VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Written examination 1

Wednesday 2 November 2022

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = 3xe^{2x}$.

Find $\frac{dy}{dx}$. 1 mark

b. Find and simplify the rule of $f'(x)$, where $f: R \to R,\ f(x) = \frac{\cos(x)}{e^x}$. 2 marks

Question 2 (4 marks)

a. Let $g:\left(\frac{3}{2}, \infty\right) \to R,\ g(x) = \frac{3}{2x-3}$.

Find the rule for an antiderivative of $g(x)$. 1 mark

b. Evaluate $\int_0^1 \left(f(x)\left(2f(x)-3\right)\right)dx$, where $\int_0^1 [f(x)]^2 dx = \frac{1}{5}$ and $\int_0^1 f(x)\,dx = \frac{1}{3}$. 3 marks

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Question 3 (3 marks)

Consider the system of equations

$$kx - 5y = 4 + k$$
$$3x + (k + 8)y = -1$$

Determine the value of k for which the system of equations above has an infinite number of solutions.

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Question 4 (5 marks)

A card is drawn from a deck of red and blue cards. After verifying the colour, the card is replaced in the deck. This is performed four times.

Each card has a probability of $\frac{1}{2}$ of being red and a probability of $\frac{1}{2}$ of being blue.

The colour of any drawn card is independent of the colour of any other drawn card.

Let X be a random variable describing the number of blue cards drawn from the deck, in any order.

a. Complete the table below by giving the probability of each outcome. 2 marks

x	0	1	2	3	4
$\Pr(X = x)$	$\frac{1}{16}$		$\frac{6}{16}$		

b. Given that the first card drawn is blue, find the probability that exactly two of the next three cards drawn will be red. 1 mark

c. The deck is changed so that the probability of a card being red is $\frac{2}{3}$ and the probability of a card being blue is $\frac{1}{3}$.

Given that the first card drawn is blue, find the probability that exactly two of the next three cards drawn will be red. 2 marks

Question 5 (5 marks)

a. Solve $10^{3x-13} = 100$ for x. 2 marks

b. Find the maximal domain of f, where $f(x) = \log_e\left(x^2 - 2x - 3\right)$. 3 marks

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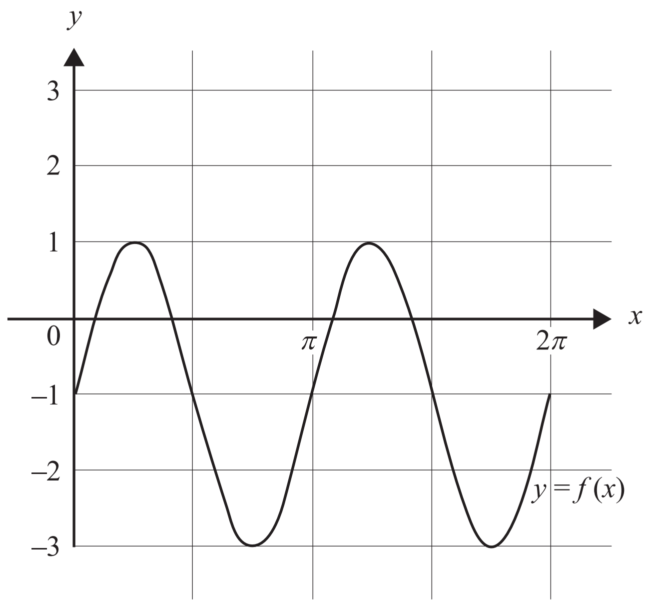
Question 6 (8 marks)

The graph of $y = f(x)$, where $f : [0, 2\pi] \to R$, $f(x) = 2\sin(2x) - 1$, is shown below.



a. On the axes above, draw the graph of $y = g(x)$, where $g(x)$ is the reflection of $f(x)$ in the horizontal axis. 2 marks

b. Find all values of k such that $f(k) = 0$ and $k \in [0, 2\pi]$. 3 marks

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Question 6 – continued

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c. Let $h : D \to R$, $h(x) = 2\sin(2x) - 1$, where $h(x)$ has the same rule as $f(x)$ with a different domain.

