Level 9 – Measurement and geometry

Overview

Task name	When three sides work
Learning intention	To explore Pythagoras's theorem and the relationship between the sides of triangles and their lengths
Duration	30 minutes

Links to the Victorian Curriculum

These work samples are linked to Level 9 of the Mathematics curriculum.

Extract from achievement standard

Apply Pythagoras's theorem … to solve problems involving … lengths in right-angled triangles.

Relevant content description

- Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (VCMMG318)

Links to NAPLAN

Minimum standards – numeracy

Year 9: Classification and properties of shapes

Students can classify 2D shapes … according to common properties including … sides, perimeters, areas …

Student work samples – Constructing triangles

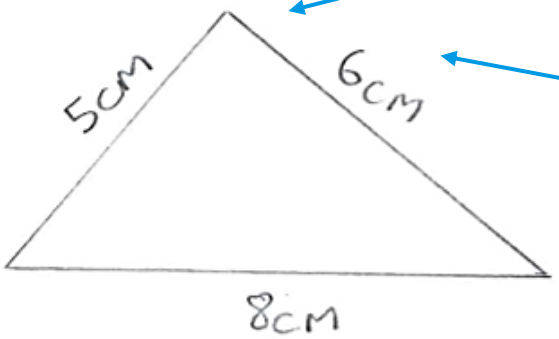
These work samples were created by students working at Level 9. Evidence of student achievement has been annotated.

Victorian Curriculum link

Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (VCMMG318)

Part 1

a. Construct a triangle with side lengths of 5, 6 and 8 centimetres.

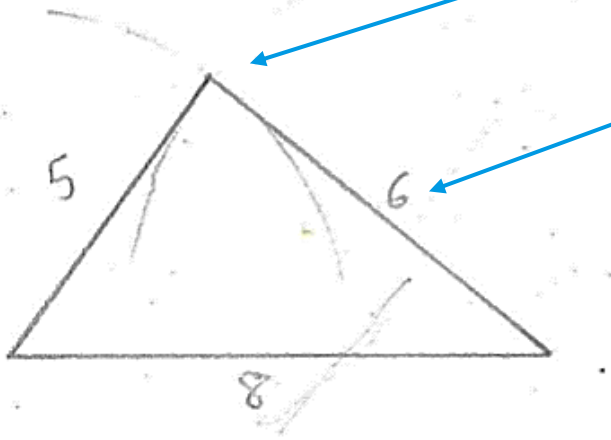


Uses a ruler to draw a triangle with required side lengths

Labels sides with correct measurements, including units

Part 1

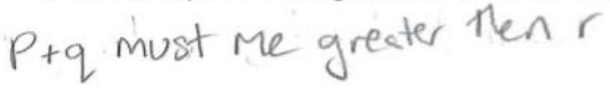
a. Construct a triangle with side lengths of 5, 6 and 8 centimetres.



Uses a ruler and compass to construct a triangle with required side lengths

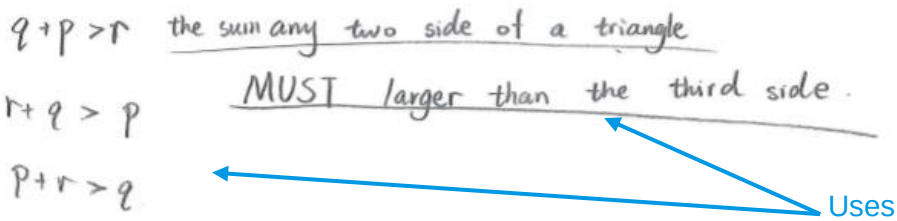
Labels sides with correct measurements but without units

b. Let p, q and r be three positive integers. Find a relation between p, q and r which means that these numbers could represent the lengths of the three sides of a triangle, for any kind of triangle.



