

2015 VCE Mathematical Methods (CAS) 1 examination report

General comments

The format and structure of the 2015 Maths Methods (CAS) 1 examination was similar to previous years and responses given by students indicated that the questions were highly accessible. Students are encouraged to view the examination as an opportunity to demonstrate their knowledge and skills.

A substantial number of students performed reasonably well. However, others made errors with simple arithmetic calculations, especially those involving fractions or decimals, or in manipulating algebraic expressions. This was particularly prevalent in Questions 1, 2, 3, 6b., 7a., 8a. and 9a.

Students need to ensure that they set work out appropriately, that figures or expressions are legible and that they use correct mathematical notation. Students are also advised to check the validity of their final answers. In Question 5a., a negative height was not a feasible solution in the context of the given problem.

Simplification of algebraic expressions, particularly if the same expression is to be applied in a subsequent part of the question, is advised. In Question 9a., students who left their answer as a sum of two products often then experienced difficulty in using their expression in part b. of the same question. It is also suggested that students practise cancellation techniques, as in Questions 1bi., 6b., 7a. and 9b.

The probability questions were generally not answered well. A relevant diagram or table in these questions can be a very useful tool, not only to clarify the question but also to assist with explanation of the reasoning used to solve the problem.

Question 10 was not answered well. Some students who experienced difficulty with parts a. and b. were still able to receive full marks for part c. Some students did not attempt Question 10d.

Areas of strength included:

- use of the chain rule (Question 1)
- sketching the graphs of a cubic polynomial function (Question 4b.)
- solving equations involving circular functions (Question 5b.)
- solving equations involving logarithms (Question 7a.)
- basic probability (Question 8a.).

Areas of weakness included:

- antidifferentiation involving negative fractional powers (Question 3)
- average value, solving equations involving exponential functions (Question 4c.)
- solving exponential equations involving negative indices (Question 7b.)
- conditional probability (Questions 6b. and 9bii.).

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	15	85	0.9

$$\frac{dy}{dx} = 35(5x+1)^6$$

Most students correctly applied the chain rule. The most common errors were arithmetical.

Question 1bi.

Marks	0	1	2	Average
%	10	18	72	1.6

$$f'(x) = \frac{1 - 2\log_e(x)}{x^3}$$

The majority of students used the quotient rule and this was the most direct method. Many students experienced difficulty in simplifying their derived expressions, to the point where some final answers involved fractions within a fraction.

Question 1bii.

Marks	0	1	Average
%	32	68	0.7

$$f'(1) = \frac{1 - 0}{1} = 1$$

This question was well handled. However, some students who correctly identified the required derivative in Question 1bi. could not evaluate $2\log_e(1) = 2(0) = 0$.

Question 2

Marks	0	1	2	3	Average
%	14	14	21	51	2.1

$$f(x) = \int \left(1 - \frac{3}{x} \right) dx = x - 3\log_e(|x|) + c$$

$$f(e) = e - 3\log_e(e) + c = -2$$

$$c = -2 + 3 - e = 1 - e$$

$$f(x) = x - 3\log_e(|x|) + 1 - e$$

Most students identified that the antiderivative involved a logarithmic expression. Evaluation of the constant of antidifferentiation caused some difficulties.

Question 3

Marks	0	1	2	Average
%	38	6	55	1.2

$$\int_1^4 \left(x^{-\frac{1}{2}}\right) dx = \left[2x^{\frac{1}{2}}\right]_1^4$$

$$= \left[(2 \times \sqrt{4}) - (2 \times \sqrt{1})\right]$$

$$= 2$$

This question was not answered well. A range of incorrect anti-derivatives were given, the majority of which involved the logarithm function.

Question 4a.

Marks	0	1	2	Average
%	14	24	62	1.5

$$f'(x) = \frac{3}{2}x(x+2) = 0$$

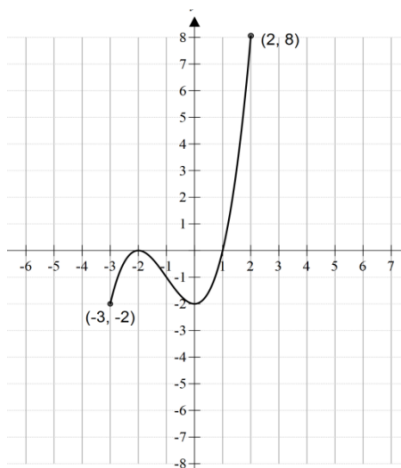
$$x = 0 \text{ or } x = -2$$

Coordinates $(0, -2), (-2, 0)$

Most students recognised the need to solve $f'(x) = 0$. Some students incorrectly stated the derivative as $3x(x+2)$. Other students used the product rule to obtain a correct derivative, though this was not the most efficient method.

