SUPERVISOR TO ATTACH
PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Specialist Mathematics Examination 2

Question and Answer Book

VCE Examination – Tuesday 11 November 2025

- Reading time is **15 minutes**: 11.45 am to 12 noon
- Writing time is **2 hours**: 12 noon to 2.00 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 24 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1

A tiger is a type of cat.

Consider the following statement.

'If I have a tiger, then I have a cat.'

The contrapositive of this statement is

A. if I do not have a tiger, then I do not have a cat.

B. if I have a cat, then I have a tiger.

C. if I do not have a cat, then I do not have a tiger.

D. if I do not have a tiger, then I have a different type of cat.

Question 2

Consider the following statement.

'If $f''(0) = 0$, then the graph of f necessarily has a point of inflection at $x = 0$.'

A counter-example that disproves this statement is when

A. $f(x) = \sin^{-1}(x)$

B. $f(x) = \dfrac{2x}{x^2 - 1}$

C. $f(x) = x^{\frac{1}{3}}$

D. $f(x) = x^4 - x$

Question 3

The graph of $y = \frac{x^2 + a}{bx + c}$ has an asymptote given by $y = -\frac{1}{2}x + \frac{1}{4}$ and a y-intercept of -2.

The values of a, b and c are

A. $a = 2, b = -2, c = -1$

B. $a = 2, b = 2, c = -1$

C. $a = -2, b = -2, c = 1$

D. $a = -2, b = -2, c = -1$

Question 4

Consider the following algorithm used to estimate a volume of revolution.

```
define f(x)
    return √(x + 1)
sum ← 0
a ← 1
b ← 3
left ← a
while left ≤ b
    volume ← π(f(left))^2
    sum ← sum + volume
    left ← left + 1
end while
print sum
```

The algorithm above will print the value

A. 5π

B. 9π

C. 14π

D. 29π

Question 5

The equation $z^3 + az^2 + bz - 52 = 0$, where $a, b \in R$ and $z \in C$, has a solution $z = 2 - 3i$.

The value of ab is

A. –232

B. –64

C. –8

D. 0

Question 6

Let $z \in C$.

Given that $|z| = 1$ and $z \neq 1$, $\text{Re}\left(\frac{1}{1-z}\right)$ is

A. $-\frac{1}{2}$

B. 0

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

Question 7

Using the substitution $u = \cos(\theta)$, $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin(2\theta)}{1+\cos(\theta)}\, d\theta$ can be expressed as

A. $\int_0^{\frac{\pi}{2}} u\sqrt{\frac{1-u}{1+u}}\, du$

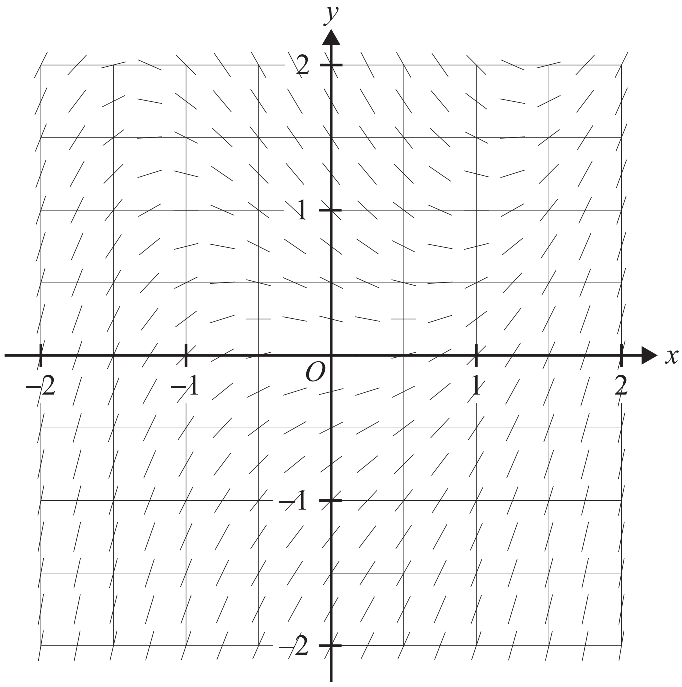
B. $\int_0^1 \left(1 + \frac{1}{1+u}\right) du$

C. $\int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{1+u}\right) du$

D. $\int_0^1 \left(1 - \frac{1}{1+u}\right) du$

Question 8

Consider the direction field below.