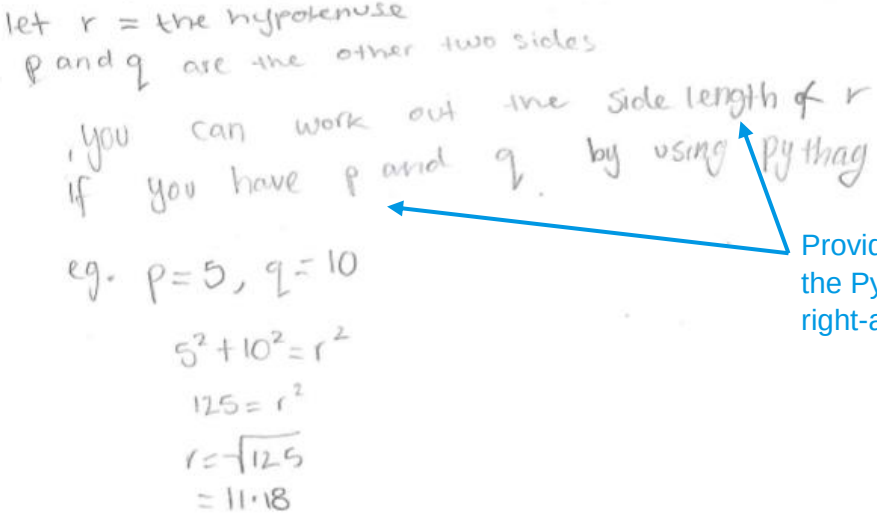
Uses symbols and words to specify the relation, assuming r corresponds to the longest side

b. Let p, q and r be three positive integers. Find a relation between p, q and r which means that these numbers could represent the lengths of the three sides of a triangle, for any kind of triangle.



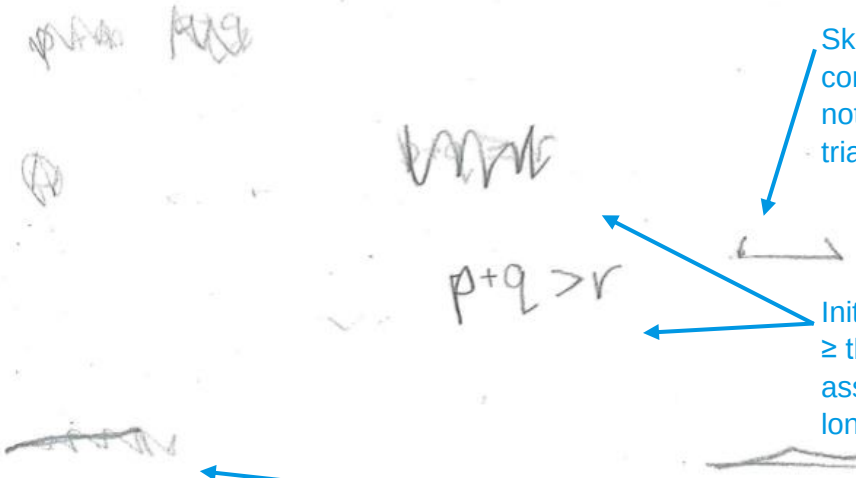
Uses symbolic expressions including inequality to write a set of relations independent of the relative magnitudes

b. Let p, q and r be three positive integers. Find a relation between p, q and r which means that these numbers could represent the lengths of the three sides of a triangle, for any kind of triangle.



Provides a specific example of the Pythagorean relation for right-angled triangles

b. Let p, q and r be three positive integers. Find a relation between p, q and r which means that these numbers could represent the lengths of the three sides of a triangle, for any kind of triangle.



Sketches a case for which the corresponding values would not form the side lengths of a triangle

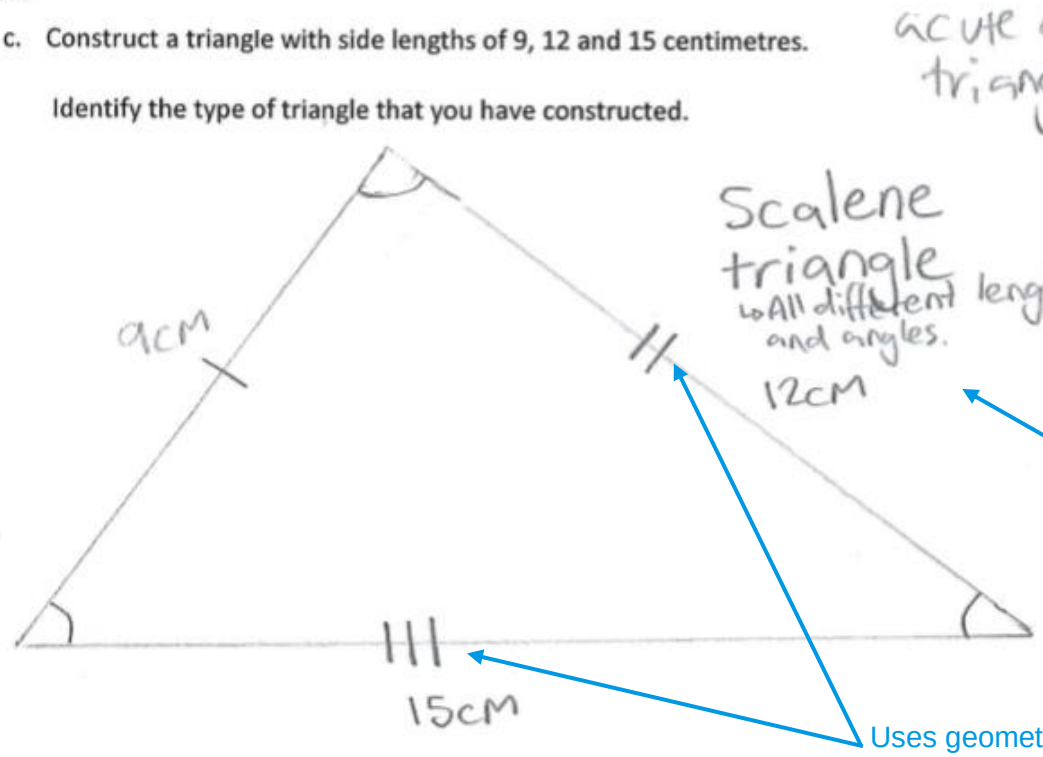
Initially writes a relation using ≥ then changes this to > assuming r corresponds to the longest side