Question 4b.

Marks	0	1	2	Average
%	17	29	54	1.4



This question was generally answered well. Students were wary of the restricted domain, though errors occurred with the calculation or the placement of the endpoints.

While labelling of intercepts and turning points was not required by this question, a correct graph was required to be awarded full marks.

Question 4c.

Marks	0	1	2	Average
%	40	19	41	1

$$\begin{aligned}
 &= \frac{1}{2-0} \int_0^2 f(x) dx \\
 &= \frac{1}{2} \times \left[\frac{1}{2} \left(\frac{x^4}{4} + x^3 - 4x \right) \right]_0^2 \\
 &= \frac{1}{4} [(4 + 8 - 8) - 0] \\
 &= 1
 \end{aligned}$$

Most students recalled the average value definition, which was not stated on the formula sheet, but then did not integrate correctly. The main error in student responses was the misplacement of $\frac{1}{2}$ in the integrand. Some students split the integration; for example, $\frac{1}{2} \left(\int_0^1 f(x) dx + \int_1^2 f(x) dx \right)$. Some students confused average value with average rate of change, instead finding a gradient.

Question 5a.

Marks	0	1	Average
%	31	69	0.7

$$h_{\min} = 14 - 8 = 6 \text{ metres}$$

This question was generally well handled. Common errors included finding the maximum height rather than the minimum, negative heights ($8 - 14$) and evaluating $h(10)$. Students who used calculus to obtain a minimum value tended to make careless errors in the differentiation.

Question 5b.

Marks	0	1	2	Average
%	25	23	53	1.3

$$14 + 8 \sin\left(\frac{\pi t}{12}\right) = 10$$

$$\sin\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$$

$$\frac{\pi t}{12} = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$t = 14, 22$$

This question was well handled. Most students set up an equation that when solved would yield the two correct answers for the restricted domain. Some students did not recognise the base angle of $\frac{\pi}{6}$.

Question 6a.

Marks	0	1	Average
%	50	50	0.5

$$\Pr(X > \mu + 2\sigma) = \Pr(Z < 0 - 2)$$

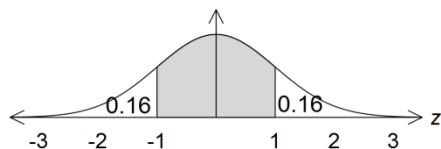
$$b = -2$$

Those students who drew a diagram of a 'normal' curve with relevant areas shaded found this helpful. An answer of + 2 was common. The answer of 1.9 was also common, and was two standard deviations below the mean of X . This question required a conversion to the standard normal curve.

Question 6b.

Marks	0	1	2	Average
%	42	22	37	1

$$\begin{aligned} \Pr(X < \mu + 1\sigma | X > \mu) &= \frac{\Pr(0 < Z < 1)}{\frac{1}{2}} \\ &= 2(0.5 - (0.16)) = 0.68 \end{aligned}$$



Most students could state the relevant rule and obtained the correct denominator of $\frac{1}{2}$ but then

failed to recognise that $\Pr(X < 2.8 | X > 2.5) = \frac{\Pr(2.5 < X < 2.8)}{\Pr(X > 2.5)}$. Probabilities greater than 1 or errors in handling decimals and/or fraction simplifications were common.

Question 7a.

Marks	0	1	2	Average
%	16	19	65	1.5

$$\log_2 \left(\frac{6-x}{4-x} \right) = 2$$

$$\frac{6-x}{4-x} = 2^2$$

$$x = \frac{10}{3}$$

Many students correctly applied logarithm laws, but others incorrectly cancelled logarithms.

Question 7b.

Marks	0	1	2	3	Average
%	49	5	10	36	1.3

$$3e^t = 5 + 8e^{-t}$$

$$3e^{2t} - 5e^t - 8 = 0$$

$$(e^t + 1)(3e^t - 8) = 0$$

$$e^t \neq -1, e^t = \frac{8}{3} \Rightarrow t = \log_e \left(\frac{8}{3} \right)$$

This question was not answered well. Many students were unable to create the quadratic equation evolved from manipulating e^{-t} . Many students solved via the quadratic formula rather than using simpler factorising techniques. The feasibility of only one answer was generally well handled.

