VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2023

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Written examination 2

Monday 6 November 2023

Reading time: 11.45 am to 12.00 pm (15 minutes)
Writing time: 12.00 pm to 2.00 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

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2023

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Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\text{E}(aX_1 + b) = a\,\text{E}(X_1) + b$ $\text{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\text{E}(X_1) + a_2\text{E}(X_2) + \ldots + a_n\text{E}(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \ldots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\text{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \ldots X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\text{E}(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$
$\frac{d}{dx}(\cot(ax)) = -a\,\text{cosec}^2(ax)$
$\frac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$
$\frac{d}{dx}(\text{cosec}(ax)) = -a\,\text{cosec}(ax)\cot(ax)$
$\frac{d}{dx}(\sin^{-1}(ax)) = \frac{a}{\sqrt{1-(ax)^2}}$
$\frac{d}{dx}(\cos^{-1}(ax)) = \frac{-a}{\sqrt{1-(ax)^2}}$
$\frac{d}{dx}(\tan^{-1}(ax)) = \frac{a}{1+(ax)^2}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\int \text{cosec}^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\int \sec(ax)\tan(ax)\, dx = \frac{1}{a}\sec(ax) + c$
$\int \text{cosec}(ax)\cot(ax)\, dx = -\frac{1}{a}\text{cosec}(ax) + c$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
$\int (ax + b)^n\, dx = \frac{1}{a(n+1)}(ax + b)^{n+1} + c, \quad n \neq -1$
$\int \frac{1}{ax + b} dx = \frac{1}{a}\log_e \lvert ax + b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{\mathrm{r}}(t) = x(t)\underset{\sim}{\mathrm{i}} + y(t)\underset{\sim}{\mathrm{j}} + z(t)\underset{\sim}{\mathrm{k}}$	$\left\lvert\underset{\sim}{\mathrm{r}}(t)\right\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{\mathrm{r}}}(t) = \dfrac{d\,\underset{\sim}{\mathrm{r}}}{dt} = \dfrac{dx}{dt}\underset{\sim}{\mathrm{i}} + \dfrac{dy}{dt}\underset{\sim}{\mathrm{j}} + \dfrac{dz}{dt}\underset{\sim}{\mathrm{k}}$
for $\underset{\sim}{\mathrm{r}}_1 = x_1\underset{\sim}{\mathrm{i}} + y_1\underset{\sim}{\mathrm{j}} + z_1\underset{\sim}{\mathrm{k}}$ and $\underset{\sim}{\mathrm{r}}_2 = x_2\underset{\sim}{\mathrm{i}} + y_2\underset{\sim}{\mathrm{j}} + z_2\underset{\sim}{\mathrm{k}}$	vector scalar product $\underset{\sim}{\mathrm{r}}_1 \cdot \underset{\sim}{\mathrm{r}}_2 = \left\lvert\underset{\sim}{\mathrm{r}}_1\right\rvert\left\lvert\underset{\sim}{\mathrm{r}}_2\right\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{\mathrm{r}}_1 \times \underset{\sim}{\mathrm{r}}_2 = \begin{vmatrix} \underset{\sim}{\mathrm{i}} & \underset{\sim}{\mathrm{j}} & \underset{\sim}{\mathrm{k}} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{\mathrm{i}} + (x_2z_1 - x_1z_2)\underset{\sim}{\mathrm{j}} + (x_1y_2 - x_2y_1)\underset{\sim}{\mathrm{k}}$
vector equation of a line	$\underset{\sim}{\mathrm{r}}(t) = \underset{\sim}{\mathrm{r}}_1 + t\underset{\sim}{\mathrm{r}}_2 = (x_1 + x_2t)\underset{\sim}{\mathrm{i}} + (y_1 + y_2t)\underset{\sim}{\mathrm{j}} + (z_1 + z_2t)\underset{\sim}{\mathrm{k}}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{\mathrm{r}}(s, t) = \underset{\sim}{\mathrm{r}}_0 + s\underset{\sim}{\mathrm{r}}_1 + t\underset{\sim}{\mathrm{r}}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{\mathrm{i}} + (y_0 + y_1s + y_2t)\underset{\sim}{\mathrm{j}} + (z_0 + z_1s + z_2t)\underset{\sim}{\mathrm{k}}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \dfrac{1}{2}(1 + \cos(2ax))$

END OF FORMULA SHEET

09999 : VICTORIAN SECONDARY COLLEGE

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

VCE SPECIALIST MATHEMATICS
Written Examination 2
ANSWER SHEET – 2023

STUDENT NAME: JOHN STUDENT

STUDENT NUMBER

9	9	1	2	3	4	5	6	A
0	0	0	0	0	0	0	0	
1	1		1	1	1	1	1	E
2	2	2		2	2	2	2	F
3	3	3	3		3	3	3	G
4	4	4	4	4		4	4	J
5	5	5	5	5	5		5	L
6	6	6	6	6	6	6		R
7	7	7	7	7	7	7	7	T
8	8	8	8	8	8	8	8	W
		9	9	9	9	9	9	X

INSTRUCTIONS: USE PENCIL ONLY

SIGN HERE IF YOUR NAME AND NUMBER ARE PRINTED CORRECTLY.

SIGNATURE: J. Student

If your name or number on this sheet is incorrect, notify the Supervisor.
Use a **PENCIL** for **ALL** entries. For each question, shade the box which indicates your answer.
All answers must be completed like **THIS** example: A ■ C D E
Marks will **NOT** be deducted for incorrect answers.
NO MARK will be given if more than **ONE** answer is completed for any question.
If you make a mistake, **ERASE** the incorrect answer – **DO NOT** cross it out.

SUPERVISOR USE ONLY

USE PENCIL ONLY

Shade the "**ABSENT**" box if the student was absent from the examination.

ABSENT ☐

SUPERVISOR'S INITIALS

OFFICE USE ONLY

☐ ☐

	ONE ANSWER PER LINE						ONE ANSWER PER LINE					
1	A	B	C	D	E	11	A	B	C	D	E	
2	A	B	C	D	E	12	A	B	C	D	E	
3	A	B	C	D	E	13	A	B	C	D	E	
4	A	B	C	D	E	14	A	B	C	D	E	
5	A	B	C	D	E	15	A	B	C	D	E	
6	A	B	C	D	E	16	A	B	C	D	E	
7	A	B	C	D	E	17	A	B	C	D	E	
8	A	B	C	D	E	18	A	B	C	D	E	
9	A	B	C	D	E	19	A	B	C	D	E	
10	A	B	C	D	E	20	A	B	C	D	E	

Please DO NOT fold, bend or staple this form.