Teaching and learning unit: 8.2.7 Pythagoras’ theorem and applications

Mathematics, Level 8

# Overview, including Victorian Curriculum F–10 links

| **Description of the teaching and learning unit** | **Cohort considerations (in relation to this teaching and learning unit)** |
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| This unit covers:* the recognition, demonstration and proof of Pythagoras’ theorem, and its use to calculate side lengths in right-angled triangles
* application of the theorem to solve practical problems involving lengths and distances in the plane
* Pythagorean triples.

The unit is planned as 8 lessons of 45–50 minutes duration over a period of approximately 2 weeks.  | This topic involves a combination of graphical, numerical and symbolic representations related to triangles, squares as shapes and the relation of these shapes to areas and side lengths involving squares and square roots of numbers.Relevant data on student background could be obtained from NAPLAN Year 7 data or DAL assessments, as well as school data on prior learning and achievement.Manipulatives, measurement, drawing and digital tools can be used to support differentiation for the diversity of student learner backgrounds and needs, as well as the inclusion of some open and rich tasks.The work in this topic uses scaled diagrams, and graph paper, as well as ‘blank pages’, and physical models can be used assist students as applicable in construction work.Where students have difficulty with computational and algebraic processes, this can be scaffolded by the complementary use of various online calculation tools that also demonstrate the working and steps in solution processes. |

## Content description links and associated achievement standard links

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| **Content description link(s)** | **Achievement standard link(s)** |
| use Pythagoras’ theorem to solve problems involving the side lengths of right-angled triangles VC2M8M06 | They use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles. |
| recognise irrational numbers in applied contexts, including *π* and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbersVC2M8N01 | By the end of Level 8, students recognise irrational numbers as numbers that cannot develop from the division of integer values by natural numbers and terminating or recurring decimals. |

## Continuum of learning

### Achievement standards

| **Level 7** | **Level 8** | **Level 9** |
| --- | --- | --- |
| Students apply knowledge of angle relationships and the sum of angles in a triangle to solve problems, giving reasons. | They use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles. | Students apply Pythagoras’ theorem and use trigonometric ratios to solve problems involving right-angled triangles. |
| They solve problems involving squares of numbers and square roots of perfect square numbers. | By the end of Level 8, students recognise irrational numbers as numbers that cannot develop from the division of integer values by natural numbers and terminating or recurring decimals. | … students recognise and use rational and irrational numbers to solve problems. |

### Content descriptions

| **Level 7** | **Level 8** | **Level 9** |
| --- | --- | --- |
| demonstrate that the interior angle sum of a triangle in the plane is 180° and apply this to determine the interior angle sum of other shapes and the size of unknown angles VC2M7M05 | use Pythagoras’ theorem to solve problems involving the side lengths of right-angled triangles VC2M8M06 | solve spatial problems, applying angle properties, scale, similarity, ratio, Pythagoras’ theorem and trigonometry in right-angled triangles VC2M9M03 |
| describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems VC2M7N01 | recognise irrational numbers in applied contexts, including π and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbers VC2M8N01 | recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers with and without using digital tools VC2M9N01 |

## Other curriculum links and notes

<Links to the Victorian Curriculum F–10 Version 2.0 cross-curriculum priorities and capabilities will be added to this example document after these curriculum areas are published.>

# Essential questions

| **Essential questions to foster inquiry, understanding and transfer of learning** |
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| When can three numbers be used as the side lengths of a triangle?What is the relationship between the side lengths of a right-angled triangle?How are distances or lengths calculated in the plane?How can right-angled triangles be used to model situations and solve practical problems?How are mathematical results shown to hold in general? |

