VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2022

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS
Written examination 1

Friday 4 November 2022

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

Consider the equation $p(z) = z^2 + 6iz - 25$, $z \in C$.

a. Express $p(z)$ in the form $p(z) = (z + ai)^2 + b$, where $a, b \in R$. 1 mark

b. Hence, or otherwise, find the solutions of the equation $p(z) = 0$. 2 marks

Question 2 (3 marks)

Solve the differential equation $\frac{dy}{dx} = -x\sqrt{4 - y^2}$ given that $y(2) = 0$. Give your answer in the form $y = f(x)$.

Question 3 (4 marks)

The time taken by a coffee machine to dispense a cup of coffee varies normally with a mean of 10 seconds and a standard deviation of 1.5 seconds.

a. Find the probability that more than 34 seconds is needed to dispense a total of four cups of coffee. Give your answer correct to two decimal places. 2 marks

b.

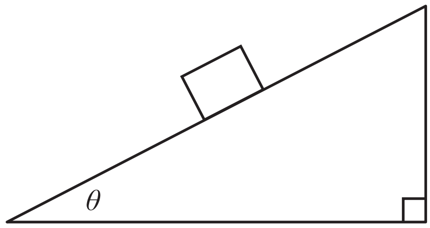
This question has been redacted following the findings of the Independent Review into the VCAA's Examination-Setting Policies, Processes and Procedures for the VCE.

Question 4 (4 marks)

Find $\int \frac{3x^2+4x+12}{x(x^2+4)}\,dx$.

Question 5 (3 marks)

A body of mass 10 kg, which is initially at rest, slides down a smooth inclined plane, as shown in the diagram below. The plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{1}{3}$.



a. Find the speed of the body after the body has been in motion for two seconds. 2 marks

b. After the body has been in motion for two seconds, a constant braking force, R newtons, is applied to the body parallel to the plane so that the body has constant velocity.

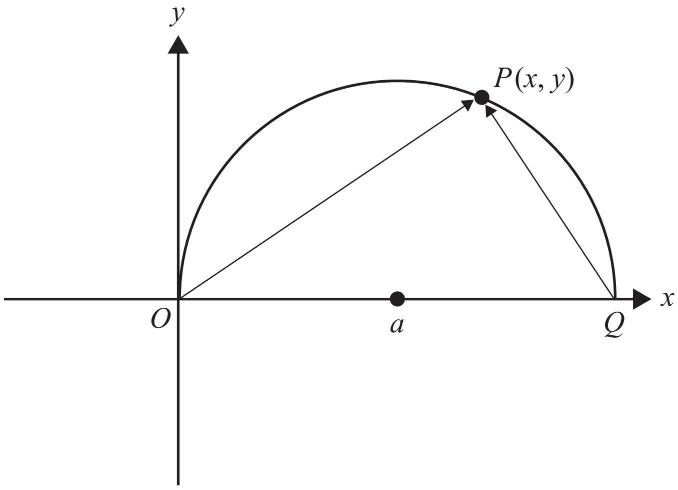
Find the value of R. 1 mark

Question 6 (6 marks)

a. Find the cosine of the acute angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$. 2 marks

Question 6 – continued

b. OPQ is a semicircle of radius a with equation $y = \sqrt{a^2 - (x-a)^2}$. $P(x, y)$ is a point on the semicircle OPQ, as shown below.



i. Express the vectors $\overrightarrow{OP}$ and $\overrightarrow{QP}$ in terms of a, x, y, $\underset{\sim}{i}$ and $\underset{\sim}{j}$, where $\underset{\sim}{i}$ is a unit vector in the direction of the positive x-axis and $\underset{\sim}{j}$ is a unit vector in the direction of the positive y-axis. 1 mark

ii. Hence, using the vector scalar (dot) product, determine whether $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{QP}$. 3 marks

TURN OVER

Question 7 (3 marks)

A curve has equation $x\cos(x+y)=\dfrac{\pi}{48}$.