Sketches a case corresponding to a collapsed triangle

Part 2

c. Construct a triangle with side lengths of 9, 12 and 15 centimetres.

Identify the type of triangle that you have constructed.



Identifies the triangle as scalene with acute angles

Uses geometric conventions to indicate angles and sides

Part 2

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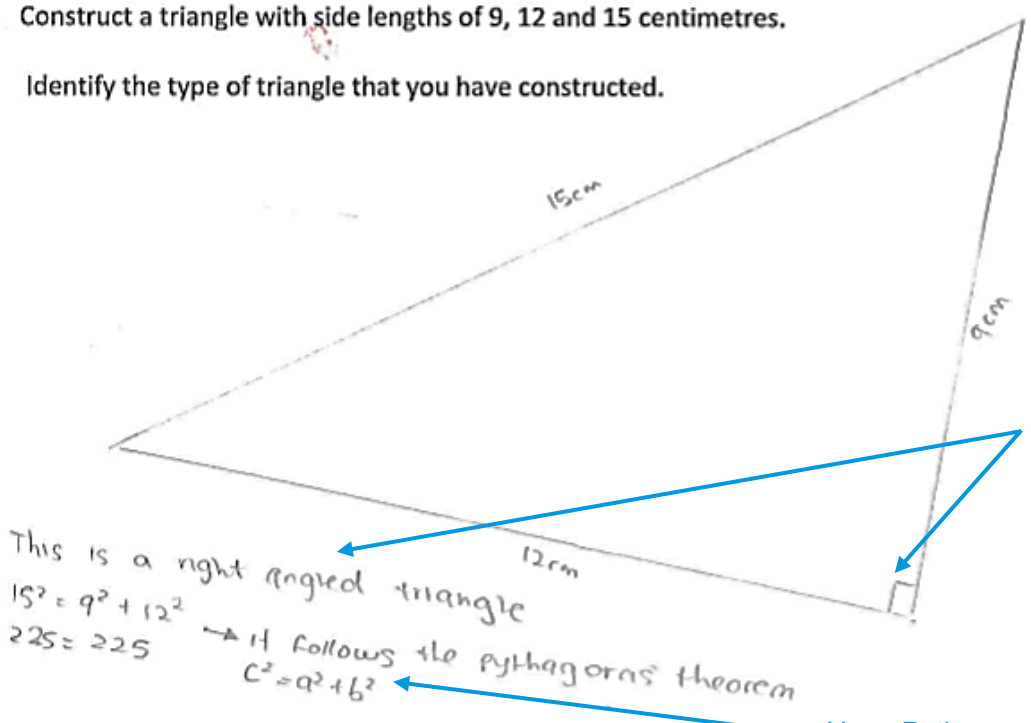
Identify the type of triangle that you have constructed.