# Assessment and learning sequence details

| **Assessment task name and type** | **Link to achievement standard** | **Moderation**  |
| --- | --- | --- |
| Activity: Triangle review, formative assessment, Lesson 1 | Level 7: They classify polygons according to their features and design an algorithm to sort and classify shapes. | NA |
| Activity: Finding lengths, formative assessment, Lesson 4 | They use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles.  | NA |
| End-of-unit task, summative assessment, Lesson 8(VCAA Level 8 example assessment task – Pythagoras’ theorem) | They use Pythagoras’ theorem to solve measurement problems involving unknown lengths of right-angled triangles.  | This task could be moderated across classes by using either a rubric with assessment criteria and comparison of samples or by using a marking scheme and cross-marking of selected samples within mark ranges. |

| **Week** | **Lesson**  | **Learning goal (e.g. learning intention and success criteria)** | **Lesson elements**  | **Differentiation and/or individualisation** | **Assessment** | **Resources** |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | **Learning intention:**We are learning to construct and label triangles and squares using their dimensions and properties.**Success criterion 1:** I can construct triangles and squares of specified dimensions using drawing tools and software.**Success criterion 2:**I can name and label these shapes accurately using mathematical conventions. | This lesson involves review of types of triangles (acute, right, oblique, equilateral isosceles, scalene) and diagram labelling conventions for showing vertices, side lengths, angles, equal sides and angles, and a right angle.Construction, using compass and straight edge and dynamic geometry software, of a triangle given the lengths of its three sides:* an equilateral triangle, for example three sides of 8 cm length
* an isosceles triangle, for example two sides of 7 cm length and a third side of 5 cm length
* a non-right-angled scalene triangle, for example side lengths of 7 cm, 8 cm and 11 cm
* a right-angled scalene triangle, for example side lengths of 5 cm, 12 cm and 13 cm.

Followed by construction of:* a perpendicular to a line
* a right-angled triangle given the lengths of its two shorter sides.
* a square of given side length, for example 9 cm.
 | A range of manipulatives, measuring and drawing tools, graph paper and dynamic geometry software can be used to provide differentiated scaffolding for students as applicable.The activities provide opportunity for guided exploration of concepts, skills and processes. | Activity: Triangle review. Involves students identifying different types of triangles and labelling them according to their properties. | [Classify Triangles by Sides and Angles – GeoGebra](https://www.geogebra.org/m/V39WMWrx)[Construction – Australian Mathematical Sciences Institute (amsi.org.au)](https://amsi.org.au/teacher_modules/Construction.html)[Constructing Triangles – NRICH (maths.org)](https://nrich.maths.org/8098) |
| 1 | 2 | **Learning intention:**We are learning to recognise when three numbers can represent the lengths of the sides of a triangle (the triangle inequality) and when they represent the lengths of the sides of right-angled triangles (Pythagoras’ theorem). **Success criterion 1:** I can recognise when three numbers can be used as the side lengths of a triangle and identify special cases of these (equilateral, isosceles, right-angled). **Success criterion 2:**I can use the squares of side lengths of triangles to identify some right-angled triangle. | This lesson involves exploration of when a set of three natural numbers can be used as the side lengths of a triangle or not (the triangle inequality: when the sum of the two smaller numbers is greater than the largest number). Include example of the various types of triangles.Comparison of the squares of side lengths for a variety of acute, right and obtuse triangles at a specified corner point of a triangle using dynamic geometry software, leading to the conjectures that:* for an acute angle, the square of the side opposite the angle is less than the sum of the squares of the two adjacent sides
* for a right angle the square of the side opposite the angle is equal to the sum of the squares of the two adjacent sides
* for an obtuse triangle the square of the side opposite the angle is greater than the sum of the squares of the two adjacent sides.
 |  |  | [Triangle Maker – GeoGebra](https://www.geogebra.org/m/JHgTXKrt)[Pythagorean Data – GeoGebra](https://www.geogebra.org/m/P3m5yqMF#material/V2SMwRr2) |
| 1 | 3 | **Learning intention:**We are learning about the connection between algebraic and geometric representations of Pythagoras’ theorem and follow a simple visual demonstration or proof of the theorem.**Success criterion 1:** I can use drawing tools and software to draw a right-angled triangle representation of Pythagoras’ theorem.**Success criterion 2:**I can follow and explain the steps in a demonstration or proof of Pythagoras’ theorem. | This lesson involves statement of the theorem, following the structured investigation of Lesson 1 leading to:* For a right angle the square of the side opposite the angle is equal to the sum of the squares of the two adjacent sides.

Algebraic representation of the theorem:Algebraic representation of the theorem: For a right angle the square of the side opposite the angle is equal to the sum of the squares of the two adjacent sides.Note that in its common equation form*a*2 + *b*2 = *c*2*c* is the side opposite the right angle (the hypotenuse), and *a* and *b* are the other two sides.Demonstration of the theorem and visual proof; for example, the following is an accessible visual proof that requires no algebra.Visual proof of the theoremThe following reference contains several animations that can be used to demonstrate the theorem (see the section ‘Proofs by dissection and rearrangement’): [Pythagorean theorem - Wikipedia](https://en.wikipedia.org/wiki/Pythagorean_theorem)View several historical representations of the Pythagorean theorem in different societies and cultures (Babylon, Egypt, China, India). |  |  | [Pythagorean Theorem and its many proofs – Cut the Knot (cut-the-knot.org)](https://www.cut-the-knot.org/pythagoras/)[Pythagorean Rearrangement – GeoGebra](https://www.geogebra.org/m/P3m5yqMF#material/crYP6qHj)[Pythagorean Theorem – History of Mathematics Project (history-of-mathematics.org)](https://www.history-of-mathematics.org/PythagoreanTheorem.html) |
| 1 | 4 | **Learning intention:**We are learning to use Pythagoras’ theorem to calculate the third side length of a right-angled triangles given the other two side lengths. **Success criterion 1:** I can calculate the length of the hypotenuse of a right-angled triangle given the lengths of the two shorter sides, using square roots, and provide a decimal approximation.**Success criterion 2:**I can calculate the length of one of the shorter sides of a right-angled triangle given the length of the hypotenuse and the length of the other short side, using square roots, and provide a decimal approximation. | This lesson involves application of the theorem – given the lengths of any two sides of a right-angled triangle, calculate the length of the third side. Examples and practice:Problem type 1: Given the lengths of the two shorter sides, calculate the length of the hypotenuse:$$c=\sqrt{a^{2}+b^{2}}$$Problem type 2: Given the length of one of the shorter sides and the length of the hypotenuse, calculate the length of the other shorter side:$a=\sqrt{c^{2}-b^{2}}$ or $b=\sqrt{c^{2}-a^{2}}$Use of substitution into the formula, rearrangement and the use of technology to calculate approximate values for resultant square roots to a specified accuracy.Use of scale diagrams of right-angled triangles to compare measured values with calculated values and numerical approximations (1 cm graph paper is helpful for this activity). |  | Activity: Finding lengths. Involves Problem type 1 and Problem type 2 questions with approximate answers.  | [Pythagoras’ theorem – Part 1 – KS3 Maths – BBC Bitesize (bbc.co.uk)](https://www.bbc.co.uk/bitesize/topics/z93rkqt/articles/zf8mp9q)[Pythagoras’ theorem – Part 2 – KS3 Maths – BBC Bitesize (bbc.co.uk)](https://www.bbc.co.uk/bitesize/topics/z93rkqt/articles/zgf8ng8?topicJourney=true) |
| 2 | 5 | **Learning intention:**We are learning to solve practical problems involving right-angled triangles using Pythagoras’ theorem.**Success criterion 1:** I can formulate a practical problem that involves finding the hypotenuse of a right-angled triangle and solve it using Pythagoras’ theorem.**Success criterion 2:**I can formulate a practical problem that involves finding one of the shorter sides of a right-angled triangle and solve it using Pythagoras’ theorem. | This lesson involves application of Pythagoras’ theorem to practical problems modelled using right-angled triangles.Solving practical problems based on Problem type 1 and Problem type 2.Plotting coordinates of points in the Cartesian plane and calculating the distance between them by drawing a suitable right-angled triangle and applying Pythagoras’ theorem. |  |  | [Pythagoras' Theorem and Trigonometry – Short Problems (maths.org)](https://nrich.maths.org/9344)[Pythagorean Theorem Calculator – CalculatorSoup (calculatorsoup.com)](https://www.calculatorsoup.com/calculators/geometry-plane/pythagorean-theorem.php) |
| 2 | 6 | **Learning intention:**We are learning to construct right-angled triangles with hypotenuse lengths such as $\sqrt{5}$ and locate the corresponding position on a number line.**Success criterion 1:** I can use drawing tools and software to construct right-angled triangles with hypotenuse of lengths such as $\sqrt{5}$**Success criterion 2:**I can locate the corresponding position of numbers such as $\sqrt{5}$ on a number line. | This lesson involves the use of the theorem and geometric constructions for square roots of natural numbers: * snail or spiral construction
* parallel line construction (with lines that are 1 unit apart)