Question 8a.

Marks	0	1	Average
%	13	87	0.9

$$\Pr(A \cap B) = \Pr(A|B) \times \Pr(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

This question was well answered.

Question 8b.

Marks	0	1	Average
%	41	59	0.6

$$\Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

A Karnaugh map was most useful for formulating a solution. Some poor manipulation of fractions was evident in responses.

Question 8c.

Marks	0	1	Average
%	72	28	0.3

If A and B are independent, then $\Pr(A|B) = \Pr(A) = \frac{3}{4}$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6}$$

Many students made little headway into solving this problem due to their lack of understanding of independent events. The addition rule was then applied using an incorrect value for $\Pr(A)$, resulting in final answers well outside the interval $[0,1]$. Students must note that a probability must lie within $[0,1]$ and is never a negative number.

Question 9a.

Marks	0	1	Average
%	47	53	0.5

$$\Pr(W) = \Pr(A) \times \Pr(W|A) + \Pr(B) \times \Pr(W|B)$$

$$\begin{aligned} \Pr(W) &= p \times \frac{3}{5} + (1-p) \times \frac{1}{5} \\ &= \frac{2p+1}{5} \end{aligned}$$

Many students made good use of a tree diagram in their formulation of a solution. Some students left their answer unsimplified as a sum of two products. A significant number of students offered a final expression not in terms of p .

Question 9bi.

Marks	0	1	2	Average
%	46	26	28	0.8

$$\Pr(B|W) = \frac{\Pr(B \cap W)}{\Pr(W)} = \frac{\frac{1-p}{5}}{\frac{2p+1}{5}} = \frac{1-p}{2p+1}$$

While most students recognised that this question involved conditional probability, many could not apply it within the context of the specific question. Algebraic fractions were not handled well.

Question 9bii.

Marks	0	1	Average
%	81	19	0.2

$$\Pr(B|W) = 0.3 = \frac{1-p}{2p+1}$$

$$0.6p + 0.3 = 1 - p$$

$$p = \frac{7}{16}$$

Many students missed the specific connection of this part with the previous part.

Question 10a.

Marks	0	1	Average
%	80	20	0.2

$$(2 + 2\cos(\theta), 2\sin(\theta))$$

The most common error in responses to this question was the oversight of + 2 for the x coordinate.

Question 10b.

Marks	0	1	Average
%	84	16	0.2

$$m_{CT} = \tan(\theta)$$

$$m_{XY} = -\frac{1}{\tan(\theta)}$$

Equivalent expressions such as $m_{XY} = -\frac{\cos(\theta)}{\sin(\theta)}$ were accepted. Some students included the

variables of b or d in their final answer. Many students found the gradient of the radius CT rather than the gradient of the line segment XT .

Question 10c.

Many students who had no success with parts a. and b. managed to attain full marks for this question. The most efficient method was to substitute the relevant points into the given equation then transpose for the variable specified.

Question 10ci.

Marks	0	1	Average
%	46	54	0.6

$$b \sin(\theta) + 2 \cos(\theta) = 2 + 2 \cos(\theta)$$

$$b = \frac{2}{\sin(\theta)}$$

Question 10cii.

Marks	0	1	Average
%	53	47	0.5

$$d \sin(\theta) + 4 \cos(\theta) = 2 + 2 \cos(\theta)$$

$$d = \frac{2 - 2 \cos(\theta)}{\sin(\theta)}$$

Question 10d.

Marks	0	1	2	3	Average
%	68	18	3	11	0.6

$$A(\theta) = \frac{b+d}{2} \times 2 = \frac{2}{\sin(\theta)} + \frac{2-2\cos(\theta)}{\sin(\theta)} = \frac{4-2\cos(\theta)}{\sin(\theta)}$$

$$A'(\theta) = \frac{2-4\cos(\theta)}{\sin^2(\theta)} = 0$$

$$\theta = \frac{\pi}{3}$$

$$A\left(\frac{\pi}{3}\right) = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

Students found this question challenging. The first step required an expression for the area of a trapezium in terms of only θ , use of calculus to determine the value of θ for which the minimum area occurred and finally to find this minimum area. Many students did not attempt this question or had difficulty in deriving the area function.