Level 9 – Number and algebra

Overview

Task name	This expression is the same as that expression when …
Learning intention	To identify equivalent forms of simple algebraic equations
Duration	30 minutes

Links to the Victorian Curriculum

These work samples are linked to Level 9 of the Mathematics curriculum.

Extract from achievement standard

Students use the distributive law to expand algebraic expressions, including binomial expressions, and simplify a range of algebraic expressions.

Relevant content description

- Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (VCMNA306)

Links to NAPLAN

Minimum standards – numeracy

Year 9: Algebra, function and pattern – Equivalence

Students can establish equivalence between algebraic expressions. For example, students can generally:

- identify equivalent forms of simple algebraic expressions.

Student work samples – Expressions (Questions a and b)

These work samples were created by students working at Level 9. Evidence of student achievement has been annotated.

Victorian Curriculum link

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (VCMNA306)

This expression is the same as that expression when …

Introduction

This task involves the equivalence between expanded and factorised forms of simple algebraic expressions.

Students should be familiar with the use of the distributive rule for expansion of simple expressions with positive integer coefficients such as: $2(x + 3) = 2x + 6$ and $(x + 3) = x^2 + 3x$.

Sample 1

a. Find a positive integer value of *c* for which $x^2 + 8x + c$ **can** be written as the product (multiplication) of two different linear expressions with positive integer coefficients.

$x^2 + 8x + 16$

$(x^2 + 4x)(+4x + 16)$

$x(x + 4)\ 4(x + 4)$

This equation wouldn't work because the two expressions are the same

$x^2 + 8x + 12$

$(x^2 + 2x) + (6x + 12)$

$x(x + 2)\ 6(x + 2)$

$(x + 2)(x + 6)$

$c = 12$

This is the correct equation because there are two different expressions

b. Explain why this is the case for this value of c.

C = 12 because there are two different expressions.

Factorises two expressions with $8x$ terms and different constant terms

Identifies $c = 12$ and compares to distinct linear factors

Identifies $c = 16$ and compares to distinct linear factors

Selects solution with two distinct linear factors

Sample 2

a. Find a positive integer value of *c* for which $x^2 + 8x + c$ **can** be written as the product (multiplication) of two different linear expressions with positive integer coefficients.

$x^2 + 8x + c$

$(x+5)(x+3)$

$= x^2 + 8x + 15$

$c = 15.$

Expands pair of distinct factors with resulting $8x$ term and constant term of 15

Identifies $c = 15$

b. Explain why this is the case for this value of *c*.

Because $5+3=8$

and $5 \times 3 = 15$

Explains in terms of pair of values satisfying coefficient relations

Sample 3

a. Find a positive integer value of *c* for which $x^2 + 8x + c$ **can** be written as the product (multiplication) of two different linear expressions with positive integer coefficients.

Expands two pairs of distinct linear factors to obtain two solutions for c

$x^2 + 8x + c$

$x(x+8)+c$

$(x+2)(x+6)$ →

$x^2 + 6x + 2x + 12$

$= x^2 + 8x + 12$

$\boxed{c = 12}$

$(x+4)(x+4)$

$x^2 + 4x + 4x + 16$

$x^2 + 8x + 16$

$c = 16$

$(x+3)(x+5)$

$x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$

$c = 15$

Identifies there are several possible solutions

b. Explain why this is the case for this value of *c*.

Once you take out the common factor, which in this case is x, you can see that both x & 8 are multiplied by x. Next you need to find out which numbers add to 8. In this situation there were multiple. There was; 6 & 2, 5 & 3, 4 & 4, 7 & 1 the list goes on.

Then one using the method of foil we can see that 'c' could be multiple different numbers But I have chosen 12.

Selects particular solution $c=12$

Mathematics – Annotated student work samples

Sample 4

a. Find a positive integer value of c for which $x^2 + 8x + c$ **can** be written as the product (multiplication) of two different linear expressions with positive integer coefficients.

$x^2 + 8x + 7$ $c = 7$

$7 \times 1 = 7$

$7 + 1 = 8$

$(x + 7)(x + 1)$

States solution and verifies using coefficient relations

States factorised form

b. Explain why this is the case for this value of c.

I chose this for the value of C as 7 multiplied by 1 equals 7 and 7 plus 1 equals 8 so it works in this problem. There are also other integers that could have worked but I chose this one.

Identifies possibility of other solutions without listing any

Sample 5

a. Find a positive integer value of c for which $x^2 + 8x + c$ **can** be written as the product (multiplication) of two different linear expressions with positive integer coefficients.

$(x + 2) \times (x + 6) = x^2 + 2x + 6x + \cancel{8}\ 12$

$c = 12$ $= x^2 + 8x + 12$

$(x + 5) \times (x + 3) = x^2 + 5x + 3x + 15$

$c = 15$ $= x^2 + 8x + 15$

$c = 16$

$c = 7$

Expands to identify solutions with $8x$ term

Lists four possible values for c including perfect square case

b. Explain why this is the case for this value of c.

$(x + 2) \times (x + 6) = x^2 + 2x + 6x + 12$

$= x^2 + 8x + 12$

$(x + 5) \times (x + 3) = x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$

Re-states working for two selected solutions

Student work samples – Expressions (Questions c and d)

These work samples were created by students working at Level 9. Evidence of student achievement has been annotated.

Victorian Curriculum link

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (VCMNA306)

Sample 1

c. Find a positive integer value of *c* for which $x^2 + 8x + c$ **cannot** be written as the product (multiplication) of two linear expressions with positive integer coefficients.

$x^2 + 8x + 4$

Writes quadratics expression $x^2 + 8x + c$ with $c = 4$

d. Explain why this is the case for this value of *c*.

It won't work because 4 doesn't have any factors which add/subtract to equal 8

Applies heuristic to explain why there is no product of linear expressions when $c = 4$

Sample 2

c. Find a positive integer value of c for which $x^2 + 8x + c$ **cannot** be written as the product (multiplication) of two linear expressions with positive integer coefficients

$x^2 + 8x + 3$

$(x +$

Writes quadratic expression $x^2 + 8x + c$ with $c = 3$

d. Explain why this is the case for this value of c.

C cannot equal 3 as 3 is a prime number So the only factors of 3 are 3 and 1 and 3+1 does not equal 8 it equals 4 ∴ it doesn't fit in the equation

Applies heuristic to explain why there is no product of linear expressions when $c = 4$

Sample 3

c. Find a positive integer value of c for which $x^2 + 8x + c$ **cannot** be written as the product (multiplication) of two linear expressions with positive integer coefficients

Writes quadratic expression $x^2 + 8x + c$ with $c = 10$

$x^2 + 8x + 10$

10 → 1×10, 5×2 → not 8

10

d. Explain why this is the case for this value of c.

Notes factors of 10 do not sum to 8, the coefficient of x

The factors of 10 doesn not sum up to 8, which is the co-efficient of x

Student work samples – Expressions (Questions e and f)

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Victorian Curriculum link

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (VCMNA306)

Sample 1

e. Find the positive integer value of c for which $x^2 + 8x + c$ can be written as the product (multiplication) of two **identical** linear expressions with positive integer coefficients.

$c = 16$

Identifies $c = 16$ as a solution

$x^2 + 8x + 16$

$(x^2 + 4x)(+4x + 16)$

$x(x+4)\ 4(x+4)$

$(x+4)(x+4)$

$(x+4)^2$

Factorises to obtain two identical linear factors of $x + 4$

Writes as a perfect square

f. Explain why this is the case for this value of *c*.

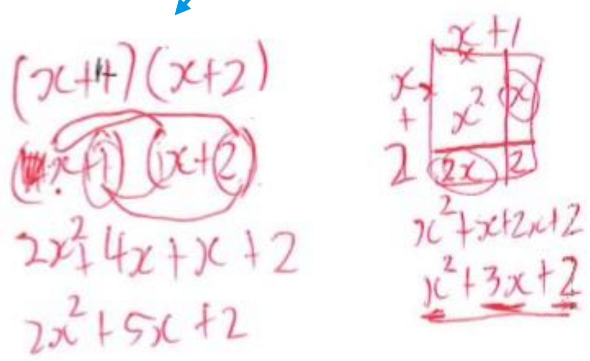
States the linear expressions are identical when $c = 16$

c=16 because there are two identical expressions.

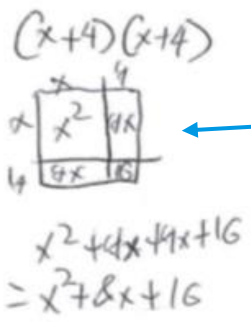
Sample 2

e. Find the positive integer value of c for which $x^2 + 8x + c$ can be written as the product (multiplication) of two **identical** linear expressions with positive integer coefficients.

Attempts two combinations of linear expressions that do not work



f. Explain why this is the case for this value of c.



Uses diagram for perfect square form $(x + 4)\,(x + 4)$ and shows expansion

Sample 3

e. Find the positive integer value of *c* for which $x^2 + 8x + c$ can be written as the product (multiplication) of two **identical** linear expressions with positive integer coefficients

$x^2 + 8x + c$

$= (x + 4)(x + 4) = (x + 4)^2$

$x^2 + 8x + 16$

$c = 16$

Factorises and states in perfect square form

States the value of c

f. Explain why this is the case for this value of *c.*

The value of c had to be a perfect square because the two linear equations had to be identical.

In this case it was $(x+4)(x+4)$ because they are both identical. and $4 + 4 = 8$

Explains that perfect square form requires identical linear expressions

Sample 4

e. Find the positive integer value of *c* for which $x^2 + 8x + c$ can be written as the product (multiplication) of two **identical** linear expressions with positive integer coefficients

$(x + 4) \times (x + 4)$

$x^2 + 8x + 16$

$c = 16$

Expands pair of identical linear expressions

States the value of c

e. Explain why this is the case for this value of *c*.

c = 16 because 4 × 4 = 16 and 4x + 4x = 8x which is what we needed to find in the product

Explains the relationship between coefficients and constant term

Where to next for the teacher?

When the task on which these annotated student work samples is based has been used as a classroom activity, there is opportunity to gather data on student achievement to help inform further teaching.

An analysis of student responses, on an individual, group or whole class basis, can be used to develop and direct student learning with respect to the following content.

For students needing to review underpinning knowledge and skills at Level 8

- Extend and apply the distributive law to the expansion of algebraic expressions (VCMNA279)

For students consolidating knowledge and skills at Level 9

- Apply set structures to solve real-world problems (VCMNA307)

For students moving on to new knowledge and skills at Level 10

- Simplify algebraic products and quotients using index laws (VCMNA330)
- Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

Resources

- Mathematics Sample Programs, Victorian Curriculum and Assessment Authority (VCAA) – This set of sample programs covering the Victorian Curriculum Mathematics: F–10 were developed *as examples* to illustrate how the Mathematics curriculum could be organised into yearly teaching and learning programs.
- Numeracy Learning Progressions, Victorian Curriculum and Assessment Authority (VCAA) – The Numeracy Learning Progressions amplify, extend and build on the numeracy skills in the Victorian Curriculum Mathematics F–10 and support the application of numeracy learning within other learning areas.
- FUSE, Victorian Department of Education and Training (DET) – The FUSE website provides access to digital resources that support the implementation of the Victorian Curriculum F–10, including an extensive range of activities and other resources for Primary Mathematics and Secondary Mathematics.
- Mathematics Teaching Toolkit, Victorian Department of Education and Training (DET)
- Mathematics Curriculum Companion, Victorian Department of Education and Training (DET)
- Victorian Numeracy Portal, Victorian Department of Education and Training (DET)
- Aligned Australian Curriculum Resources (Mathematics), Australian Curriculum, Assessment and Reporting Authority (ACARA)