to generate lengths for the sequence$\sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3 …$ and place them exactly on a number line. |  |  | [Spiral of Pythagoras – GeoGebra](https://www.geogebra.org/m/cfYqRyye)[Pythagoras’ Theorem – Australian Mathematical Sciences Institute (amsi.org.au)](https://amsi.org.au/teacher_modules/pythagoras_theorem.html) |
| 2 | 7 | **Learning intention:**We are learning to generate triples of natural numbers that satisfy Pythagoras’ theorem.**Success criterion 1:** I can use a scale factor to generate new triples from a given triple.**Success criterion 2:**I can use a formula to generate a new triple from two given non-zero natural numbers and verify the result. | This lesson involves investigation of Pythagorean triples.Note. in this section the term ‘triple’ is taken to refer to a set of three natural numbers {*a*, *b*, *c*}, where 0 < *a* < *b* < *c*, that satisfies Pythagoras’ theorem.Scaling of existing triples, for example (3, 4, 5) to (6, 8, 10) by a factor of 2, and verification that these satisfy the theorem.Application of a provided formula to a pair of natural numbers to generate a triple – for example, the formula *a* = *m*2 – *n*2, *b* = 2*mn*, c = *m*2 + *n*2 where 0 < *n* < *m,* for example, *m* = 5 and *n* = 3 – and verification that these satisfy the theorem. |  |  | [Pythagorean triple – Wikipedia](https://en.wikipedia.org/wiki/Pythagorean_triple) |
| 2 | 8 | NA | Summative assessment task |  | End-of-unit task(VCAA Level 8 example assessment task – Pythagoras’ theorem) |  |

# Unit reflection

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| **Reflection questions:*** How does the teaching and learning unit provide evidence of student learning and progress?
* Does the teaching and learning unit:
* specify the content descriptions addressed in the unit
* specify the achievement standards addressed in the unit
* include the resources and activities used to develop knowledge and skills
* provide for a range of student abilities
* specify the assessments used to monitor and progress student learning
* provide guidance about the approximate time required for the unit?
* Considering your responses to the questions above and other relevant reflections, how can the teaching and learning unit be improved?
* How will the evidence of student learning from this teaching and learning unit influence the subsequent teaching and learning unit?

**Responses:*** This teaching and learning unit specifies the two content descriptions and the two achievement standard sections addressed (at Level 8), as well as the content descriptions and achievement standard sections at level, below and above.
* It describes the learning activities used to develop mathematical concepts, knowledge, skills and processes and provides links to related resources.
* It caters for a range of student abilities by connecting numerical, graphical and algebraic approaches with the use of manipulatives, measuring and drawing tools, and dynamic geometry applications. Using multiple mechanisms for students to engage is something the team wants to continue to focus on for future units.
* The timing (8 lessons) was well suited to the students’ needs. It provided enough time for students to develop an understanding and demonstrate their learning throughout and at the conclusion.
* The unit would benefit from a pre-test. The team used the data collected from the end-of-year exam from last year and at times more current data would have been useful, particularly for students identified as working above level.
* The unit specifies two formative assessments and one summative assessment to monitor and progress student learning. The team would not add any more assessment as this would be well paced with a pre-test.
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