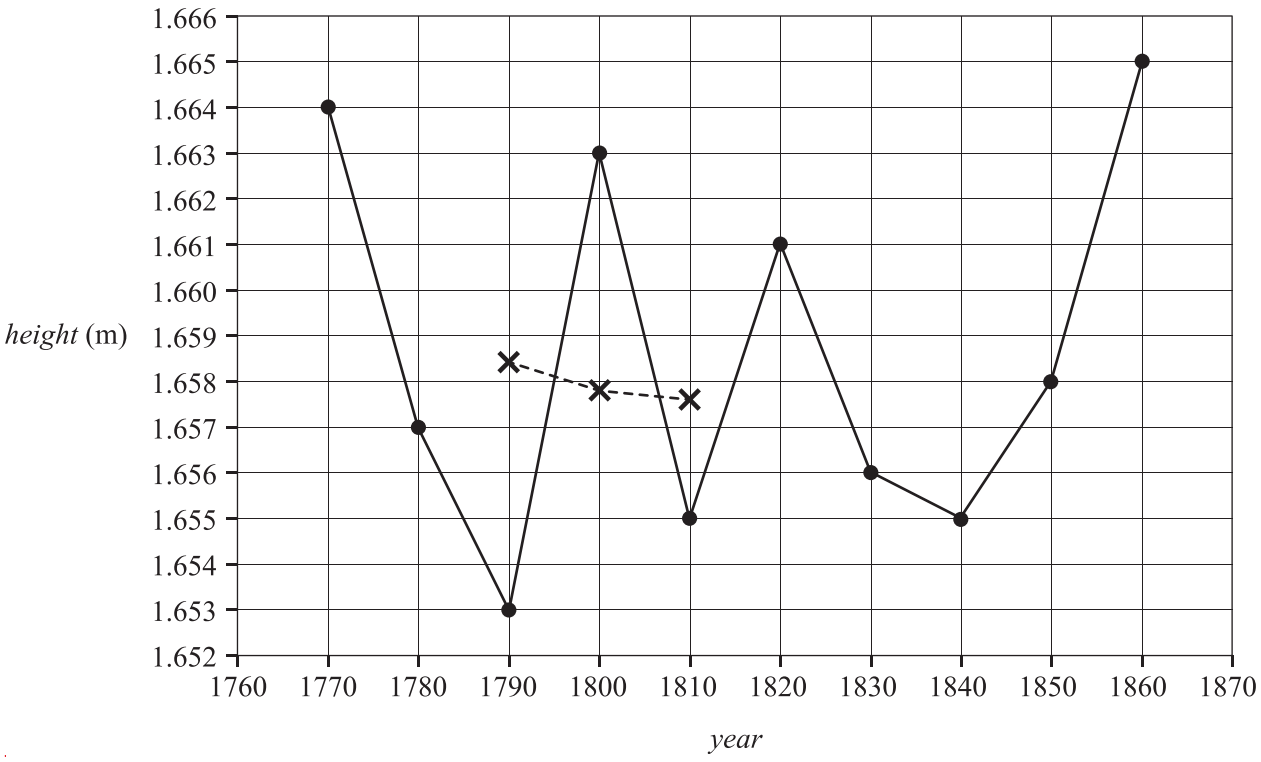
The coefficient of determination for this sample of mother–son pairs is 9.5%

The correlation coefficient for the father–son pairs sample is 0.3744

By referring to appropriate statistics, identify which variable, *father's height* or *mother's height*, has the stronger association with *son's height*. 1 mark

Question 5 (3 marks)

The time series plot below shows the estimated average height of adult males, in metres, in a European country between the years 1770 and 1860.



Data: adapted from <https://ourworldindata.org/human-height>

a. Five-mean smoothing will be used to smooth the time series plot above.

The first three smoothed points are shown as crosses (✖).

A line joining each of these points is shown as a dashed line (- - -).

Complete the five-mean smoothing by calculating the missing smoothed values and marking these smoothed values with crosses (✖) joined by a dashed line (- - -) on the time series plot above. 2 marks

(*Answer on the time series plot above.*)

b. The average height of adult males in this country in 1860 was estimated to be 1.665 m.

The average height of adult males in this country in 1960 was estimated to be 1.822 m.

Assuming the rate of increase in average adult male height remains the same as the average increase in height from 1860 to 1960, what would be the estimated average height of adult males, in metres, in this European country in 2040?

Round your answer to three decimal places. 1 mark

Do not write in this area.

NHT

Do not write in this area.

Recursion and financial modelling

Question 6 (3 marks)

Maria owns a mobile catering company.

She has purchased a new van for $80 000.

The value of the van will be depreciated using the flat rate method.

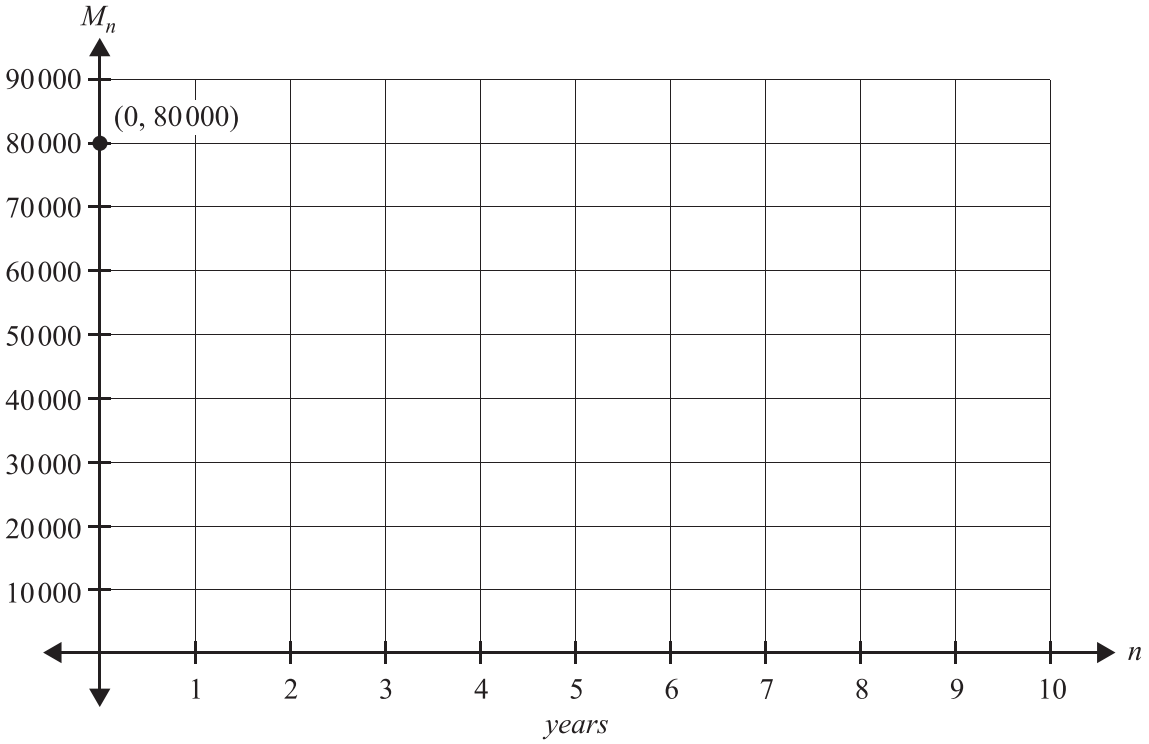
The recurrence relation below can be used to model the value of the van, M_n, after n years.

$$M_0 = 80\,000, \qquad M_{n+1} = M_n - 7200$$

a. What is the annual percentage flat rate of depreciation used? 1 mark

b. On the axes below, the point representing M_0 is shown.

Mark in the point that represents M_5, the value of the van after 5 years, with a point (●) and **label** the coordinates. 1 mark



c. Maria will sell the van when its value is $8000.

For how many years will Maria own the van? 1 mark

Question 7 (4 marks)

Maria invests $300 000 into an annuity.