The direction field best represents the differential equation

A. $\dfrac{dy}{dx} = x^2 - y$

B. $\dfrac{dy}{dx} = x - y^2$

C. $\dfrac{dy}{dx} = y - x$

D. $\dfrac{dy}{dx} = x - y$

Question 9

A parametric curve is given by $x = kt$, $y = e^{kt}$, where k is a positive constant. The curve is rotated about the x-axis from $t = a$ to $t = b$, where $b > a$, to form a surface of revolution.

The area of this surface is given by

A. $2\pi \int_a^b e^{kt} \sqrt{k^2 t^2 + e^{2kt}}\, dt$

B. $2\pi \sqrt{k} \int_a^b e^{kt} \sqrt{1 + e^{kt}}\, dt$

C. $2\pi \int_{e^{ka}}^{e^{kb}} \sqrt{1 + u^2}\, du$

D. $2\pi \int_{e^{ka}}^{e^{kb}} u\sqrt{1 + u^2}\, du$

Question 10

The region bounded by the curve given by $y = 3\cos^{-1}(x)$, for $0 \le y \le a$, where $a > 0$, and the line $x = 0$ is rotated about the y-axis to form a solid of revolution. The volume of the solid is $\dfrac{\pi\left(4\pi + 3\sqrt{3}\right)}{8}$.

The value of a is

A. $\dfrac{\pi}{4}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{\pi}{2}$

D. π

Question 11

Given that $y(x)$ is a solution to the differential equation $\frac{dy}{dx} = x^2 y^3$, where $y(1) = 3$, the domain of y is

A. $x \le \left(\frac{7}{6}\right)^{\frac{1}{3}}$

B. $x < \left(\frac{7}{6}\right)^{\frac{1}{3}}$

C. $x \ge \left(\frac{7}{6}\right)^{\frac{1}{3}}$

D. $x > \left(\frac{7}{6}\right)^{\frac{1}{3}}$

Question 12

A particle moves along a straight line with constant acceleration. It passes through a point A with velocity u m s^{-1} and then through a point B with velocity v m s^{-1}.

The velocity of the particle at the midpoint of the line segment AB is given by

A. $\frac{u+v}{2}$

B. $u + \frac{u+v}{2}$

C. $\frac{u^2+v^2}{2}$

D. $\sqrt{\frac{u^2+v^2}{2}}$

Question 13

From an open window, a person projects a ball vertically up using an outstretched arm so the ball does not strike any part of the building. The point of projection of the ball is 50 m above the ground and its velocity of projection is 20 m s^{-1}.

The time, in seconds, it takes for the ball to reach the tray of a truck that is 1 m above the ground directly below the point of projection is closest to

A. 1.72

B. 5.80

C. 5.83

D. 1.75

Question 14

For non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, if $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a} \times \underset{\sim}{b}|$, then the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

Question 15

Consider the two planes described by the equations $2x + 2y + z = 2$ and $ax + 4z = 1$, where a is a positive constant.

The angle between the two planes is $\cos^{-1}\left(\frac{2}{3}\right)$.

The value of a satisfies the equation

A. $a + 2 = \sqrt{a^2 + 16}$

B. $\frac{2a+4}{3\sqrt{a^2+16}} = \frac{3}{2}$

C. $2a + 4 = 3\sqrt{a^2 + 16}$

D. $\frac{2a+4}{\sqrt{a^2+16}} = \frac{2}{3}$

Question 16

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = ne^{-2t}\underset{\sim}{i} - t^2\underset{\sim}{j}$, where n is a positive constant.

For what value of n is the particle's acceleration perpendicular to its velocity when $t = \frac{1}{2}$?

A. $2e$

B. $\frac{e^{0.5}}{2}$

C. $\frac{e}{2}$

D. $\frac{e}{2\sqrt{2}}$

Question 17

The acceleration vector of a particle that starts from rest is given by
$\underset{\sim}{a}(t) = 4\cos(2t)\underset{\sim}{i} + 10\sin(2t)\underset{\sim}{j} - 6e^{-2t}\underset{\sim}{k}$, where $t \geq 0$.

The velocity vector of the particle, $v(t)$, is given by

A. $\underset{\sim}{v}(t) = 2\sin(2t)\underset{\sim}{i} - 5\cos(2t)\underset{\sim}{j} + 3e^{-2t}\underset{\sim}{k}$

B. $\underset{\sim}{v}(t) = 2\sin(2t)\underset{\sim}{i} - 5\left(\cos(2t) + 1\right)\underset{\sim}{j} + 3(e^{-2t} + 1)\underset{\sim}{k}$

C. $\underset{\sim}{v}(t) = 2\sin(2t)\underset{\sim}{i} - 5\left(\cos(2t) - 1\right)\underset{\sim}{j} + 3(e^{-2t} - 1)\underset{\sim}{k}$

D. $\underset{\sim}{v}(t) = -8\sin(2t)\underset{\sim}{i} + 20\cos(2t)\underset{\sim}{j} + 12e^{-2t}\underset{\sim}{k}$

Question 18

The lines given by $\underset{\sim}{r}_1(\lambda) = 2\underset{\sim}{i} + r\underset{\sim}{j} - 3\underset{\sim}{k} + \lambda\left(\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}\right)$ and $\underset{\sim}{r}_2(\mu) = \underset{\sim}{i} + s\underset{\sim}{k} + \mu\left(\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)$ intersect at the point $(4, 3, t)$, where $\lambda, \mu \in R$ and r, s and t are real constants.

The values of r, s and t respectively are

A. 2, 3 and 5

B. 5, 3 and 5

C. 5, 5 and 8

D. 5, 8 and 5

Question 19

The plane with equation $x + y + z = a$, where $a \in R$, intersects the coordinate axes at three points that form the vertices of a triangle.

The area of this triangle is given by

A. $\dfrac{a^2\sqrt{3}}{4}$

B. $\dfrac{a^2\sqrt{3}}{2}$

C. $\dfrac{a^2}{4}$

D. $a^2\sqrt{3}$

Question 20

Let $P \sim \mathrm{N}\left(-2, 2^2\right)$, $Q \sim \mathrm{N}\left(3, 3^2\right)$, $R \sim \mathrm{N}\left(5, 6^2\right)$ and $Z \sim \mathrm{N}(0, 1)$.

Given that P, Q and R are independent random variables, $\Pr\left(3P + 2Q - R > 25\right)$ is equal to

A. $\Pr\left(Z > \frac{5\sqrt{3}}{3}\right)$

B. $\Pr(Z > 5)$

C. $\Pr\left(Z > \frac{5\sqrt{66}}{11}\right)$

D. $\Pr\left(Z > \frac{30}{7}\right)$

End of Section A

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Examination continues on the next page.

Section B

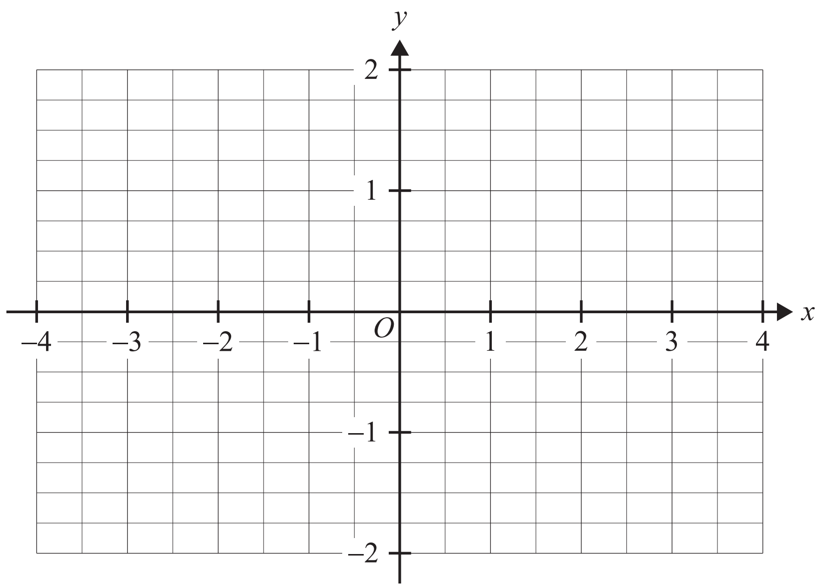
Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required for each question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1 (10 marks)

a. Sketch the graph of $y(x) = \dfrac{3x}{x^3 + x + 2}$ on the axes below.

Label the asymptotes with their equations, and label the turning point and the point of inflection with their coordinates. Give the coordinates of the point of inflection correct to one decimal place. 3 marks



b. The region bounded by the graph of $y = \dfrac{3x}{x^3 + x + 2}$, the coordinate axes and the line $x = 2$ is rotated about the x-axis to form a solid of revolution.

i. Write down a definite integral that, when evaluated, will give the volume of the solid of revolution. 1 mark

ii. Find the volume of the solid of revolution correct to two decimal places. 1 mark

c. Find the equations of the vertical asymptotes of the curve given by $y = \dfrac{3x}{x^3 - 5x + 2}$. 1 mark

d. A family of curves is given by $y(x) = \dfrac{3x}{x^3 + ax + 2}$, where $a \in R$.

i. Consider the case where the graph has a stationary point P.

Find the y-coordinate of P in terms of a. 1 mark

ii. For a given value of a, the graph has no stationary points.

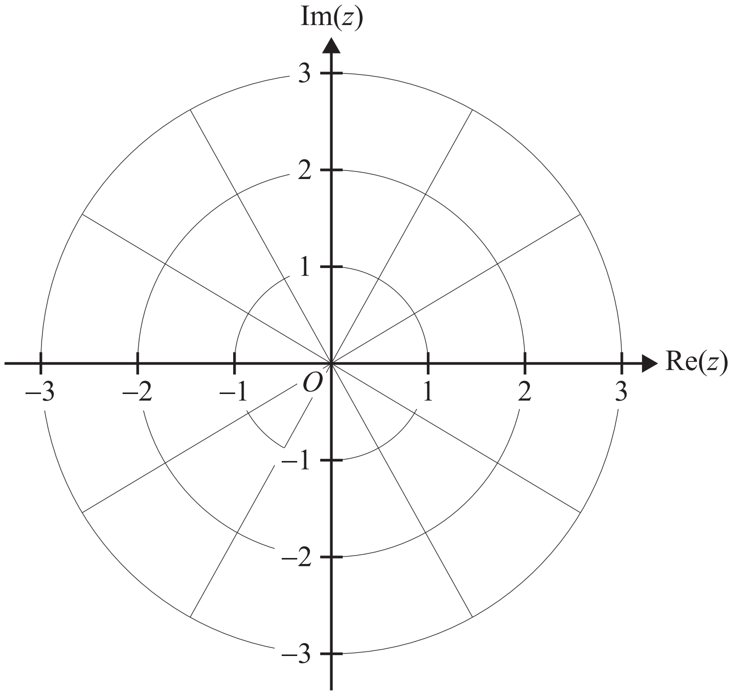
Find the equations of the vertical asymptotes of the graph in this case. 1 mark

iii. For a given value of a, the graph will have a point of inflection at $x = 2$.

Find the value of a. 2 marks

Question 2 (10 marks)

a. Sketch $\{z : z\bar{z} = 4, z \in C\}$ on the Argand plane below. 1 mark



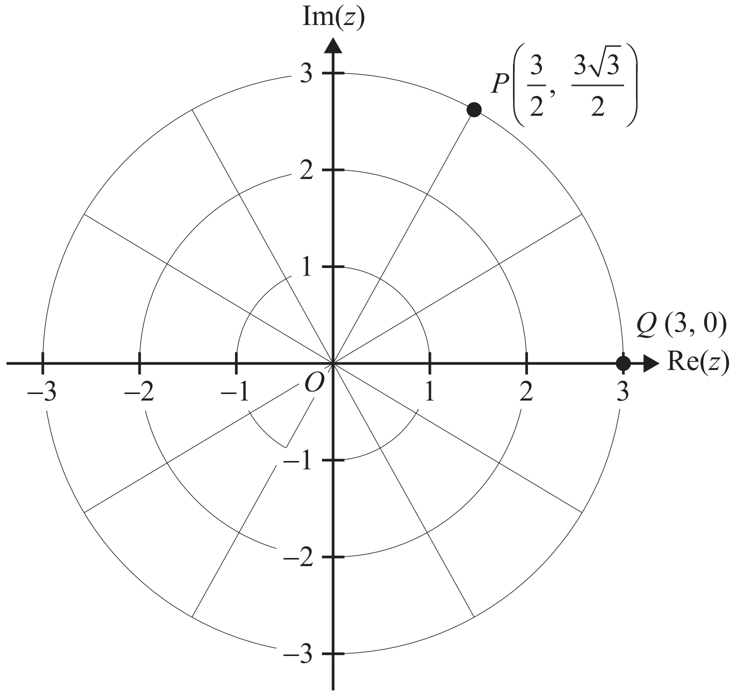
b. **i.** Show that $\left\{z : |z - 2i| = \left|z - \sqrt{3} - i\right|, z \in C\right\}$ may be expressed as $y = \sqrt{3}x$. 2 marks

ii. Sketch $\left\{z : |z - 2i| = \left|z - \sqrt{3} - i\right|, z \in C\right\}$ on the Argand plane in **part a**. 1 mark

c. **i.** Find the points of intersection of the curves defined in **part a** and in **part b.i**, expressing your answers in the form $a + ib$, where $a, b \in R$. 2 marks

ii. Label these points on the Argand plane in **part a**. 1 mark

Consider the points P and Q labelled on the Argand plane below.



d. A ray originating at point P and passing through point Q has the equation $\text{Arg}(z - z_0) = \theta$, where θ is a radian measure.

Write down the values of z_0 and θ. 1 mark

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e. Find the area of the minor segment bounded by the chord connecting the points P and Q and the circle given by $|z| = 3$.

Give your answer in the form $c\pi + d$, where $c, d \in R$. 2 marks

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Question 3 (10 marks)

A tank initially contains 5 kg of salt dissolved in 3000 litres of water. Salty water that contains 0.1 kg of salt per litre of water enters the tank at a rate of 20 litres per minute. The solution is kept thoroughly mixed and drains from the tank via a tap at the same rate of 20 litres per minute.

a. By considering concentration, explain whether the quantity of salt in the tank increases with time. 1 mark

b. Let Q denote the quantity of salt, in kilograms, in the tank at time t minutes.

Show that Q satisfies the differential equation $\frac{dQ}{dt} = \frac{300 - Q}{150}$. 1 mark

c. Using Euler's method with a step size of 15 minutes, find $Q(30)$, the approximate quantity of salt in the tank after 30 minutes.

Give your answer in kilograms, correct to two decimal places. 2 marks

d. Use calculus to solve the differential equation $\frac{dQ}{dt} = \frac{300-Q}{150}$, expressing Q in terms of t. 3 marks

e. What value does the quantity of salt in the tank approach as time approaches infinity?

Give your answer in kilograms. 1 mark

f. Find the time taken for the quantity of salt in the tank to reach 100 kg. 1 mark

g. When the quantity of salt in the tank reaches 100 kg, the tap draining the tank is turned off. Assume that the tank does not overflow and there is no change to the inflow rate.

After the tap is turned off, how many minutes does it take for the concentration of salt in the tank to reach $\frac{1}{20}$ kg L^{-1}? 1 mark

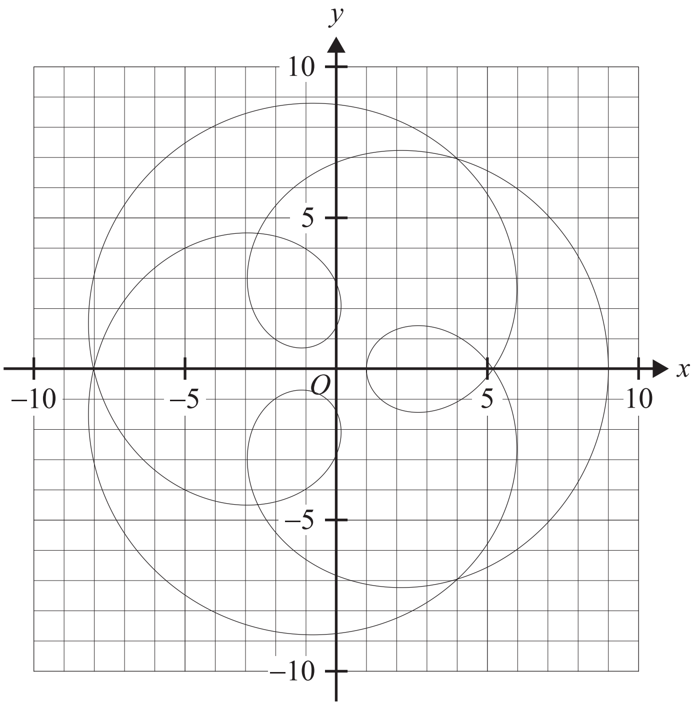
Question 4 (10 marks)

The path of a moving particle with position vector

$$\underset{\sim}{r}(t) = \left(5\cos(t) - 4\cos\left(\frac{5t}{2}\right)\right)\underset{\sim}{i} + \left(5\sin(t) - 4\sin\left(\frac{5t}{2}\right)\right)\underset{\sim}{j}$$

is shown below for time $t \geq 0$.

All lengths are in metres and time is measured in seconds.



a. Write down the coordinates of the particle's starting point. 1 mark

b. On the graph above, draw an arrow from the point (9, 0) to indicate the direction of motion of the particle. 1 mark

c. Find the value of t for which the particle will first return to its starting point. 1 mark

d. Show that the speed of the particle, in m s^{-1}, at time t can be expressed as $\sqrt{125 - 100\cos\left(\frac{3t}{2}\right)}$. 3 marks

e. What is the maximum speed of the particle in m s^{-1}? 1 mark

f. On the graph on page 18, trace the path of the particle for $t \in [0, \pi]$. 1 mark

g. Find the length of the path traced in **part f**, giving your answer in metres, correct to one decimal place. 2 marks

Question 5 (10 marks)

Consider three planes defined by the equations $\Pi_1 : 2x + 9z = 8$, $\Pi_2 : 3x + 6y + 5z = 7$ and $\Pi_3 : x + 9y - 3z = 7$.

a. Find the point of intersection of the three planes. 1 mark

b. **i.** Find a vector that gives the direction of the line of intersection of the planes Π_2 and Π_3. 2 marks

ii. Find a set of parametric equations that give the coordinates of the points that lie on this line of intersection. 1 mark

c. Find the shortest distance from the point (1, 1, 2) to the plane Π_3. 2 marks

d. Consider a family of planes, Ψ, with equation $6x + 27z = m$, where $m \in N$.

i. Show that the plane Π_1 is parallel to each member of Ψ. 1 mark

ii. Find all values of m for which the shortest distance between plane Π_1 and the plane of the form $6x + 27z = m$ is $\dfrac{23}{3\sqrt{85}}$. 3 marks

Question 6 (10 marks)

The volume of water, V mL, consumed by a student during a school day may be assumed to be normally distributed with a mean of 1000 mL and a standard deviation of 80 mL.

a. **i.** Write down the mean and standard deviation of the sampling distribution for the average volume of water consumed by randomly selected samples of 25 students.

Give your answers in millilitres. 1 mark

ii. What is the probability, correct to four decimal places, that the average volume of water consumed by a random sample of 25 students on a particular school day is more than 970 mL? 1 mark

The canteen at a particular school stocks two brands of water in bottles, Wasser and Apa.

The manufacturer of Wasser bottled water knows that the volume of water dispensed into bottles may be assumed to be normally distributed with a standard deviation of 5 mL. Engineers at the company take a random sample of 30 bottles and measure the volume of water in each bottle. The sample mean is found to be 750 mL.

b. Find a 95% confidence interval for the mean volume of water dispensed into each Wasser bottle.

Give your values in millilitres, correct to one decimal place. 1 mark

c. The engineers decide to take 300 random samples, each containing 30 bottles, and calculate the respective 95% confidence intervals. All samples are independent.

In how many of these confidence intervals would the engineers expect the value of the true mean volume dispensed to be included? 1 mark

d. What is the minimum size of the sample required to ensure that the difference between the sample mean and the mean volume dispensed is no more than 1 mL at the 95% confidence level? 1 mark

The volume of water dispensed into Apa water bottles may be assumed to be normally distributed with a mean of 750 mL and a standard deviation of 5 mL. After a service, a random sample of 50 bottles gave a sample mean of 748 mL. The company now claims that the mean volume of water dispensed is less than the stated mean of 750 mL.

A one-tailed statistical test at the 1% level of significance is proposed.

e. Write down the null and alternative hypotheses that will be used in testing the company's claim. 1 mark

f. **i.** Determine the p value for this test.

Give your answer correct to four decimal places. 1 mark

ii. Is the company's claim correct?

Explain your conclusion in terms of the p value. 1 mark

g. At the 1% level of significance for a sample size of 50 bottles, find the critical value of the sample mean, below which a sample mean value would support the conclusion that the mean volume of water dispensed is now less than 750 mL.

Give your answer correct to three decimal places. 1 mark

h. Assume that, after the service, the true mean volume of water in the Apa bottles was found to be 747.5 mL and that the population standard deviation, σ, is 5 mL.

At the 1% level of significance, for a sample size of 50, find the probability that the company will conclude that the service has not reduced the mean volume of water in an Apa bottle.

Give your answer correct to three decimal places. 1 mark

End of Question and Answer Book

Specialist Mathematics Examination 2

2025 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$	
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = n x^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\cot(ax)\right) = -a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\, dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}\left(\mathrm{cosec}(ax)\right) = -a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\, dx = -\frac{1}{a}\mathrm{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\, dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \quad n \neq -1$
	$\int \frac{1}{ax+b} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = u\,v - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x,\ y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f\left(x_n, y_n\right)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1 - \cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1 + \cos(2ax)\right)$

End of Formula Sheet