Find the gradient of the curve at the point $\left(\dfrac{\pi}{24}, \dfrac{7\pi}{24}\right)$. Give your answer in the form $\dfrac{a\sqrt{b}-\pi}{\pi}$, where $a, b \in Z$.

DO NOT WRITE IN THIS AREA

Question 8 (4 marks)

A body moves in a straight line so that when its displacement from a fixed origin O is x metres, its acceleration, a, is $-4x$ ms^{-2}. The body accelerates from rest and its velocity, v, is equal to -2 ms^{-1} as it passes through the origin. The body then comes to rest again.

Find v in terms of x for this interval.

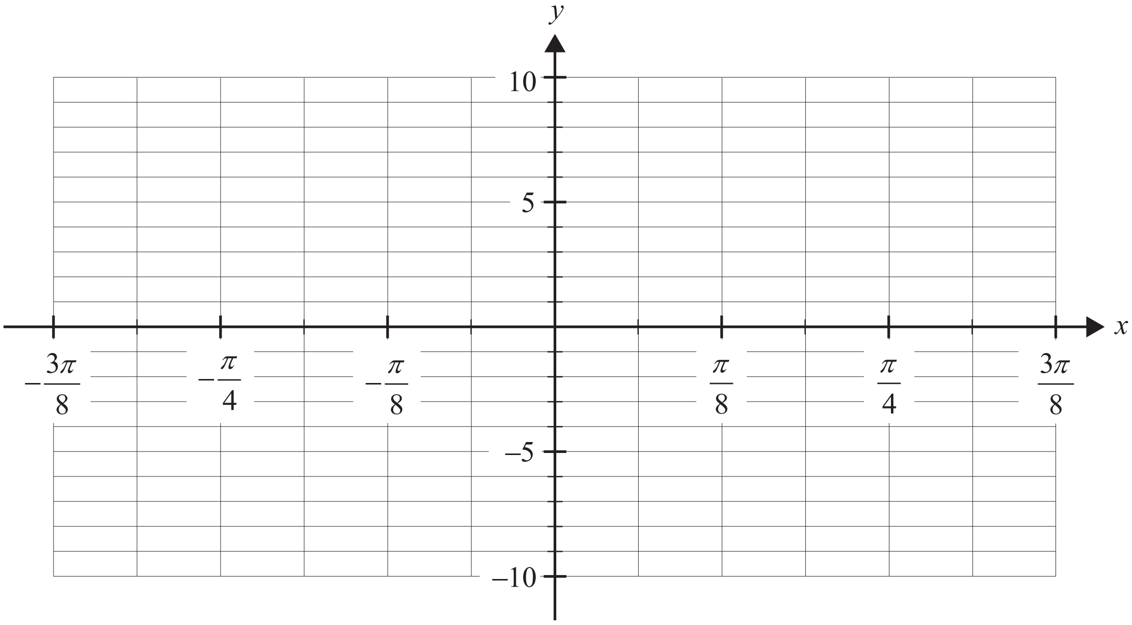
Question 9 (4 marks)

Given that $f'(x) = \dfrac{\cos(2x)}{\sin^3(2x)}$ and $f\left(\dfrac{\pi}{8}\right) = \dfrac{3}{4}$, find $f(x)$.

Question 10 (6 marks)

Let $f(x) = \sec(4x)$.

a. Sketch the graph of f for $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ on the set of axes below. Label any asymptotes with their equations and label any turning points and the endpoints with their coordinates. 3 marks



Question 10 – continued

b. The graph of $y = f(x)$ for $x \in \left[-\frac{\pi}{24}, \frac{\pi}{48}\right]$ is rotated about the x-axis to form a solid of revolution.

Find the volume of this solid. Give your answer in the form $\frac{\left(a - \sqrt{b}\right)\pi}{c}$, where a, b, $c \in R$. 3 marks

END OF QUESTION AND ANSWER BOOK

DO NOT WRITE IN THIS AREA

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\mathrm{E}\left(\overline{X}\right) = \mu$ variance $\operatorname{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \|x\| + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \|ax+b\| + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+\left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\|\underset{\sim}{r}\| = \sqrt{x^2+y^2+z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET