2006 Specialist Maths GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat the 2006 examination was 5210, compared to 5625 in 2005. This year was the first of the new structure in which students answered 22 multiple-choice questions, and then five extended answer questions. The time allowed – two hours for the exam, which allowed students access to notes – seemed adequate, and most students attempted all parts of the paper. Students seemed well prepared for the standard questions but, as has been the case in previous years, questions or parts of questions which were 'outside the square' proved to be a challenge.

The mean and median total scores were 44 and 45 out of 80 respectively. Five students scored full marks compared with 13 in 2005. Although there were fewer perfect scores than in previous years, there seemed to be more accessible marks on the paper as a whole, with the middle 90% of the 2006 cohort scoring from 14 to 72 out of 80. In Section 2 the average score for the five questions, expressed as a percentage of the marks available for each question, was 53%, 54%, 57%, 39% and 45% respectively. This compares with 49%, 52%, 47%, 43%, and 53% in 2005. More detailed statistical information is published on the VCAA website.

This year there were eight 'show that' questions in Section 2 of the paper. Teachers should remind their students that this instruction is intended to help keep them on track and enable access to subsequent marks for later parts of the same question, even if the student could not 'show' a given result. For this type of question it is essential that students show all steps which lead to the given answer – the assessor needs to be convinced that the student has independently arrived at the stated result.

The paper also contained six 'hence' type questions. With this type of question, students **must** use the result obtained in the previous question part to answer the subsequent part of a question. To gain full marks, this instruction must be strictly complied with. Where a question states 'hence or otherwise' students are free to use the previous result or any other relevant method of their choice, as applicable.

It is again necessary to remind students that appropriate working must be shown in questions worth more than one mark, and that assessors must be able to see the steps used to reach an answer. Marks are usually allocated to a valid method which is clearly shown, not just to the final answer. It is not sufficient to just write down an answer for a question worth two or more marks.

New aspects to the course, such as direction fields and implicit differentiation, were not well handled. The examination also revealed other common areas of weakness experienced by many students. These included:

- understanding key definitions, such as 'equation of motion', and that the angle between two vectors is measured when the vectors are placed 'tail to tail'
- finding a vector which connects two given cartesian points
- finding the reciprocal of a sum of algebraic fractions
- basic skills in the areas of plotting complex numbers, their negatives and conjugates, applying double angled and related formulae and manipulating surds
- the requirements of 'hence', 'show that' and 'prove' type questions.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	9	9	71	8	3	1	
2	0	1	2	5	92	0	
3	4	4	2	88	2	0	
4	6	6	74	5	9	0	
5	28	39	12	9	12	1	Roots are separated by an argument of $\frac{2\pi}{5}$. So the real root is $a^{\frac{1}{5}}\text{cis}\left(\frac{5\pi}{5}\right)$, which is option B.
6	4	62	26	5	3	0	
7	9	5	16	6	63	0	
8	60	9	18	10	2	1	
9	12	2	80	5	1	0	
10	4	47	12	10	26	1	At time t the mass of undissolved chemical is $8-x$, so $\frac{dx}{dt}=(8-x)\times 5\%$, hence option B.
11	31	15	40	6	7	1	Only option C gives zero gradients of line segments on the y axis and correct signs of the gradients of the line segments in all four quadrants.
12	52	4	7	35	1	0	$v\frac{dv}{dx}=\sin(2x)\times 2\cos(2x)=\sin(4x)$ which is option D.
13	1	8	2	6	83	0	
14	42	50	3	3	2	0	
15	4	10	6	74	6	0	
16	2	16	8	49	24	1	Options D and E (only) represent unit vectors, and D is the one which is perpendicular, seen by evaluating the scalar product.
17	3	85	6	4	2	0	
18	81	12	2	3	3	0	
19	62	3	26	2	6	0	
20	6	28	10	50	5	1	
21	68	8	13	6	5	0	
22	8	8	4	6	74	0	

The mean score for the multiple-choice section was 14.26 out of 22 and the standard deviation was 4.57.

Just five questions (Questions 5, 10, 11, 12 and 16) were answered correctly by less than 50% of students, with Question 12 being the only question where the majority of students selected a particular option which was incorrect. This could have occurred where students simply differentiated the given expressions with respect to u, confusing $\frac{dv}{dx}$ with acceleration, instead of using $a=\frac{dv}{dt}=\frac{dv}{dx}\times\frac{dx}{dt}=v\frac{dv}{dx}$. This form is given on the formula sheet.

Section 2

Question 1a.

Marks	0	1	2	Average
%	13	8	80	1.7

$$\int_0^5 \pi\left(\frac{6x}{\sqrt{1+x^3}}\right)^2 dx$$

This question was quite well done. Common errors were the omission of π, neglecting to square y, and the occasional omission of dx or the terminals.

Question 1b.

Marks	0	1	2	Average
%	24	25	51	1.3

$$\pi\int_1^{126} \frac{12}{u}\,du$$

This question was reasonably well done. Many students did not change the terminals, some retained a mix of x and u in their answers and du was occasionally left out.

Question 1c.

Marks	0	1	Average
%	48	52	0.6

182

This was well done by those who set up a correct integral. A few students did not give the answer numerically, correct to the nearest cubic cm. Some evaluated the integral of Question 1b. directly, whilst others used the numerical integration feature of their calculators to evaluate the integral of Question 1a.

Question 1d.

Marks	0	1	Average
%	25	75	0.8

$$\frac{dy}{dt} = \frac{6-3x^3}{\left(1+x^3\right)^{\frac{3}{2}}}\times 2$$

Most students applied the chain rule correctly; however, some divided by two rather than multiplying.

Question 1e.

Marks	0	1	2	3	Average
%	65	8	5	22	0.9

$$\frac{dA}{dt} = \frac{24\pi x\left(6-3x^3\right)}{\left(1+x^3\right)^2}$$

This question was not very well done. Many students did not realise that y gave the radius of the circular surface area of the wine. Some did not use 'hence' and tried to use $\frac{dA}{dx}\times\frac{dx}{dt}$ to find $\frac{dA}{dt}$, which was a far more complicated approach.

Question 1f .

Marks	0	1	Average
%	72	28	0.3

$\sqrt[3]{2}$

Many students did not attempt this last part of Question 1. Some gave the answer as $\sqrt{2}$. Many did not realise that the answer could be obtained by equating $\frac{dA}{dt}$ to zero given in part e.

Question 2a.

Marks	0	1	2	Average
%	16	14	70	**1.6**

$\overrightarrow{AC} = 6\underset{\sim}{i}$, $\overrightarrow{BD} = 10\underset{\sim}{j}$ A scalar product of these two vectors gives zero.

Quite a few students had difficulties finding $\overrightarrow{AC}$ and $\overrightarrow{BD}$. A common error was to say $\overrightarrow{AC} = \underset{\sim}{a} - \underset{\sim}{c}$ and so on.

$\overrightarrow{AC} = \overrightarrow{OA} + \overrightarrow{OC}$ was also sometimes seen. Many students just stated that $\overrightarrow{AC}$ and $\overrightarrow{BD}$ were perpendicular without demonstrating or explicitly saying why. On the issue of notation, quite a few students used the standard multiplication sign ($\times$) to show a scalar product instead of the correct 'dot' symbol.

Question 2b.

Marks	0	1	2	3	Average
%	16	10	13	62	**2.3**

$\cos(\angle ADC) = \frac{4}{5}$

This question was generally well done. Quite a few students proceeded further to find $\angle ADC$. Some did not state $\cos(\angle ADC)$ at all, but instead gave an approximation to $\angle ADC$. A number of students had difficulty obtaining correct expressions for $\overrightarrow{DA}$ and $\overrightarrow{DC}$ to use in their scalar product formula.

Question 2c.

Marks	0	1	2	Average
%	47	38	16	**0.7**

$\cos(\angle ABC) = -\frac{4}{5}$

Many students experienced the same sorts of difficulties detailed above when attempting to find $\cos(\angle ABC)$. Few managed to *show* $\angle ABC$ and $\angle ADC$ were supplementary. Most resorted instead to finding numerical approximations of the two angles in an attempt to show they were supplementary.

Some capable students answered $\cos^{-1}\left(-\frac{4}{5}\right) = \frac{\pi}{2} + \sin^{-1}\left(\frac{4}{5}\right)$ (as $\angle ABC$ is in the second quadrant), so

$\cos^{-1}\left(-\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2} + \sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$, hence the angles are supplementary.

Question 2d.

Marks	0	1	2	3	Average
%	44	33	3	20	**1.0**

$\cos(\angle APC) = \frac{7}{25}$ and $\cos(2\times\angle ADC) = 2\times\left(\frac{4}{5}\right)^2 - 1 = \frac{7}{25}$

Many students found that $\cos(\angle APC) = \frac{7}{25}$, but only a minority attempted to use a double angle formula to show the given result. A large number again resorted to finding numerical approximations to the two angles to try to *prove* $\angle APC = 2\times\angle ADC$.

Question 3ai.

Marks	0	1	Average
%	7	93	**1.0**

$105\,600 = 48\,000a$, so $a = 2.2 \text{ ms}^{-2}$

This question was very well done, with most students getting the mark.

Question 3aii.

Marks	0	1	2	Average
%	9	3	88	**1.9**

31.8 s

This question was also very well done with nearly all students able to apply the basic kinematics formula. A minority gave their answer to the nearest second instead of the nearest one tenth of a second.

Question 3aiii.

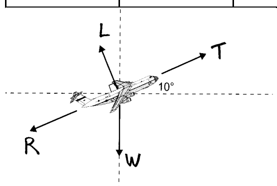
Marks	0	1	2	Average
%	12	26	62	**1.6**

1114 m

The major error in this question was using the **rounded** answer from part ii. An exact answer for *t*, or at least one with more accuracy than one decimal place, was needed to give the answer correct to the nearest metre.

Question 3bi.

Marks	0	1	Average
%	10	90	**0.9**



This question was well done by most. A few students had *R* and *T* interchanged. Some others insisted on introducing a μN term in various places. *Wg* was sometimes used erroneously instead of *W*.

Question 3bii.

Marks	0	1	2	Average
%	39	27	34	**1.0**

$T - W\sin(10^\circ) - R = 0$, $L - W\cos(10^\circ) = 0$

This question was not particularly well done. Rather than resolving parallel and perpendicular to the direction of motion of the jet, many students resolved in the vertical and horizontal directions, yielding a more complicated pair of equations. A number of students had their left sides equal to *ma*, ignoring the fact that the jet was travelling at constant velocity. $R = T$ was a common error. Interchanging of sine and cosine also occurred.

Question 3biii.

Marks	0	1	Average
%	58	42	**0.5**

463 254

Those who managed to deduce the second of the equations in part bii. generally got this mark. Otherwise, a lot of extended algebra was attempted to determine this answer to this single mark question. A number of students used a weight force of $W = 48\,000$, omitting *g*.

Question 3ci.

Marks	0	1	Average
%	62	38	0.4

$-80\,000-5v^2-500(80-v)=48\,000a$

A substantial number of students did not seem to know what was meant by 'equation of motion'. A common error was omitting the negative sign from each of the terms on the left side of the equation. Extra terms involving '80' were sometimes introduced.

Question 3cii.

Marks	0	1	2	3	Average
%	58	6	18	18	1.0

$\int_{80}^{10}\frac{9600v}{100v-v^2-24000}\,dv$ or equivalent integral.

It was pleasing to see that a large number of students realised that they needed to replace acceleration in their equation of motion with $v\frac{dv}{dx}$. However, a lesser number rearranged their terms to express x as an integral in terms of v. Frequent errors involved the terminals from 0 to 10, or from 10 to 80. Sign errors and reciprocal errors were also prevalent.

Question 3ciii.

Marks	0	1	Average
%	80	20	0.2

1385m

Those students who set up a correct integral in part cii. generally managed to evaluate it numerically. However many did not attempt this last part of Question 3.

Question 4a.

Marks	0	1	2	Average
%	20	18	63	1.5

$t=\int\frac{1}{1-y}\,dy$, which gives the stated result.

This was generally well done; however, a number of students just threw in the modulus sign at the very end as an afterthought.

Most students realised that they needed to invert the differential equation to integrate; however, sometimes the negative sign was omitted from the result.

Question 4b.

Marks	0	1	2	Average
%	41	7	52	1.2

$\frac{dx}{dt}=\frac{1}{y}$, $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ $=(1-y)\times y$

The majority of students managed to write down $\frac{dx}{dt}=\frac{1}{y}$, but linking it in with the chain rule to *show* the given result was a challenge for many.

Question 4ci.

Marks	0	1	Average
%	79	21	0.2

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\left(y-y^2\right)}{dy} \times y(1-y) = (1-2y)y(1-y)$$

Most students struggled with this 'show that' question. Implicit differentiation was not handled at all well, and quite a number of students tried to erroneously apply $\frac{d^2y}{dx^2} = 1 \div \frac{d^2x}{dy^2}$ in their workings.

Question 4cii.

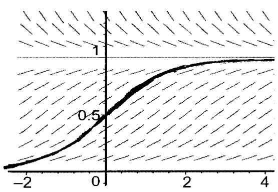
Marks	0	1	2	Average
%	69	23	8	0.4

$y = 0.5$ only

A large number of students included $y = 0$ and $y = 1$ as the y coordinates of the points of inflection. Only a few attempted to verify the point of inflection. Most thought that it was sufficient to show that the second derivative needed to be zero in order to establish a point of inflection.

Question 4d.

Marks	0	1	Average
%	72	28	0.3



Most students did not have a sufficient understanding of the concept of 'slope field'. A large number drew a horizontal line through $y = 0.5$. Others had extra curves drawn in, and some had their curve crossing the horizontal asymptotes at $y = 0$ and $y = 1$.

Question 4e.

Marks	0	1	2	Average
%	62	22	16	0.6

$\frac{21}{16}$

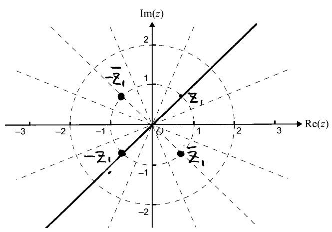
Given $x_0 = 0,\ y_0 = 2,\ h = \frac{1}{4}$ and $\frac{dy}{dx} = y(1-y)$, then $y_1 = 2 + \frac{1}{4} \times 2 \times (-1) = 1.5$ and $y_2 = 1.5 + \frac{1}{4} \times \frac{3}{2} \times \left(-\frac{1}{2}\right)$

$= \frac{21}{16} = 1.3125$

A large number of students made attempts using Euler's method, but most encountered difficulties because the differential equation was in terms of y. This was a two mark question where a single answer on its own was not sufficient.

Question 5ai.

Marks	0	1	2	Average
%	23	20	57	1.4



This question was reasonably well done, with the most common errors being points not correctly labelled, points placed on the wrong circle and points represented as rays from the origin.

Question 5aii.

Marks	0	1	Average
%	85	15	**0.2**

$|z - \bar{z}_1| = |z + \bar{z}_1|$

Only a few students obtained the correct answer. A number struggled with other cartesian forms of the line, but many just did not make an attempt at this part.

Question 5b.

Marks	0	1	2	3	Average
%	44	7	20	29	**1.4**

Solving the equation $\cos\left(\frac{\pi}{4}\right) = 2\cos^2\left(\frac{\pi}{8}\right) - 1$ for $\cos\left(\frac{\pi}{8}\right)$ gave the stated result.

A large number of students made an attempt at using the double angle formula, but often angles were confused, with $\frac{\pi}{8}$ and $\frac{\pi}{16}$ appearing instead of $\frac{\pi}{4}$ and $\frac{\pi}{8}$. Many did not explain adequately why the negative solution to their quadratic in $\cos\left(\frac{\pi}{8}\right)$ was rejected. A number of students tried to use an addition formula.

Question 5c.

Marks	0	1	2	Average
%	51	13	37	**0.9**

$\sin^2\left(\frac{\pi}{8}\right) = 1 - \cos^2\left(\frac{\pi}{8}\right)$ yields the stated result.

A large number of students ignored the ‘hence’ requirement of this question and did not use the answer for $\cos\left(\frac{\pi}{8}\right)$.

This was another ‘show that’ question where the correct steps in manipulating surds needed to be seen.

Question 5d.

Marks	0	1	2	Average
%	41	11	49	**1.1**

$\text{cis}\left(\frac{7\pi}{8}\right)$

A significant number of students saw the connection of this part with parts b. and c., although some found the polar form from scratch before applying De Moivre’s theorem. A minority expressed their answer back in cartesian form.

Question 5e.

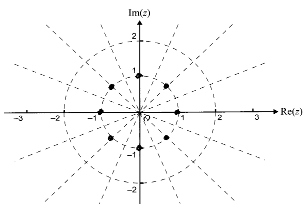
Marks	0	1	2	Average
%	65	23	12	**0.5**

$n = 8k,\ k \in Z$ (the set of integers)

This question proved to be difficult for most students. Some realised that the imaginary part of the expression would have to be zero, and others managed to find a few values by trial and error. A large number of students who did manage to do something with this question found only those *n* values which were positive integers or zero.

Question 5f.

Marks	0	1	2	Average
%	40	7	54	**1.2**



This question was handled reasonably well. Common errors involved placing points on the wrong circle, using rays from the origin to show complex numbers, and not plotting all roots. A number of students simply did not answer this question.