Interest is calculated fortnightly and Maria receives fortnightly payments of $1570.58 from the annuity.

Three complete lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	300 000.00
1	1570.58	750.00	820.58	299 179.42
2	1570.58	747.95	822.63	298 356.79
3				

a. Write a calculation to show that the interest rate on the annuity is 0.25% per fortnight. 1 mark

b. Determine the principal reduction associated with payment number 3.

Round your answer to the nearest cent. 1 mark

c. Let the balance of Maria's annuity, in dollars, after n fortnights be represented by V_n.

Write a recurrence relation, in terms of V_0, V_{n+1} and V_n, that can model the value of the annuity from one fortnight to the next. 1 mark

d. How many fortnightly payments will Maria receive over the life of the annuity? 1 mark

Question 8 (3 marks)

Maria took out a reducing balance loan of $100 000 to fund improvements to her business.

Interest is charged at the rate of 5.4% per annum, compounding monthly.

Maria intends to repay the loan with monthly repayments of $1285.12 over eight years.

a. The final repayment would need to be slightly higher to fully repay the loan.

Determine the final repayment.

Round your answer to the nearest cent. 1 mark

b. After five years Maria decides to increase her monthly repayment so she can fully repay the loan in a further 18 months.

Let B_n be the balance of Maria's loan n months after the change to the repayment.

Write a recurrence relation, in terms of B_0, B_{n+1} and B_n, that can model this balance from month to month.

Round values to the nearest cent where appropriate. 2 marks

Question 9 (2 marks)

At the beginning of 2024, Maria invested a sum of money that earns interest compounding weekly.

Halfway through 2024 the investment had grown to $25 098.84 and at the end of 2024 it had grown to $25 452.60

What was the interest rate per annum of Maria's investment, as a percentage to one decimal place, and what amount did Maria originally invest, rounded to the nearest dollar?

Assume there are exactly 52 weeks in one year.

interest rate = ☐

original investment = ☐

Matrices

Question 10 (4 marks)

Three friends from Melbourne, Australia, are planning an overseas holiday to Phuket, Thailand.

Matrix F below shows the base cost, in dollars, of flying either one-way (O) or return (R) from Melbourne to Phuket on three separate airlines, Airline B, Airline E and Airline L.

$$F = \begin{matrix} \\ B \\ E \\ L \end{matrix} \begin{matrix} O & R \\ \end{matrix} \begin{bmatrix} 460 & 890 \\ 520 & 950 \\ 610 & 1100 \end{bmatrix}$$

a. Use the information in matrix F to complete the table below. 1 mark

Airline	One-way base cost ($)	Return base cost ($)
Airline B		
Airline E		
Airline L		

b. There is an additional fee for each airfare.

One-way flights have an additional fee of $30 and return flights have an additional fee of $60.

The cost with the additional fee can be calculated as follows

$$F + A = \begin{bmatrix} 490 & 950 \\ 550 & 1010 \\ 640 & 1160 \end{bmatrix}$$

i. Write down matrix A in the space below. 1 mark

$A =$

Do not write in this area.

ii. There will also be a 2% payment processing fee.

The total amount to be paid per flight, including this fee, is given by the following calculation

$$p \times \begin{bmatrix} 490 & 950 \\ 550 & 1010 \\ 640 & 1160 \end{bmatrix}$$

Determine the value of p. 1 mark

c. Airline *B* has optional upgrades as shown in the table below. Prices are per person per one-way flight.

Seat selection ($)	Extra baggage ($)	Priority boarding ($)
30	35	45

The three friends will select upgrades for their flights.

The total cost of the upgrades is $130.

The following calculation is performed to determine the total cost of all their upgrades.

$$\begin{bmatrix} 30 & 35 & 45 \end{bmatrix} \times U = 130$$

Where U is a matrix representing the total number of each type of upgrade selected by the three friends.

Write down matrix U in the space below. 1 mark

$U =$

Question 11 (2 marks)

At Melbourne Airport, the three friends want to exchange their Australian currency (Australian dollars) for Thai currency (Thai baht).