Identifies a right-angled triangle

Uses Pythagoras's theorem to verify the triangle is right-angled

Part 2

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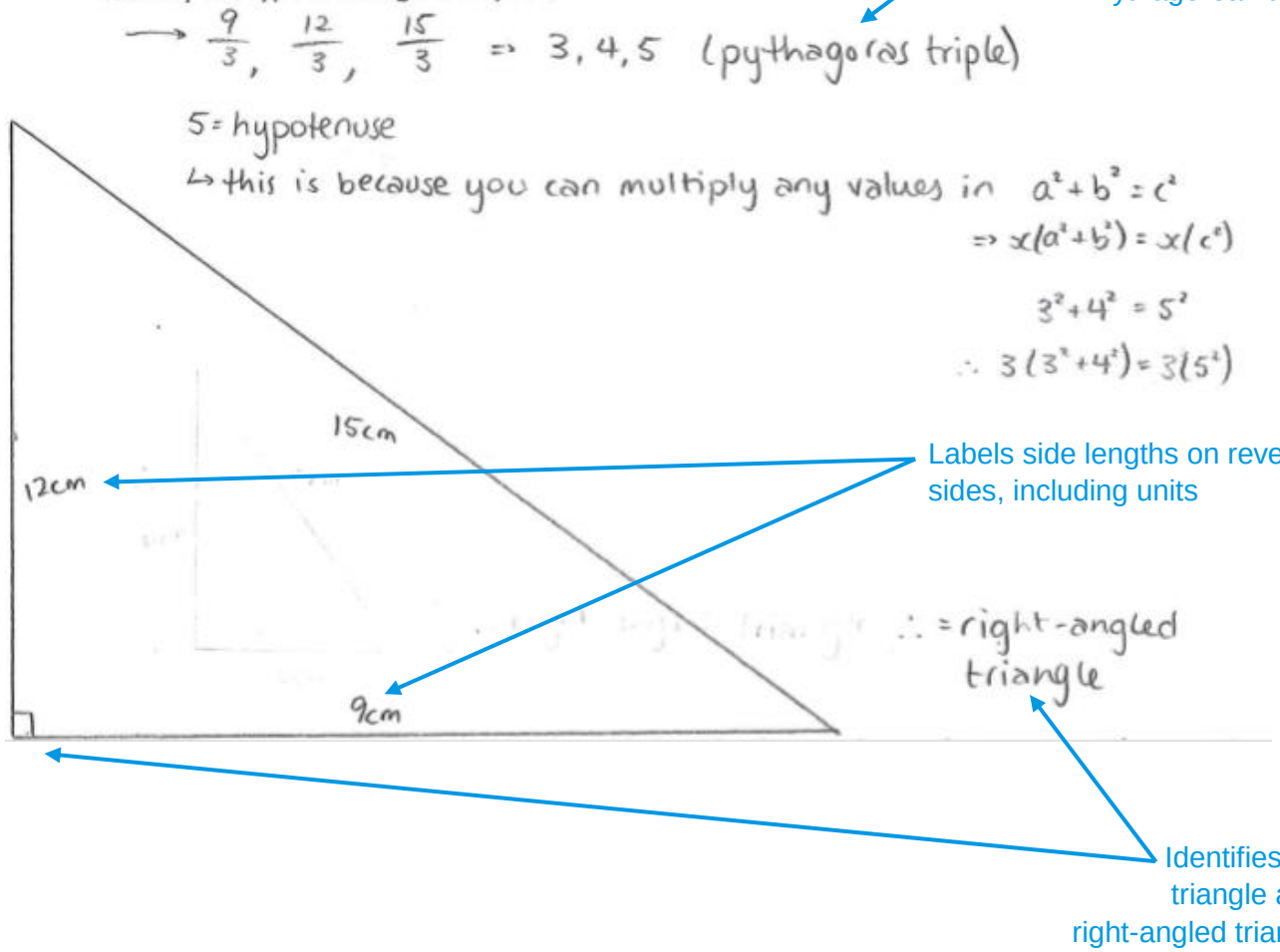

Identifies and verifies the triangle as a scaling of the 3, 4, 5 Pythagorean triple

Labels side lengths on reverse sides, including units

Identifies the triangle as a right-angled triangle

Part 2

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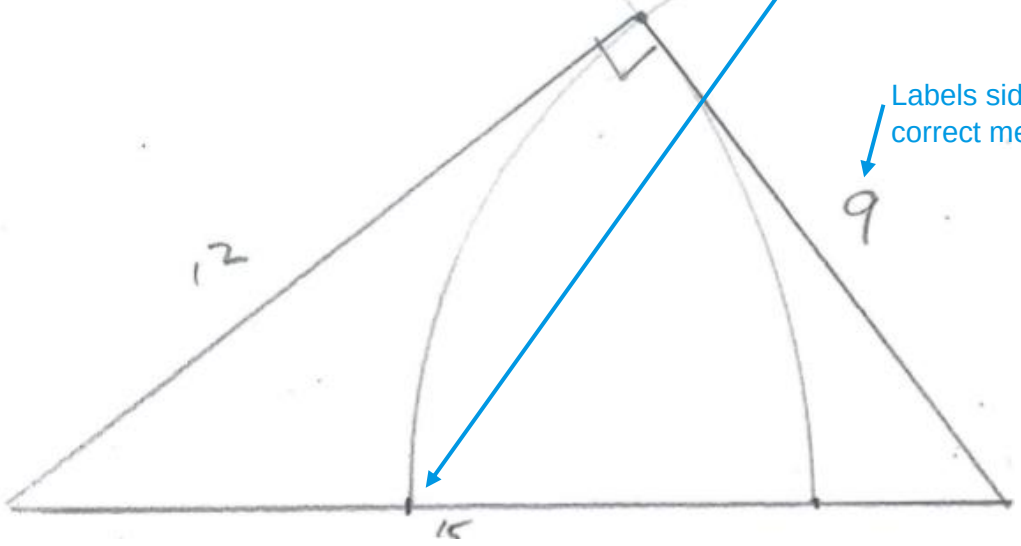
Identify the type of triangle that you have constructed.



Uses a compass and ruler to construct a right-angled triangle with right angle at the apex and hypotenuse at the base

Labels side lengths with correct measurements

d. Find some other combinations of integer side lengths that can be used to construct the same type of triangle and explain why this is the case.

$3^2 + 4^2 = 5^2$ — we know that this is true

$\therefore (3\times2)^2+(4\times2)^2 = (5\times2)^2$

$3^2\times4 + 4^2\times4 = 5^2\times4$

this can then be factorised:

$4(3^2+4^2) = 4(5^2)$

$3^2+4^2 = 5^2$

∴ we can multiply all sides by the same number, and still have a pythagoras triple:

×2 ⇒ 6, 8, 10 cm

×4 ⇒ 12, 16, 20 cm

×5 ⇒ 15, 20, 25 cm

×6 ⇒ 18, 24, 30 cm

etc. (every set of lengths of form $3x, 4x, 5x$) where x is any positive value

Uses a particular example to show the original relation is true with scaling

Lists a set of triangles based on scaling from a 3, 4, 5 triangle

Provides a general form

d. Find some other combinations of integer side lengths that can be used to construct the same type of triangle and explain why this is the case.

$9^2 + 12^2 = 15^2$

9, 12, 15 are part of the 3,4,5 type of pythagorean triads.

Assuming similar triangles:

3,4,5

6,8,10

12,16,20

15,20,25

etc

other pythagorean triads

6,12,13 etc

8,15,17 etc

7,24,25 etc

Identifies a set of similar right-angled triangles based on scaling from a 3, 4, 5 triangle

Identifies other combinations (Pythagorean triads) that can similarly generate other related triangles

Where to next for the teacher?

When the task on which these annotated student work samples is based has been used as a classroom activity, there is opportunity to gather data on student achievement to help inform further teaching.

An analysis of student responses, on an individual, group or whole class basis, can be used to develop and direct student learning with respect to the following content.

For students needing to review underpinning knowledge and skills at Level 8

- Develop the conditions for congruence of triangles (VCMMG292)
- Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (VCMMG293)

For students consolidating knowledge and skills at Level 9

- Apply trigonometry to solve right-angled triangle problems (VCMMG320)
- Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (VCMMG319)

For students moving on to new knowledge and skills at Level 10

- Solve right-angled triangle problems including those involving direction and angles of elevation and depression (VCMMG346)

Resources

- Numeracy Learning Progressions, Victorian Curriculum and Assessment Authority (VCAA) – The Numeracy Learning Progressions amplify, extend and build on the numeracy skills in the Victorian Curriculum F–10: Mathematics and support the application of numeracy learning within other learning areas.
- FUSE, Victorian Department of Education and Training (DET) – The FUSE website provides access to digital resources that support the implementation of the Victorian Curriculum F–10, including an extensive range of activities and other resources for Primary Mathematics and Secondary Mathematics.
- Mathematics Curriculum Companion, Victorian Department of Education and Training (DET)
- Aligned Australian Curriculum Resources (Mathematics), Australian Curriculum, Assessment and Reporting Authority (ACARA)