The graph of $y = h(x)$ is translated a units in the positive horizontal direction and b units in the positive vertical direction so that it is mapped onto the graph of $y = g(x)$, where $a, b \in (0, \infty)$.

i. Find the value for b. 1 mark

ii. Find the smallest positive value for a. 1 mark

iii. Hence, or otherwise, state the domain, D, of $h(x)$. 1 mark

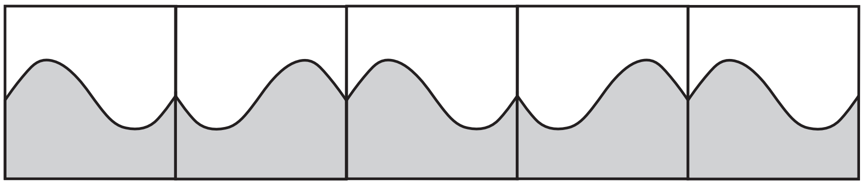
Question 7 (7 marks)

A tilemaker wants to make square tiles of size 20 cm × 20 cm.

The front surface of the tiles is to be painted with two different colours that meet the following conditions:

- Condition 1 – Each colour covers half the front surface of a tile.
- Condition 2 – The tiles can be lined up in a single horizontal row so that the colours form a continuous pattern.

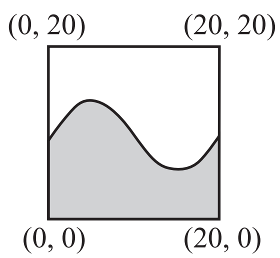
An example is shown below.



There are two types of tiles: Type A and Type B.

For Type A, the colours on the tiles are divided using the rule $f(x) = 4\sin\left(\frac{\pi x}{10}\right) + a$, where $a \in R$.

The corners of each tile have the coordinates (0, 0), (20, 0), (20, 20) and (0, 20), as shown below.



a. **i.** Find the area of the front surface of each tile. 1 mark

ii. Find the value of a so that a Type A tile meets Condition 1. 1 mark

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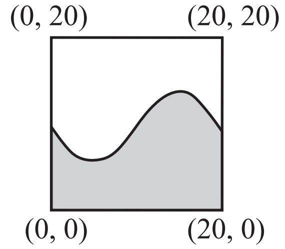
Type B tiles, an example of which is shown below, are divided using the rule $g(x) = -\frac{1}{100}x^3 + \frac{3}{10}x^2 - 2x + 10$.

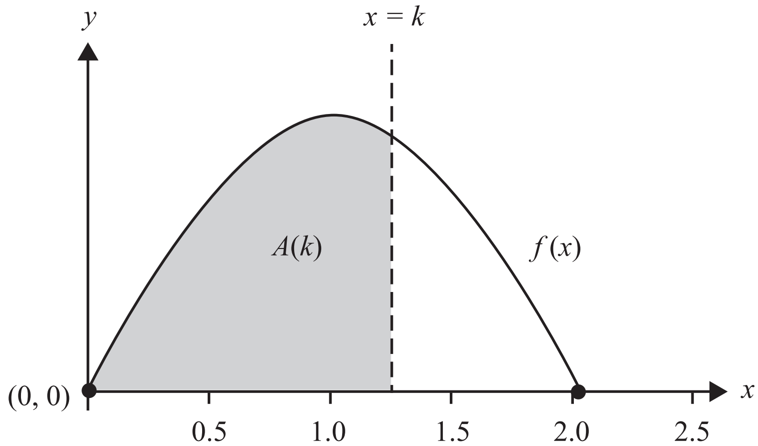


b. Show that a Type B tile meets Condition 1. 3 marks

c. Determine the endpoints of $f(x)$ and $g(x)$ on each tile. Hence, use these values to confirm that Type A and Type B tiles can be placed in any order to produce a continuous pattern in order to meet Condition 2. 2 marks

Question 8 (5 marks)

Part of the graph of $y = f(x)$ is shown below. The rule $A(k) = k\sin(k)$ gives the area bounded by the graph of f, the horizontal axis and the line $x = k$.



a. State the value of $A\left(\frac{\pi}{3}\right)$. 1 mark

b. Evaluate $f\left(\frac{\pi}{3}\right)$. 2 marks

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Question 8 – continued

c. Consider the average value of the function f over the interval $x \in [0, k]$, where $k \in [0, 2]$.

Find the value of k that results in the maximum average value. 2 marks

END OF QUESTION AND ANSWER BOOK

Victorian Curriculum and Assessment Authority

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2022

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET