# VCE Mathematical Methods

## Written examination 2 – End of year

## Sample questions

These sample questions are intended to demonstrate how new aspects of Units 3 and 4 of VCE Mathematical Methods may be examined in written examination 2. They do **not** constitute a full examination paper.

### SECTION A – Multiple-choice questions

**Question 1**

$$x - 2y = 3$$
$$2y - z = 4$$

Which one of the following correctly describes the general solution to the system of linear equations given above?

**A.** $x = k,\ \ y = \frac{1}{2}(k+3),\ \ z = k-1,\ \ \text{for all } k \in R$

**B.** $x = k,\ \ y = \frac{1}{2}(k+3),\ \ z = k+1,\ \ \text{for all } k \in R$

**C.** $x = k,\ \ y = \frac{1}{2}(k-3),\ \ z = k+7,\ \ \text{for all } k \in R$

**D.** $x = k,\ \ y = \frac{1}{2}(k-3),\ \ z = k-7,\ \ \text{for all } k \in R$

**E.** $x = k,\ \ y = \frac{1}{2}(k+3),\ \ z = k-7,\ \ \text{for all } k \in R$

**Question 2**

Newton's method is being used to approximate the non-zero $x$-intercept of the function with the equation $f(x)=\dfrac{x^3}{5}-\sqrt{x}$. An initial estimate of $x_0 = 1$ is used.

Which one of the following gives the first estimate that would correctly approximate the intercept to three decimal places?

**A.** $x_6$

**B.** $x_7$

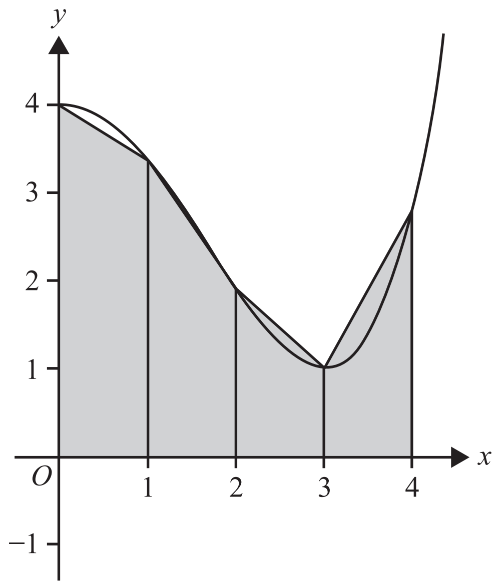
**C.** $x_8$

**D.** $x_9$

**E.** The intercept cannot be correctly approximated using Newton's method.

**Question 3**

The area between the curve $y=\dfrac{1}{27}(x-3)^2(x+3)^2+1$ and the $x$-axis on the interval $x \in [0, 4]$ has been approximated using the trapezium rule, as shown in the graph below.




Using the trapezium rule, the approximate area calculated is equal to

**A.** $\dfrac{1}{2}\left(4+\dfrac{91}{27}+\dfrac{52}{27}+1+\dfrac{76}{27}\right)$

**B.** $\dfrac{1}{2}\left(4+\dfrac{182}{27}+\dfrac{104}{27}+2+\dfrac{76}{27}\right)$

**C.** $\dfrac{1}{2}\left(8+\dfrac{182}{27}+\dfrac{104}{27}+2+\dfrac{152}{27}\right)$

**D.** $\dfrac{1}{2}\left(\dfrac{182}{27}+\dfrac{104}{27}+2+\dfrac{76}{27}\right)$

**E.** $\dfrac{1}{2}\left(8+\dfrac{182}{27}+\dfrac{104}{27}+2\right)$

**Question 4**

The probability density function $f$ of a random variable $X$ is given by

$$f(x)=\begin{cases}\dfrac{x+1}{20} & 0\le x<4\\ \dfrac{36-5x}{64} & 4\le x\le 7.2\\ 0 & \text{elsewhere}\end{cases}$$

The value of $a$ such that $\Pr(X\le a)=\dfrac{5}{8}$ is

**A.** $\dfrac{4\left(\sqrt{15}-9\right)}{5}$

**B.** $\sqrt{26}-1$

**C.** $\dfrac{36-4\sqrt{15}}{5}$

**D.** $\dfrac{4\sqrt{15}+9}{5}$

**E.** $\dfrac{4\sqrt{15}+36}{5}$

**Question 5**

The algorithm below, described in pseudocode, estimates the value of a definite integral using the trapezium rule.

```
Inputs: f(x), the function to integrate
        a, the lower terminal of integration
        b, the upper terminal of integration
        n, the number of trapeziums to use

Define trapezium(f(x),a,b,n)
     h ← (b − a) ÷ n
     sum ← f(a) + f(b)
     x ← a + h
     i ← 1
     While i < n Do
           sum ← sum + 2 × f(x)
           x ← x + h
           i ← i + 1
     EndWhile
     area ← (h ÷ 2) × sum
     Return area
```

Consider the algorithm implemented with the following inputs.

$$\texttt{trapezium}(\log_e(x),1,3,10)$$

The value of the variable `sum` after **one** iteration of the **`While`** loop would be closest to

**A.** 1.281

**B.** 1.289

**C.** 1.463

**D.** 1.617

**E.** 2.136

**Question 6**

Consider the algorithm below, which uses the bisection method to estimate the solution to an equation in the form $f(x) = 0$.

```
Inputs: f(x), a function of x, where x is in radians
        a, the lower interval endpoint
        b, the upper interval endpoint
        max, the maximum number of iterations

Define bisection(f(x),a,b,max)
     If f(a) × f(b) > 0 Then
          Return "Invalid interval"
     i ← 0
     While i < max Do
          mid ← (a + b) ÷ 2
          If f(mid) = 0 Then
               Return mid
          Else If f(a) × f(mid) < 0 Then
               b ← mid
          Else
               a ← mid
          i ← i + 1
     EndWhile
     Return mid
```

The algorithm is implemented as follows.

```
bisection(sin(x),3,5,2)
```

Which value would be returned when the algorithm is implemented as given?

**A.** −0.351
**B.** −0.108
**C.** 3.25
**D.** 3.5
**E.** 4

**Question 7**

One way of implementing Newton's method using pseudocode, with a tolerance level of 0.001, is shown below.

The pseudocode is incomplete, with two missing lines indicated by an empty box.

```
Inputs: f(x), a function of x
        x0, an initial estimate for the x-intercept of f(x)

Define newton(f(x), x0)
    df(x) ← the derivative of f(x)
    i ← 0
    prev_x ← x0
    While i < 1000 Do
        next_x ← prev_x − f(prev_x) ÷ df(prev_x)
        [                                    ]

        Else
            prev_x ← next_x
            i ← i + 1
    EndWhile
```

Which one of the following options would be most appropriate to fill the empty box?

**A.**
```
If next_x − prev_x < 0.001 Then
    Return prev_x
```

**B.**
```
If next_x − prev_x < 0.001 Then
    Return next_x
```

**C.**
```
If prev_x − next_x < 0.001 Then
    Return next_x
```

**D.**
```
If -0.001 < next_x − prev_x < 0.001 Then
    Return prev_x
```

**E.**
```
If -0.001 < next_x − prev_x < 0.001 Then
    Return next_x
```

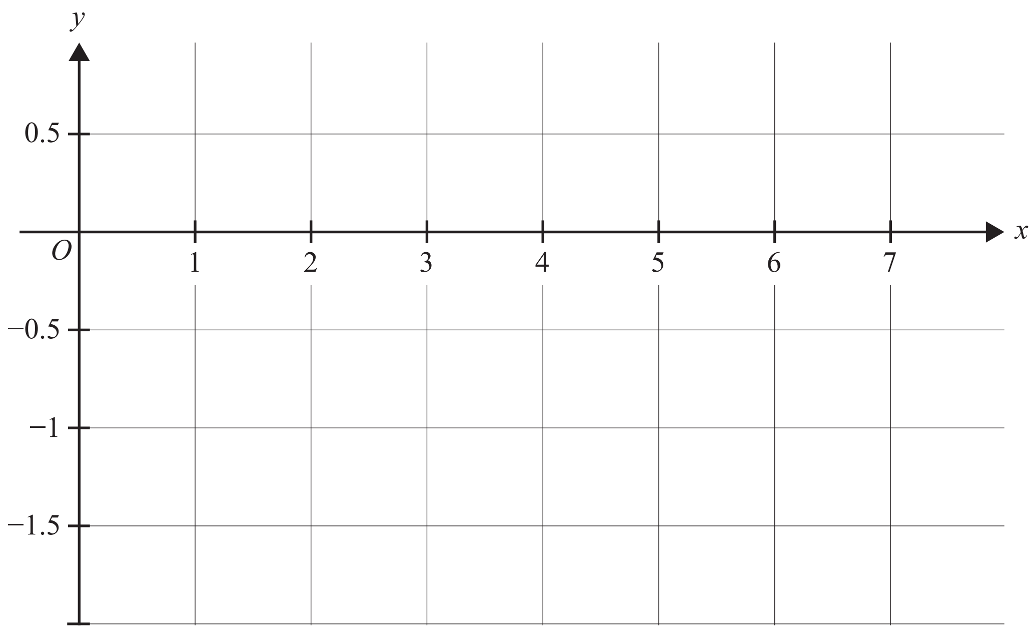
## SECTION B

**Question 1** (10 marks)

The function $g$ is defined as follows.

$$g : (0, 7] \to R, g(x) = 3 \log_e(x) - x$$

**a.** Sketch the graph of $g$ on the axes below. Label the vertical asymptote with its equation, and label any axial intercepts, stationary points and endpoints in coordinate form, correct to three decimal places. 3 marks




**b.** **i.** Find the equation of the tangent to the graph of $g$ at the point where $x = 1$. 1 mark

**ii.** Sketch the graph of the tangent to the graph of $g$ at $x = 1$ on the axes in **part a.** 1 mark

Newton's method is used to find an approximate $x$-intercept of $g$, with an initial estimate of $x_0 = 1$.

**c.** Find the value of $x_1$. 1 mark

**d.** Find the horizontal distance between $x_3$ and the closest $x$-intercept of $g$, correct to four decimal places. 1 mark

**e.** **i.** Find the value of $k$, where $k > 1$, such that an initial estimate of $x_0 = k$ gives the same value of $x_1$ as found in **part c.** Give your answer correct to three decimal places. 2 marks

**ii.** Using this value of $k$, sketch the tangent to the graph of $g$ at the point where $x = k$ on the axes in **part a.** 1 mark

**Question 2** (12 marks)

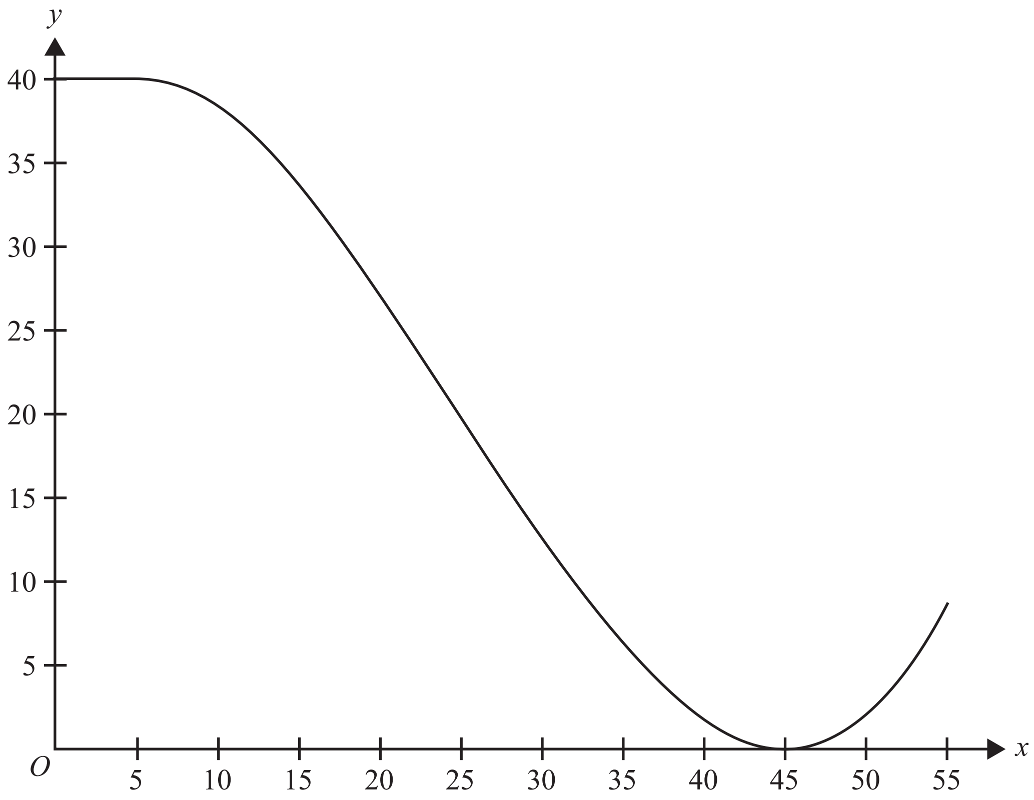
Jac and Jill have built a ramp for their toy car. They will release the car at the top of the ramp and the car will jump off the end of the ramp.

The cross-section of the ramp is modelled by the function $f$, where

$$f(x)=\begin{cases} 40 & 0 \le x < 5 \\ \dfrac{1}{800}\left(x^3-75x^2+675x+30\,375\right) & 5 \le x \le 55 \end{cases}$$

$f(x)$ is both smooth and continuous at $x = 5$.

The graph of $y = f(x)$ is shown below, where $x$ is the horizontal distance from the start of the ramp and $y$ is the height of the ramp. All lengths are in centimetres.



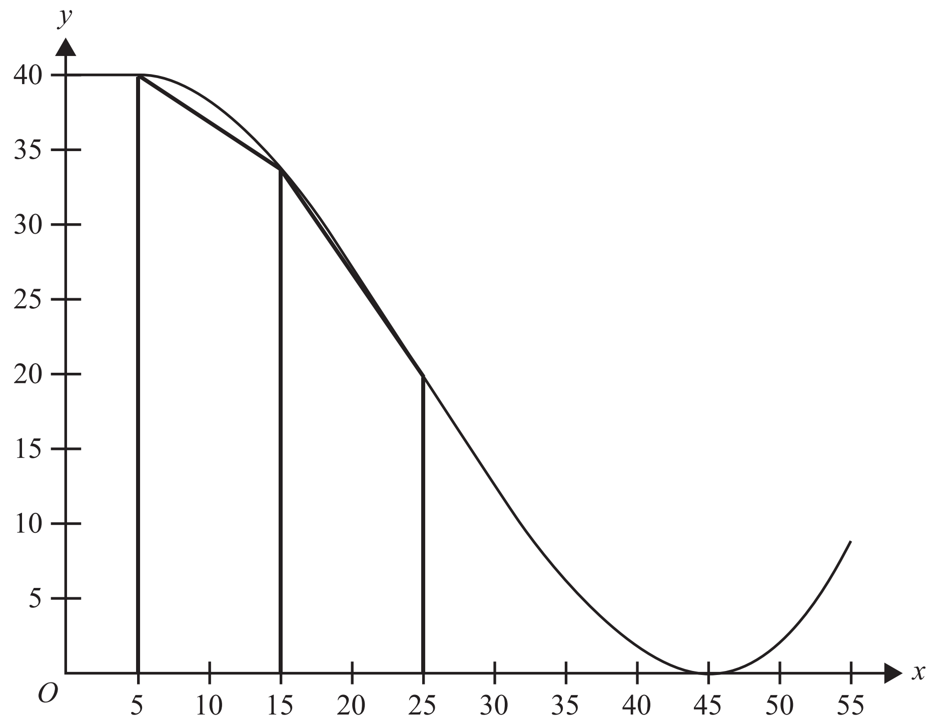

**a.** Find $f'(x)$ for $0 < x < 55$. 2 marks

**b.** **i.** Find the coordinates of the point of inflection of $f$. 1 mark

**ii.** Find the interval of $x$ for which the **gradient function** of the ramp is strictly increasing. 1 mark

**iii.** Find the interval of $x$ for which the **gradient function** of the ramp is strictly decreasing. 1 mark

Jac and Jill decide to use two trapezoidal supports, each of width 10 cm. The first support has its left edge placed at $x = 5$ and the second support has its left edge placed at $x = 15$. Their cross-sections are shown in the graph below.




**c.** Determine the value of the ratio of the area of the trapezoidal cross-sections to the exact area contained between $f(x)$ and the $x$-axis between $x = 5$ and $x = 25$. Give your answer as a percentage, correct to one decimal place. 3 marks

**d.** Referring to the gradient of the curve, explain why a trapezium rule approximation would be greater than the actual cross-sectional area for any interval $x \in [p, q]$, where $p \geq 25$. 1 mark

**e.** Jac and Jill roll the toy car down the ramp and the car jumps off the end of the ramp. The path of the car is modelled by the function $P$, where

$$P(x) = \begin{cases} f(x) & 0 \leq x \leq 55 \\ g(x) & 55 < x \leq a \end{cases}$$

$P$ is continuous and differentiable at $x = 55$, $g(x) = -\frac{1}{16}x^2 + bx + c$, and $x = a$ is where the car lands on the ground after the jump, such that $P(a) = 0$.

**i.** Find the values of $b$ and $c$. 2 marks

**ii.** Determine the horizontal distance from the end of the ramp to where the car lands. Give your answer in centimetres, correct to two decimal places. 1 mark

## Answers to multiple-choice questions

| Question | Answer |
|---|---|
| 1 | D |
| 2 | C |
| 3 | B |
| 4 | C |
| 5 | C |
| 6 | D |
| 7 | E |