An exchange rate is the rate at which one country's currency can be traded for another country's currency.

The table below shows the currency exchange rate for $1 (Australian dollar) for Thai baht and three other currencies.

Currency	Thai baht	Japanese yen	Indian rupee	Chinese yuan
Exchange rate for $1 (Australian dollar)	22.3	94.1	56.4	4.7

Teena wants to know for how much she could exchange $100 (Australian dollars) in each currency.

a. A matrix equation that will calculate the exchanged amount, in each currency, is given below.

$$\begin{bmatrix}100 & 100 & 100 & 100\end{bmatrix} \times Q = \begin{bmatrix}2230 & 9410 & 5640 & 470\end{bmatrix}$$

Write down matrix Q in the space below. 1 mark

$Q =$

Do not write in this area.

b. The following table shows how many Australian dollars each of the three friends exchanged for Thai baht.

	Australian dollars	**Thai baht**
Teena	100	2230
Alison	350	7805
Sally	n	4906

The exchange in currency can be represented by the following matrix equation:

$$m \times \begin{bmatrix} 100 \\ 350 \\ n \end{bmatrix} = \begin{bmatrix} 2230 \\ 7805 \\ 4906 \end{bmatrix}$$

Determine the values of m and n. 1 mark

$m =$

$n =$

Question 12 (2 marks)

The friends decide that their next holiday destination will be Nuuk in Greenland.

The communication matrix, G, below, shows the flights that connect the five cities of Melbourne (M), Perth (P), Dubai (D), Copenhagen (C), and Nuuk (N).

$$G = \begin{array}{c} \\ M \\ P \\ D \\ C \\ N \end{array} \begin{array}{c} \begin{array}{ccccc} M & P & D & C & N \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In the matrix G:

- the '1' in row M, column P indicates a direct flight between Melbourne and Perth
- the '0' in row M, column C indicates no direct flight between Melbourne and Copenhagen.

a. What is the most direct route from Melbourne to Nuuk? 1 mark

b. The two-step communication matrix, G^2, is shown below.

$$G^2 = \begin{array}{c} \\ M \\ P \\ D \\ C \\ N \end{array} \begin{array}{c} \begin{array}{ccccc} M & P & D & C & N \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

The element in row 3, column 3 of G^2 is 3.

Identify the flight pathways associated with this element. 1 mark

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Examination continues on the next page.

Question 13 (4 marks)

The three friends are staying at a resort classified as 'Luxury'.

At the start of every year, a classification board classifies resorts as either Luxury (L), Mid-range (M), Standard (S) or Budget (B).

The transition matrix, T, shows the expected proportions of resorts that remain in the same classification or change classification from one year to the next.

$$T = \begin{array}{c} \\ L \\ M \\ S \\ B \end{array} \begin{array}{c} \begin{array}{cccc} \textit{this year} & & & \\ L & M & S & B \end{array} \\ \begin{bmatrix} 0.9 & 0.3 & 0.1 & 0 \\ 0.1 & 0.6 & d & f \\ 0 & 0.1 & e & 0.1 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} L \\ M \\ S \\ B \end{array} \quad \textit{next year}$$

a. Interpret the meaning of element t_{14}. 1 mark

b. In the long term, what is predicted to happen to the number of resorts classified as Budget? 1 mark

Do not write in this area.

c. Let S_0 be the state matrix that shows the number of resorts classified as either L, M, S or B at the start of 2024.

Let S_1 be the state matrix that shows the expected number of resorts classified as either L, M, S or B at the start of 2025.

$$S_0 = \begin{bmatrix} 40 \\ 120 \\ 90 \\ 70 \end{bmatrix} \qquad S_1 = \begin{bmatrix} 81 \\ 119 \\ 64 \\ 56 \end{bmatrix}$$

Using this information, determine the value of d in transition matrix T. 1 mark

d. What is the expected percentage growth in the number of Luxury resorts from the start of 2024 to the start of 2026?

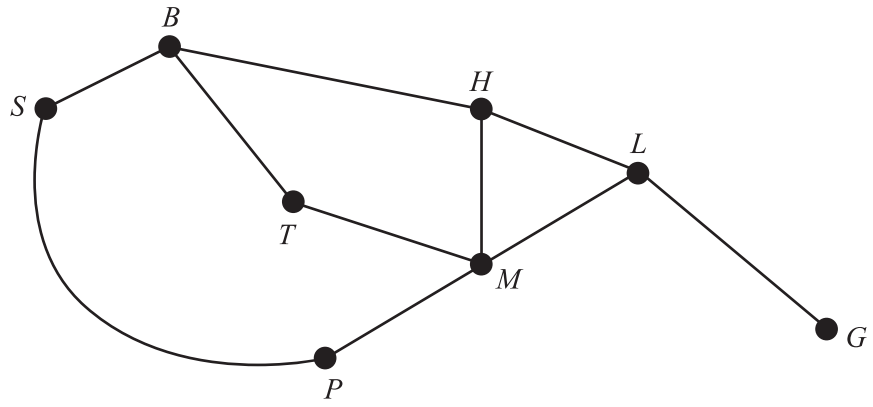
Write your answer correct to one decimal place. 1 mark

Networks and decision mathematics

Question 14 (4 marks)

A site has eight ancient landmarks that tourists can visit: Baths (*B*), Shop (*S*), Temple (*T*), Park (*P*), Hall (*H*), Marketplace (*M*), Living Quarters (*L*) and Gate (*G*).

The diagram below shows a network of paths at the site, with the vertices representing the different landmarks.



a. Alice is standing at the Park, *P*.

She wants to visit every other landmark once only.

i. Write down a route she could take. 1 mark

ii. What is the mathematical term for this route? 1 mark

b. What information does the degree of vertex *B* give about the connections between the Baths and other landmarks at the site? 1 mark

c. The management of the site wants to ensure that tourists can walk down every path once only and finish back where they started.

They will do this by building two new paths between landmarks that are not already directly connected.

The two new paths that will need to be built are between:

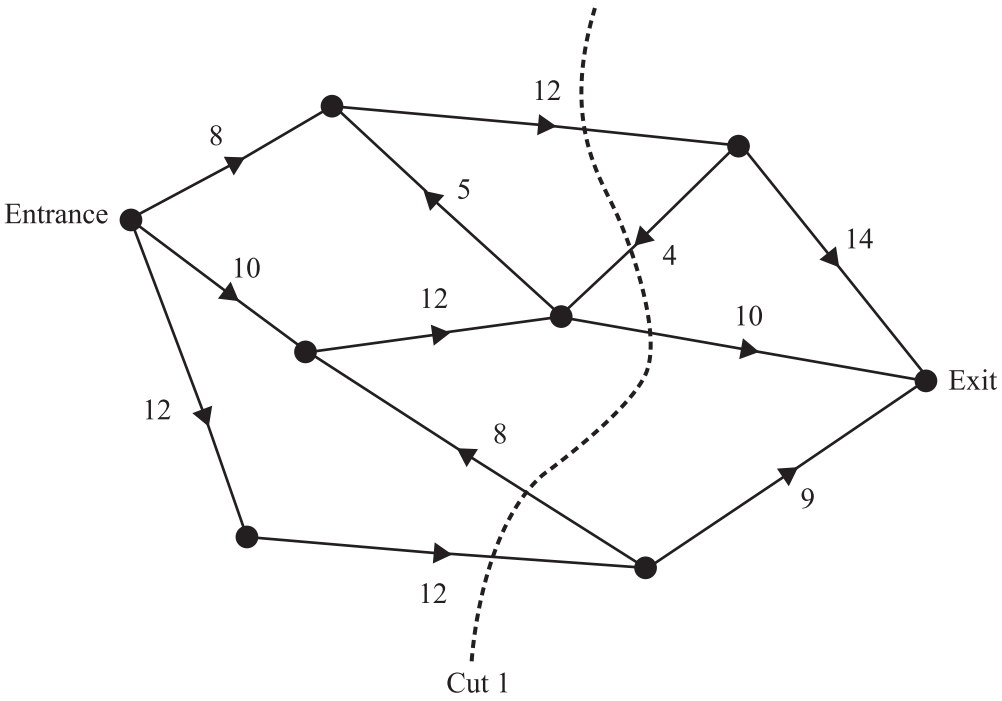
- vertex ☐ and vertex ☐
- vertex ☐ and vertex ☐

1 mark

Question 15 (2 marks)

The network below shows the one-way paths between the entrance and the exit of the Marketplace. The vertices represent the intersections of the one-way paths.

The number on each edge is the maximum number of visitors who are allowed to travel along that path per minute.



a. When considering the possible flow of visitors through the Marketplace, many different cuts can be made.

Determine the capacity of Cut 1, shown above. 1 mark

b. Determine the maximum number of visitors through the Marketplace from the Entrance to the Exit per minute. 1 mark

N
H
T

Do not write in this area.

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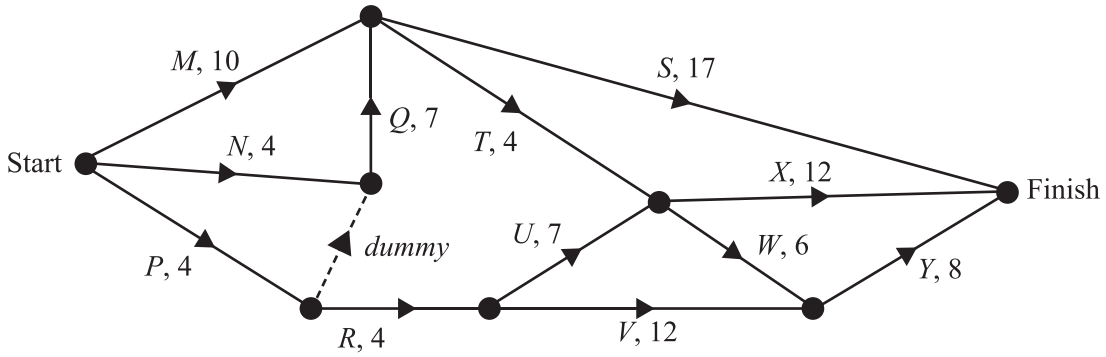
Examination continues on the next page.

Question 16 (6 marks)

The Temple is being restored.

This project involves 12 activities *M*, *N*, *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W*, *X* and *Y*.

The network below shows these activities and their completion times in weeks.



a. The precedence table for the project is shown below. This table is incomplete.

Activity	Immediate predecessor(s)
M	–
N	–
P	–
Q	
R	
S	
T	*M, Q*
U	*R*
V	*R*
W	*T, U*
X	*T, U*
Y	*V, W*

Complete the table above by writing the missing immediate predecessor(s) for activities *Q*, *R* and *S*. 1 mark

b. What is the earliest start time of activity W? 1 mark

c. What is the latest start time of activity X? 1 mark

d. Which activities have a float time that is **not** zero? 1 mark

e. The management of the site wants to reduce the time taken to complete this project.

They can reduce the completion time of any activity by exactly one week. The cost to reduce the completion time of any activity by one week is $2000 per activity.

i. What is the new minimum completion time of the project in weeks? 1 mark

ii. In order to achieve the new minimum completion time, what is the minimum cost of these reductions? 1 mark

End of Question and Answer Book

2025
NHT

General Mathematics Examination 2

2025 Formula Sheet

You may keep this Formula Sheet.

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Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $\text{Q1} - 1.5 \times \text{IQR}$ upper $\text{Q3} + 1.5 \times \text{IQR}$
least squares line of best fit	$y = a + bx$, where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = Ru_n + d$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \qquad \text{where} \qquad \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \qquad S_{n+1} = T S_n + B$
Leslie matrix recurrence relation	$S_0 = \text{initial state}, \qquad S_{n+1} = L S_n$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$

End of Formula Sheet